University of Pisa

Trajectory optimization of multibody systems with contacts

Silvia Manara

IMTEK, University of Freiburg, July 5, 2016

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Pisa

University of Pisa

One of the eldest universities in Italy (founded in 1343)
45985 students (roughly half of the number of inhabitants)

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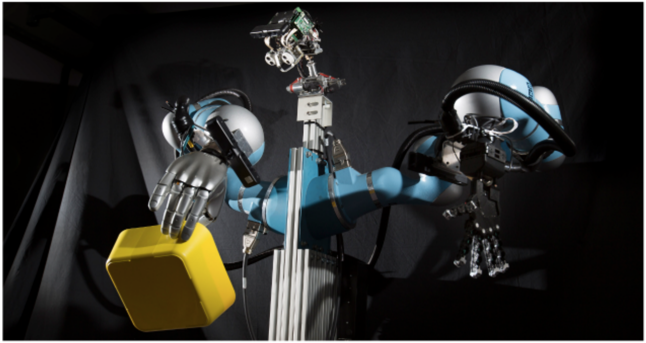
Probabilistic and Compositional Representations for Object Manipulation



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Objective

Grasping under uncertainty

How to make robotic manipulation effective in unstructured environments, where objects are new and their properties (pose, mass, friction...) are unknown?

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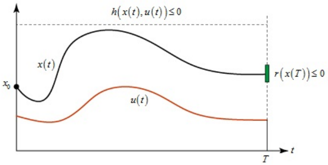
How to make robotic manipulation effective in unstructured environments, where objects are new and their properties (pose, mass, friction...) are unknown?

Towards numerical optimal control

Formulate the problem as an optimal control one, (ideally) to be solved online in order to control the system.

OCP formulation



$$\begin{aligned}
\min_{x(\cdot),u(\cdot)} \quad & \int_0^T L(x(t),u(t))dt & \\
\text{subject to} \quad & x(0) = x_0 & \text{(initial configuration)} \\
& g(x,\dot{x},u) = 0 & \text{(dynamics)} \\
& h(x,u) \leq 0 & \text{(path constraints)} \\
& r(x(T)) \leq 0 & \text{(terminal constraints)}
\end{aligned}$$

Direct transcription

- Discretize the time horizon $[0, T]$ into N time intervals
- Approximate objective function and constraints
- **Finite dimensional NLP**

$$\begin{aligned} \min_{w} \quad & f(w) \\ \text{s.t.} \quad & c_{\text{min}} \leq c(w) \leq c_{\text{max}} \\ & w_{\text{min}} \leq w \leq w_{\text{max}} \end{aligned}$$

Decision variables

$$w = (q_0, \dot{q}_0, u_0, f_{C_0}, \ldots, q_k, \dot{q}_k, u_k, f_{C_k}, \ldots, q_N, \dot{q}_N)$$

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Numerical solution

- Calculate derivatives through Algorithmic Differentiation → **CasADi**
- Solve with IPOPT

Challenges

Problems to be faced

- Multibody dynamics: fast and highly nonlinear
- Discontinuities due to contacts and friction

↓

Analysed aspects

- Different formulations of the dynamics
- Modeling of the contact phenomenon

Description of the system dynamics

Geometric approach based on Lie theory: rigid body motion can be described through objects belonging to differentiable manifolds, therefore having some special properties.

$$R \in SO(3) \qquad \hat{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \in so(3)$$

Dynamics description through Newton-Euler approach:

Equations of motion

$$\begin{pmatrix} m_i I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & \mathbb{J}_{G_i} \end{pmatrix} \begin{pmatrix} \dot{v}_i \\ \dot{\omega}_i \end{pmatrix} + \begin{pmatrix} 0_{3\times 3} \\ \hat{\omega}_i \mathbb{J}_{G_i} \omega_i \end{pmatrix} = \begin{pmatrix} F_i^s \\ M_i^b \end{pmatrix}$$

Unconstrained MBS

The state space of an unconstrained MBS can be modeled using these tools:

