Systems and Control 2 (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 3: Lyapunov stability - Steady State Errors
(Thursday 12.11.2015 at 15:00 in Room SR 00 014)

Dr. Jörg Fischer, Prof. Dr. Moritz Diehl, and Jochem De Schutter

1. Analyze the stability of the following LTI-SISO systems. Determine whether the systems are BIBO-stable and/or asymptotically Lyapunov stable. If the latter is not the case, are the systems then marginally Lyapunov stable?

   (a) $\ddot{y}(t) + 10\dot{y}(t) + 36y(t) = u(t)$

   (b) $2\ddot{y}(t) + 5\dot{y}(t) - 6y(t) = 2\dot{u}(t) - 3u(t)$

   (c) $\ddot{y}(t) + \dot{y}(t) - 2y(t) = 4\dot{u}(t) + 3u(t)$

   (d) $\ddot{y}(t) + 3\dot{y}(t) = 5\dot{u}(t)$

   (e) $\dddot{y}(t) + 8\ddot{y}(t) + 16y(t) = \ddot{u}(t) + 2\dot{u}(t) + u(t)$

2. Examine the stability of the closed-loop system that is constituted by the block diagram shown in Fig. 1. The plant and the controller in this system are described by the transfer functions

$$G(s) = \frac{0.1}{(s+2)(s+2)} \quad \text{and} \quad K(s) = 10 \quad . \tag{1}$$

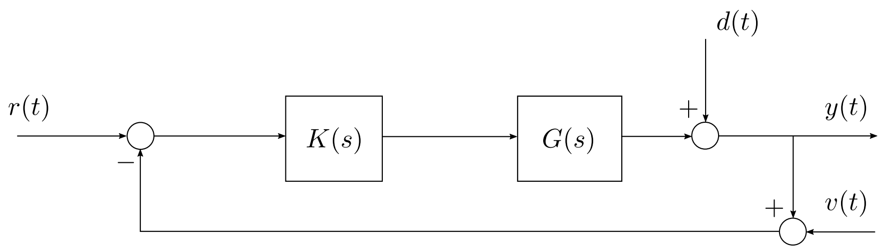

Figure 1: Closed-loop system under consideration.

   (a) Sketch the Bode Diagram of the open-loop system $G_0(s) = G(s)K(s)$ and verify it via MATLAB.

   (b) Sketch the Nyquist plot of $G_0(s)$ and verify via MATLAB.

   (c) Is the closed-loop transfer function $G_\mathrm{r}(s) = \frac{Y(s)}{U(s)}$ BIBO-stable according to the Nyquist criterion?

   (d) Verify your answer by calculating the closed-loop poles of the transfer function $G_\mathrm{r}(s)$.

   (e) Are the disturbance-to-output transfer functions $\frac{Y(s)}{D(s)}$ and $\frac{Y(s)}{V(s)}$ BIBO-stable?

   (f) Is the system internally stable?

3. Consider the closed-loop system shown in Fig. 2, where the plant $G(s)$ is a DC motor that is described by the simplified transfer function $G(s) = \frac{1}{s(s+1)}$.

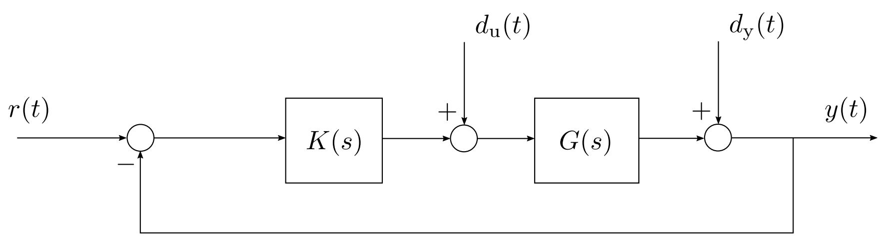

Figure 2: Closed-loop system of a DC motor.

   (a) Somebody designed a proportional controller $K(s) = 5$ for the DC-motor. Assume there are no disturbances ($d_\mathrm{u}(t) = 0$ and $d_\mathrm{y}(t) = 0$). What is the steady state error due to a step reference input ($r(t) = \sigma(t)$)?

   (b) Still assuming there are no disturbances, what is the steady state error due to a ramp reference input ($r(t) = t$)?

(c) Still assuming there are no disturbances, what is the steady state error if the DC-motor is accelerated, i.e. if the reference input is parabolic ($r(t) = t^2$)?

(d) Assume that there is a constant disturbance at the output of the DC-motor: $d_{\mathrm{y}}(t) = \sigma(t)$. How does this influence the steady state error?

(e) Assume that there is also a constant disturbance at the input of the DC-motor: $d_{\mathrm{u}}(t) = \sigma(t)$. How does this influence the steady state error?

(f) How can $K(s)$ be changed so that a zero steady state error can be achieved in the presence of a permanent input disturbance $d_{\mathrm{u}}(t) = \sigma(t)$?
Hint: Use the inner-model-principle. Do not forget to check whether the attained closed-loop system is BIBO-stable or not.