Systems and Control 2 (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 2: Review of control theory - Loopshaping, BIBO-stability, robust stability

(Thursday 05.11.2015 at 15:00 in Room SR 00 014)

Dr. Jörg Fischer, Prof. Dr. Moritz Diehl and Jochem De Schutter

A DC motor, as shown in Fig. 1, has a rotor shaft with an inertia moment J and a viscous friction coefficient c. The control signal $u(t)$ is the voltage applied to the DC-motor. When inductance effects are neglected, the relation between the input voltage and the motor torque $\tau_{\mathrm{m}}(t)$ is approximately described by

$$\tau_{\mathrm{m}}(t) = a_1 u(t) - a_2 \dot{\theta}(t) \quad ,$$

where $\theta(t)$ is the rotor position and a_1 and a_2 are constants. The motor drives a load that exerts a torque $\tau_{\mathrm{l}}(t)$ on the rotor shaft. The shaft is assumed to be stiff, so that distortion effects can be neglected.

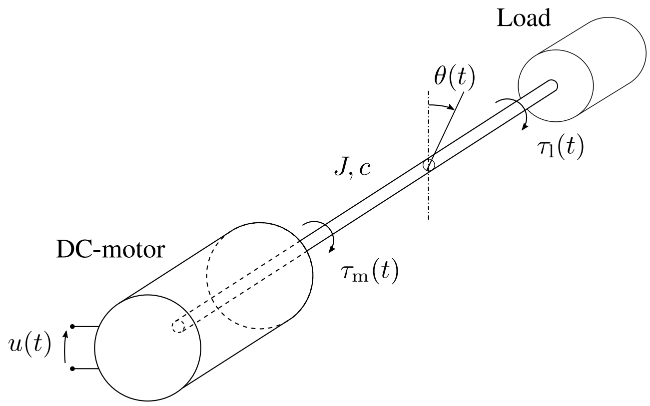

Figure 1: Setup of a DC-motor with load.

1. Elaborate a system model that can be used to design a controller for this setup.

   (a) Formulate the general ODE that describes the dynamics of the motor position $\theta(t)$ as a function of the input voltage $u(t)$ and the load torque $\tau_{\mathrm{l}}(t)$.

   (b) Transform the general ODE model into a transfer function model.

   (c) Examine the stability of the DC-motor setup for $\tau_{\mathrm{l}}(t) = 0$. Is this system BIBO-stable? Is it Lyapunov stable? And if so, is it asymptotically or marginally Lyapunov stable?

2. Design a controller $K(s)$ for the DC-motor system, using the 'loopshaping' method. In case there is no load ($\tau_{\mathrm{l}}(t) = 0$), we wish to obtain for a unit-ramp input a steady-state error $e_{\mathrm{ss}} \leq 0.1$ rad and an overshoot $M_{\mathrm{p}} < 25\%$. Values of the inertia moment, the friction coefficient and the motor constants are $J = 1\mathrm{kg}\cdot\mathrm{m}^2$, $c = 0.3\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}}$, $a_1 = 1\frac{\mathrm{Nm}}{\mathrm{V}}$ and $a_2 = 0.7\frac{\mathrm{Nm}\cdot\mathrm{s}}{\mathrm{rad}}$.

   Hint: An adequate controller type would be a lead compensator of the form $K(s) = K_{\mathrm{P}}\frac{Ts+1}{\alpha Ts+1}$. Use the information given in Fig. 2 on the relation between overshoot and phase margin, as well as the information given in Fig. 3 on the relation of α and the maximum phase increase of the closed-loop.

   (a) Draw the block diagram of the control setup, with the controller $K(s)$, the plant $G(s)$ and with all relevant in- and outputs.

   (b) Determine the open-loop gain K_{P} so that the error requirements are met.

   (c) Evaluate the phase-margin (PM) of the uncompensated system using the value of K_{P} from the previous step, and determine the needed phase lead $\phi_{\max}$ (allow for an extra phase margin of 15°).