- Configuration belongs to a Lie group $\mathcal{G}$:

$$q = (r_1, R_1, \ldots, r_k, R_k) \in \mathcal{G} = \mathbb{R}^3 \times SO(3) \times \ldots \times \mathbb{R}^3 \times SO(3)$$

- Element of the Lie algebra $\mathfrak{g}$:

$$v = (v_1, \hat{\omega}_1, \ldots, v_k, \hat{\omega}_k) \in \mathfrak{g} = \mathbb{R}^3 \times so(3) \times \ldots \times \mathbb{R}^3 \times so(3)$$

Kinematic reconstruction is an ODE on the Lie group:

$$\begin{pmatrix} \dot{r}_i \\ \dot{R}_i \end{pmatrix} = \begin{pmatrix} v_i \\ R_i \hat{\omega}_i \end{pmatrix}$$

Description of the dynamics:

$$M\dot{v} = Q(q, v)$$

Constrained MBS

Description of the dynamics + holonomic constraints

DAE index 3

$$M\dot{v} + J^T \lambda = Q(q, v)$$
$$\Phi(q) = 0$$

Constrained MBS

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Index reduction yields:

DAE index 2

$$M\dot{v} + J^T\lambda = Q(q, v)$$
$$J(q)v = 0$$

+ hidden constraints on positions

$$\Phi(q) = 0$$

Constrained MBS

Description of the dynamics + holonomic constraints

DAE index 3

$$M\dot{v} + J^T\lambda = Q(q, v)$$
$$\Phi(q) = 0$$

Index reduction yields:

DAE index 1

$$\begin{bmatrix} M & J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q(q, v) \\ \xi(q, v) \end{bmatrix}$$

+ hidden constraints on positions and velocities

$$J(q)v = 0$$
$$\Phi(q) = 0$$

Constrained MBS

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DAE index 3

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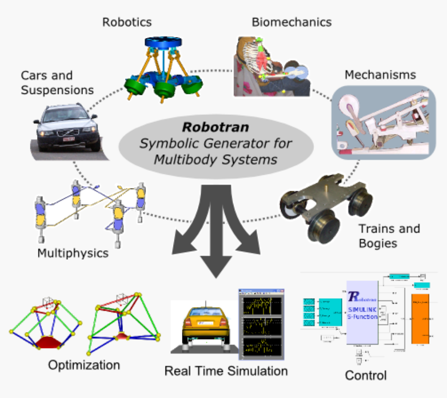
...Any advantage in optimal control?

Robotran

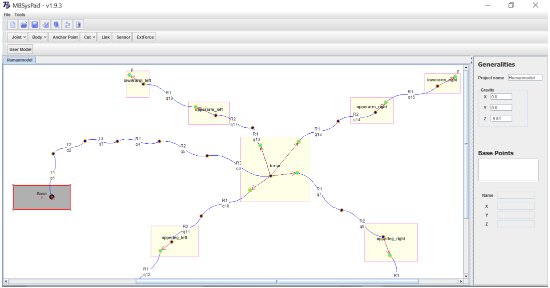
Robotran (www.robotran.be) is a software for generating kinematic and dynamical equations of multibody systems, developed by the multibody research group of the Center for Research in Mechatronics (CEREM) - UCL, Belgium.



How to use it

The Robotran software contains:

- a graphical editor for designing the model and entering your data
- a symbolic equation generator that automatically write the equations of your system in the desired programming language (Matlab, Python, C/C++)



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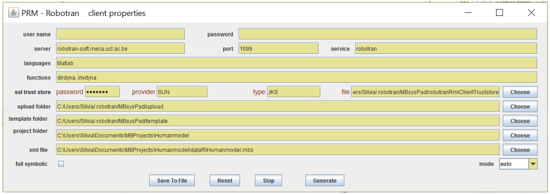


Contact

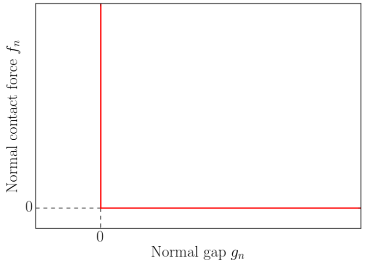
A priori unscheduled contact sequence ⇒ We don't know when the impact is going to occur!

