$\hat{\ }$ 16 & 17. $\boxed{\hat{\theta} = \Phi_N^+ Y_N}$ $\qquad (\Phi_N^T \Phi_N)^{-1} \Phi_N^T = \Phi_N^+$

$$Y_N = \Phi_N \theta_0 + \varepsilon_N$$

$$\hat{\theta} = \Phi_N^+ (\Phi_N \theta_0 + \varepsilon_N) = \theta_0 + \Phi_N^+ \cdot \underline{\underline{\varepsilon_N}}$$

$$\Sigma_{\hat{\theta}} = (\Phi_N^+)\left(\underline{\underline{\Sigma_{\varepsilon_N}}}\right)(\Phi_N^+)^T$$

16: $\Sigma_{\varepsilon_N} = \sigma^2 \cdot I \Longrightarrow \underline{\Sigma_{\hat{\theta}} = \sigma^2 (\Phi_N^T \Phi_N)} \Longrightarrow$ d)

17. $\left[W = \Sigma_{\varepsilon_N}^{-1} \right] \Longrightarrow \hat{\theta} = \arg\min_{\theta} \| Y_N - \Phi_N \theta \|_{\underline{\underline{W}}}^2$

$$\Sigma_{\hat{\theta}} = (\Phi_N^T W \Phi_N)^{-1}$$

$$\|x\|_W^2 = x^T W x$$

18. $\theta = 40\%$??

$p(\text{heads} \mid \theta) = \theta \qquad 40\times$

$p(\text{tails} \mid \theta) = 1 - \theta \qquad 60\times$

$$Y_N = \begin{bmatrix} \text{heads} \\ \vdots \\ \text{heads} \\ \text{tails} \\ \vdots \\ \text{tails} \end{bmatrix}$$

$$L(\theta) = p(Y_N \mid \theta) = \underline{\prod_{i=1}^{100} p(y_i \mid \theta)} = \prod_{i=1}^{40} \theta \cdot \prod_{i=1}^{60} (1 - \theta)$$

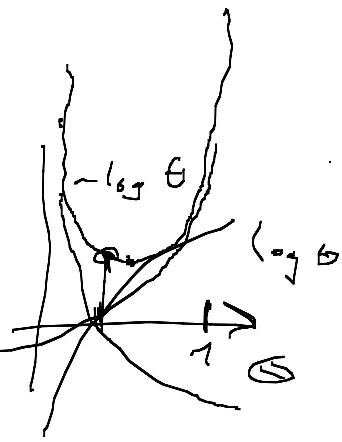
$$-\log L(\theta) = \sum_{i=1}^{40} -\log \theta + \sum_{i=1}^{60} -\log(1-\theta)$$

$$= -40 \log \theta - 60 \log(1-\theta) \Rightarrow c)$$

$$=: f(\theta)$$

$$\frac{\partial f}{\partial \theta} = \frac{-40}{\theta} + \frac{60}{1-\theta} \overset{!}{=} 0$$

$$\Leftrightarrow \frac{40}{\theta} = \frac{60}{1-\theta} \Leftrightarrow \frac{40}{60} = \frac{\theta}{1-\theta}$$



$$\boxed{\theta = 0.4}$$