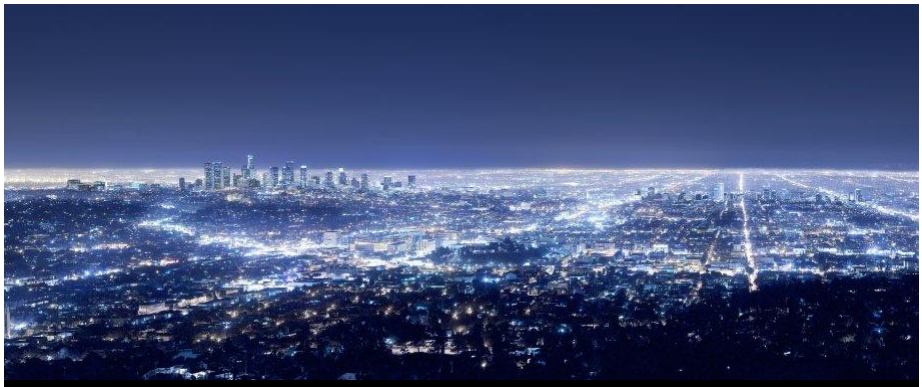


Hans Joachim Ferreau, ABB Corporate Research, Switzerland

qpOASES
Past, Present, Future

Power and productivity for a better world™

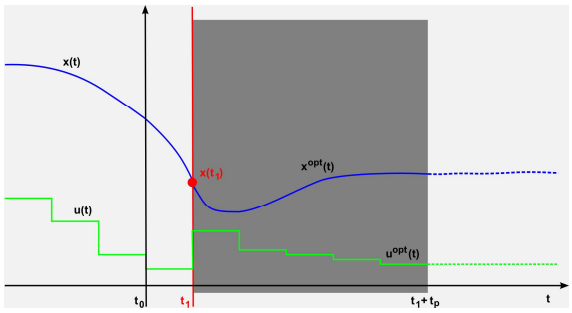
Outline

1. Introduction
2. Current Status of qpOASES
3. Future Developments

Outline

1. **Introduction**
2. Current Status of qpOASES
3. Future Developments

Motivation
Fast QP Solution for Model Predictive Control



- At each sampling instant, the following specially structured QP problem needs to solved:

$$\mathrm{QP}(x_0): \quad \min_{z\in\mathbb{R}^n} \quad \frac{1}{2}z'Hz + z'g(x_0)$$
$$\text{s.t.} \quad Gz \leq b(x_0)$$

Linear MPC in a Nutshell
What makes it difficult?

- Linear MPC algorithms need to **solve QP optimization problems in real-time**

$$\min_z \frac{1}{2} z^T H z + g^T z$$

$$s.t. \quad \underline{z} \leq Az \leq \overline{z}$$

- A main advantage of MPC is the ability to **handle inequality constraints** on inputs and outputs

- The main difficulty in solving QPs are… **inequality constraints**

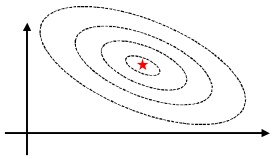
  (difficult means: solution takes more time)

ABB

Linear MPC in a Nutshell
Mind the inequalities

- **unconstrained** QP

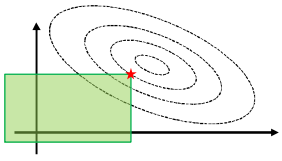
$$\min_z \frac{1}{2} z^T H z + g^T z$$



- QP with **simple constraints**

$$\min_z \frac{1}{2} z^T H z + g^T z$$

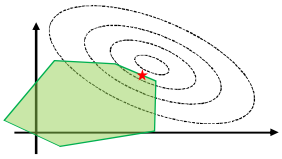
$$s.t. \quad \underline{z} \leq z \leq \overline{z}$$



- QP with **general constraints**

$$\min_z \frac{1}{2} z^T H z + g^T z$$

$$s.t. \quad \underline{z} \leq Az \leq \overline{z}$$



ABB

Linear MPC in a Nutshell
QP algorithms (to handle inequality constraints)

- **First-order methods:**
  - compute step towards solution of unconstrained QP
  - project to feasible set (difficult for general constraints)
- **Active-set methods:**
  - guess which inequalities hold with equality at solution
  - solve resulting equality-constrained QP (trivial)
  - check if guess was correct, update guess if not
- **Interior-point methods:**
  - remove inequalities, but penalize constraint violations in objective function (non-quadratic term, e.g. logarithmic)
  - solve resulting (equality-constrained) NLP with Newton's method
- **Explicit methods and others**

Linear MPC Algorithms
Why is there a whole zoo of them?