   (d) Determine T so that $\omega_{\max}$ is at the crossover frequency.

   (e) Compute the closed-loop transfer function. Is the closed-loop system (internally) stable?

   (f) Assume now that there is a step change of the load torque from $\tau_{\mathrm{l}}(t) = 0$ to a value $\tau_{\mathrm{l}}(t) \neq 0$. What is the steady state error of the system?

   (g) What is the steady state error if the load torque is linearly growing over time, i.e., if $\tau_{\mathrm{l}}(t)$ is a ramp input?

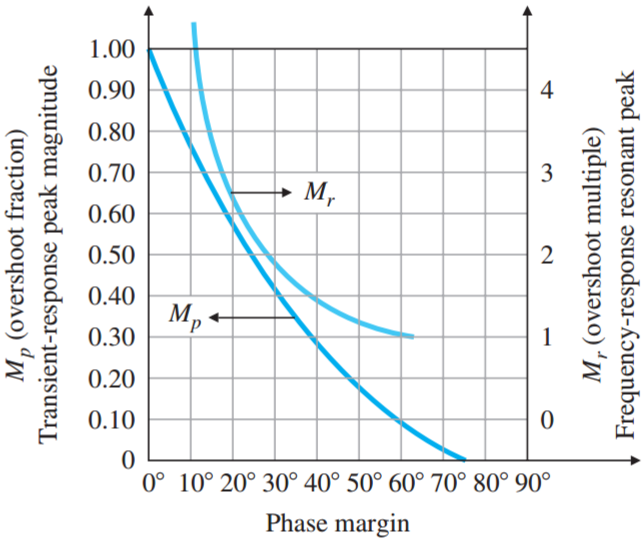

Figure 2: Transient-response overshoot (M_p) and frequency-response resonant peak (M_r), vs. phase margin (PM) for $T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, i.e., a general second order lag system [2]

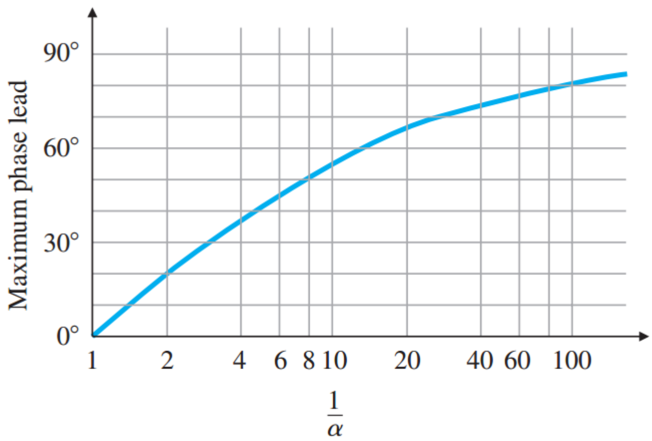

Figure 3: Maximum phase increase for lead compensation [2]

3. The DC motor drives different loads, that not only operate as ‘inputs’ to the system, but that also influence the inertial moment J of the motor shaft. The change of J changes the dynamic behavior of the system and hence leads to different transfer function models of the closed loop system. We want to analyze if the designed controller robustly stabilizes the DC motor system, under consideration of all expected load conditions. The load has a variation span of $J \in [0.7, 1.2]\,\mathrm{kg}\cdot\mathrm{m}^2$. Inspect whether the controller $K(s)$ designed for the nominal inertial moment ($\hat{J} = 1\mathrm{kg}\cdot\mathrm{m}^2$) robustly stabilizes the closed loop system.

   (a) Find the upper bound model $\bar{G}(j\omega, \bar{J})$ and the corresponding inertial moment $\bar{J}$.

   (b) Plot the upper bound model together with the robust stability limit. Is the closed loop robustly BIBO-stable?

[2] Source: G. F. Franklin, J. D. Powell, A. Emami-Naeini (2015). Feedback Control of Dynamic Systems. Pearson