Exercises for Lecture Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2017

Exercise 9: Parameter Estimation for Dynamic Systems

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In this exercise you will consider different dynamic system models, find the parameters of the 2D robot from measurements, and predict the future wind at the Feldberg windfarm. For the the MATLAB exercises, create a MATLAB script called `main.m` with your code, possibly calling other functions/scripts. From running this script, all the necessary results and plots should be clearly visible.

Theoretical Exercises

1. **Discrete Dynamic System Models** In this task, please match some simple system descriptions with appropriate model types and equation-forms. (*Hint: if necessary, use a forward Euler scheme to discretize in time.*) **Describe**

   (a) the average velocity of a model train over one minute. The train's axles are driven at a constant angular velocity on a slippery, straight track, such that the contact between the wheels and the track is not constant.

   (b) the temperature of a homogeneous liquid with constant specific heat, as measured by an imprecise thermometer. The liquid is in a well insulated container with a controllable heat source. ($\frac{1}{2}$ pt)

   (c) the velocity of a uniform ball, falling vertically down an evacuated tube, after being released from rest.

   (d) the volumetric flow-rate of water (with constant temperature) through the cross-section of a long, straight, uniformly-circular pipe. The pressure difference between the ends of the pipe can be controlled instantaneously.

   Model Types

   (a) Autoregressive (AR)

   (b) Finite Impulse Response (FIR)

   (c) Autoregressive, Exogenous Inputs (ARX)

   (d) Autoregressive, Moving Average (ARMA)

   Model Equations, where $q \in \mathbb{R}$ is a scalar constant; y, the output; u, the controls; and e, the error.

   (a) $y_t + \sum_{n=0}^{N} b_n u_{t-n} + e_t = 0$

   (b) $\sum_{m=0}^{M} a_m y_{t-m} + q = 0$

   (c) $\sum_{m=0}^{M} a_m y_{t-m} + \sum_{l=0}^{L} c_l e_{t-l} + q = 0$

   (d) $\sum_{m=0}^{M} a_m y_{t-m} + \sum_{n=0}^{N} b_n u_{t-n} + e_t + q = 0$

Computer Exercises

1. **Parameter estimation for output error minimization** You operate a two-wheeled robot with unknown dimensions (left wheel radius R_L, right wheel radius R_R, and axle length L), as simulated in Exercise 8. After observing the movement of the robot, you would like to estimate these dimensions $\theta = (R_L, R_R, L)^\top$, with `lsqnonlin`[1]. Assuming that the robot system has only output errors, and

[1] `lsqnonlin` takes as input a vector function $f(\theta) = [f_1(\theta), \ldots, f_N(\theta)]$, and minimizes $\|f(\theta)\|_2^2$ with respect to $\boldsymbol{\theta}$.

that these errors are Gaussian with zero mean and variances $\sigma_x^2 = 1.6 \cdot 10^{-3}$ m^2 and $\sigma_y^2 = 4 \cdot 10^{-4}$ m^2, then the Maximum Likelihood Estimation problem to estimate θ is:

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta} \in \mathbb{R}^3}{\arg\min} \left|\left| \boldsymbol{x}^\top - M(t, \boldsymbol{u}, \boldsymbol{x}_0, \boldsymbol{\theta}) \right|\right|^2_{\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}},$$

where $\boldsymbol{x} \in \mathbb{R}^{N \times 2}$ is the measured position of the robot, containing x and y positions; M is the modelled position; t is the time, measured at increments of 0.01s; $\boldsymbol{u} \in \mathbb{R}^{N-1 \times 2}$ is the control, containing the left angular velocity ω_L and the right angular velocity ω_R; and $\boldsymbol{x}_0$ is the initial location $(x_0, y_0, \beta_0) = (0, 0, 0)$. (2.5 pts)

(a) Implement a function `[r] = residual(param, x0, u, t, xData, sigmaData)` which computes the residual vector between the given measured locations $\boldsymbol{x}$ and the modelled locations $M(t, \boldsymbol{u}, \boldsymbol{x}_0, \boldsymbol{\theta})$ obtained with forwards-Euler integration. Use the $\boldsymbol{u}$ and $\boldsymbol{x}$ from the dataset `ex9_robotData.mat`. ($\frac{1}{2}$ pt)

(b) Adapt your function `residual` in order to incorporate the measurement variances correctly, i.e. weight the cost function in the right way. ($\frac{1}{2}$ pt)

(c) Use `lsqnonlin` to estimate θ^*. ($\frac{1}{2}$ pt)

(d) Plot the simulated model with θ^* versus the measurements. ($\frac{1}{2}$ pt)

(e) Find a estimate for the covariance of your estimator θ^*. *(Hint: linearize your residual function and use it to give an approximation of the covariance.)* ($\frac{1}{2}$ pt)

2. **Prediction error minimization for forecasting** You, the operator of the planned Feldberg windfarm from Exercise 6, need to predict what the mean wind speed will be during the next hour. Please use the 1000 datapoint subset of the full historical data `WindData.mat`, to model the wind speed time series as an autoregressive (AR), discrete-time dynamic system. (2.5 pts)

(a) Formulate the prediction error minimization problem (PEM) with which you can find the $M+1$ factors of the AR shifting polynomial $(a_0, ..., a_M)$ and the constant q. *(Hint: see Task 1. What is a_0?)* ($\frac{1}{2}$ pt)

(b) Use the backslash operator to solve the normal equations for the parameters $\theta_M \in \mathbb{R}^M$, where $\theta_M = (q, a_0, ..., a_M)$. Do this for $M = 1, 2, ..., 20$. ($\frac{1}{2}$ pt)

(c) Estimate the covariance of the parameters, Σ_{θ_M}, for each M. *(Hint: consider which error you need to use.)* ($\frac{1}{2}$ pt)

(d) Now, we need to decide which of these possible AR models to use. To do this, let's determine the partial autocorrelation function of the wind-speed AR model. Plot the value of a_M for each of the M AR models, vs. the AR model order M. Add onto this plot, the 95% confidence interval that a_M is non-zero, according to the standard normal tables. This 95% confidence interval is approximately $\pm 1.96/\sqrt{N}$, with $N = 1000$ data points. Based on this plot of the partial autocorrelation, what order AR model (M) should be used to represent the wind speed? ($\frac{1}{2}$ pt)

(e) Make a plot that compares the predictions made by the model selected in (2d) to the reported data values. Use a prediction horizon of one; ie. predict only one data-point ahead into the future. Then, predict the mean wind speed during the next hour outside of the data-file. ($\frac{1}{2}$ pt)