First-order	gradient method, primal FGM, dual FGM, GPAD, FiOrdOs
Active-set	quadprog (primal), QLD (dual), qpOASES (parametric)
Interior-point	primal barrier, CVXGEN (primal-dual), FORCES (primal-dual)
Others	Brand's algorithm, qpDUNES (Newton-type), ADMM, MPT (explicit methods)

- Tailored to different problem classes
- Different numerical properties
- Amount of sorce code
- Suitability for parallelization
- Suitability for FPGA implementations

```
/* 5) Termination criterion. */
nV = getNV( );
nC = getNC( );

homotopyLength = getRelativeHomotopyLength( g_new,lb_new,u
if ( homotopyLength <= options.terminationTolerance )
{
    status = QPS_SOLVED;

    THROWINFO( RET_OPTIMAL_SOLUTION_FOUND );

    if ( printIteration( iter,BC_idx,BC_status,BC_isBound,
         THROWERROR( RET_PRINT_ITERATION_FAILED ); /* do nc

    nWSR = iter;
    if ( cputime != 0 )
        *cputime = getCPUtime( ) - starttime;

    delete[] delta_yAC; delete[] delta_yFX; delete[] delta
    delete[] delta_ub; delete[] delta_lb; delete[] delta_u

    return SUCCESSFUL_RETURN;
}
```

The Online Active Set Strategy
Main Idea and Advantages

- *Main Idea*[1]*:*
  Introduce homotopy path from solution of current QP(x_0) to that of QP(x_0^{new}) and identify corresponding active set

- *Advantages:*
  - **Reduced number of iterations** by exploiting parametric nature of MPC problem
  - **Hot-starts** with full solution information of previous QP
  - Re-use of matrix factorizations
  - Allows for a **real-time variant** in case solution procedure has to stop prematurely

[1] Ferreau, Bock, Diehl. *An online active set strategy to overcome the limitations of explicit MPC.* International Journal of Robust and Nonlinear Control, 18 (8), pp. 816-830, 2008.

Outline

1. Introduction
2. **Current Status of qpOASES**
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qpOASES
An Online QP Solver

- qpOASES is an **object-oriented C++ implementation** of the online active set strategy with dense linear algebra[2]
- **Reliable and efficient code** for solving small- to medium-scale, dense QPs
- **Self-contained code:** no additional software packages required (but BLAS/LAPACK can be linked)
- Distributed as **open-source software** under the GNU LGPL (see www.qpOASES.org)

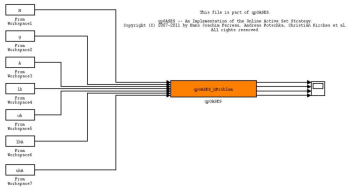
[2] Ferreau, Kirches, Potschka, Bock, Diehl. *qpOASES: A parametric active-set algorithm for quadratic programming.* Mathematical Programming Computation, 2014 (to appear).

qpOASES
Interfaces

- Interfaced to several third-party software packages:
  - **Matlab**, Octave, Scilab

```
[x,fval,exitflag,iter,lambda] = qpOASES( H,g,A,lb,ub,lbA,ubA )
```

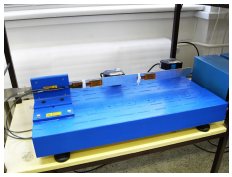
  - **Simulink (dSPACE, xPC Target)**

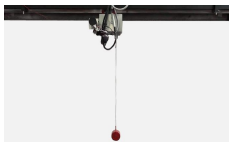


  - YALMIP, ACADO Toolkit, CasADi
  - Python

qpOASES
A Few Real-World Applications

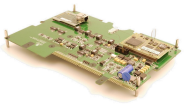
- MPC of a **Diesel engine testbench** at University of Linz, Austria (on dSPACE, 50 ms sampling time)
- MPC of **beam tip vibrations** at Slovak University of Technology, Bratislava (on xPC target, 10 ms sampling time)
- **Trajectory planning for a boom crane** at University of Stuttgart, Germany (on dSPACE, 100 ms sampling time)
- **Time-optimal control of machine tools** at KU Leuven (on dSPACE/xPC target, 4 ms sampling time)





qpOASES
Selected Industrial Applications

- qpOASES has been used within the Advanced Engine Controller by **Hoerbiger Control Systems:**
  - MPC of integral gas engines
  - **Successful field test at SoCalGas compressor station**, NO_X peaks could be reduced significantly
- qpOASES has been integrated into the **INCA MPC Engine by IPCOS**
- qpOASES is used for whole body control of **humanoid NAO robotor at Aldebaran Robotics**
- Used within **LMS** Imagine.Lab for **vehicle dynamics** applications



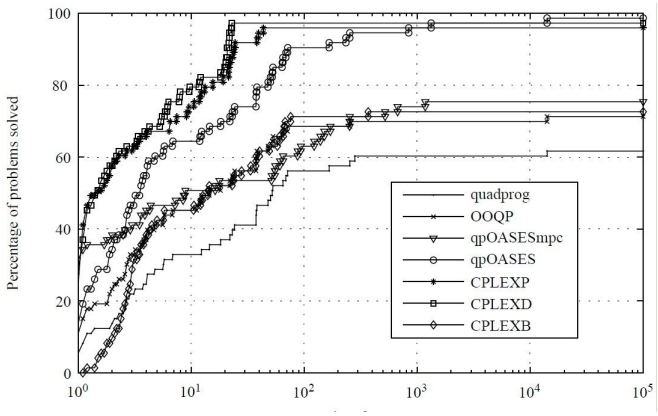
Numerical Performance
Reliability

- **Reliable code:** qpOASES solves all 70 small- to medium-scale test problems from the challenging Maros-Mészáros QP test set with high accuracy
- Comparison with other popular QP solvers with default settings (joint work with A. Potschka, U Heidelberg):

Solver Name:	Fraction of QPs solved:
quadprog	62 %
OOQP	70 %
CPLEX-IP	73 %
CPLEX-Primal	96 %
CPLEX-Dual	97 %
qpOASES 3.0beta	99 %

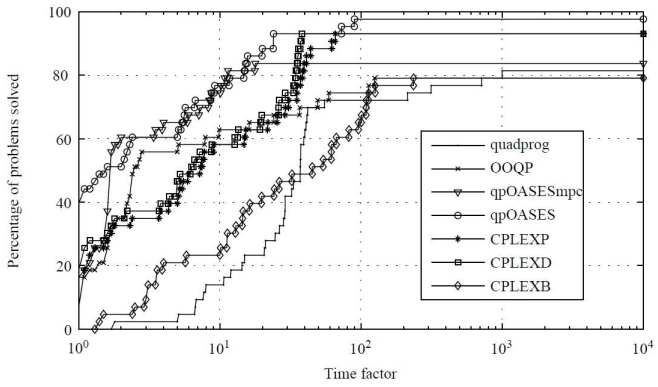
Numerical Performance
Computational Speed

- All **70 problems with less than 1000 QP variables** of challenging Maros-Mészáros QP test set



Numerical Performance
Computational Speed (cont.)

- All **43 problems with less than 250 QP variables** of challenging Maros-Mészáros QP test set



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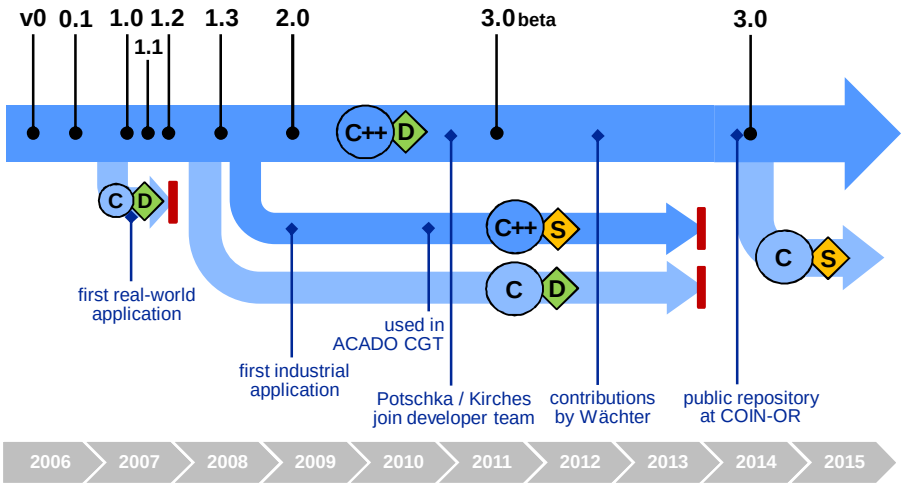
3. **Future Developments**

qpOASES
Development Timeline



Summary

- The open-source software package **qpOASES is a mature and reliable QP solver** (used in many applications)
- Efficient due to plenty of **structure-exploiting features**
- Easy to use through various third-party interfaces, **download from** www.qpOASES.org and try yourself

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