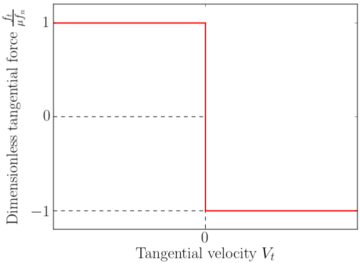
Complementarity

$$f_n g_n = 0, \text{ with } g_n \geq 0, \ f_n \geq 0$$

Coulomb model

$$\|f_t\| \leq \mu f_n$$



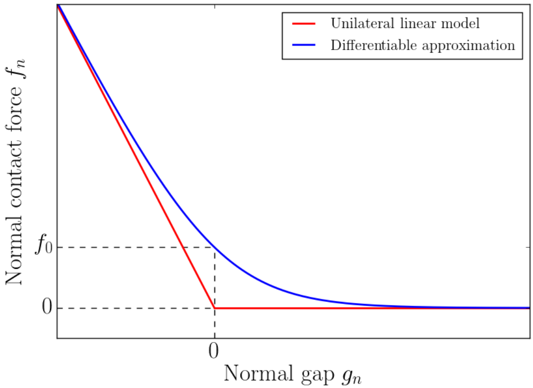


How to make a gradient-based algorithm deal with these discontinuities?

Contact force model

Normal component

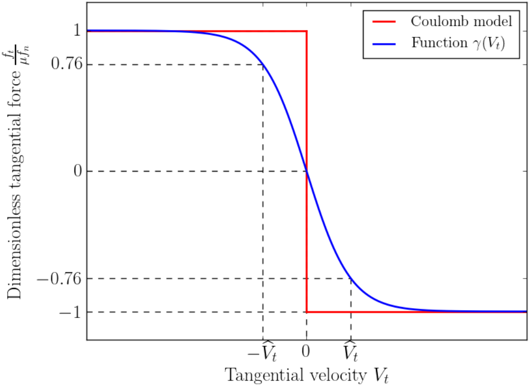
$$f_n(g_n) = f_0 \log_2 \left(1 + 2^{-\frac{\kappa}{f_0} g_n}\right)$$



Contact force model

Tangential component

$$f_t(g_n, V_t) = \mu f_n(g_n)\gamma(V_t), \quad \text{with} \quad \gamma(V_t) = -\tanh\left(\frac{V_t}{\widehat{V}_t}\right)$$



WALK-MAN project

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KIT Karlsruhe Institute of Technology
EPFL ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
UNIVERSITÀ DI PISA
UCL Université catholique de Louvain

Fukushima, 2011

Whole-body Adaptive Locomotion and Manipulation

Objective

The WALK-MAN project aims to develop a humanoid robot showing human levels of locomotion, balance and manipulation, to operate outside the laboratory environment, in disaster sites.

New skills:

- Dexterous, powerful manipulation skills - e.g. turning a heavy valve of lifting collapsed masonry.
- Robust balanced locomotion - walking, crawling over a debris pile, and physical sturdiness - e.g. operating conventional hand tools such as pneumatic drills.
- Sufficient cognitive ability to allow it to operate autonomously or under tele-operation in case of severe communication limitations for remote control.

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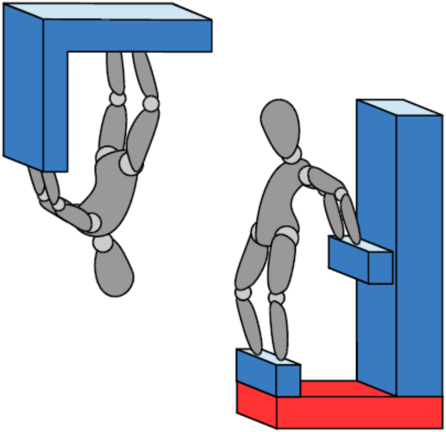
It's a different problem... or not?



Agile whole-body locomotion

Walking in highly unstructured environment and balancing in a human-like manner may require exploitation of the environment through additional contacts, involving also upper limbs...

Agile whole body locomotion

Bipedal locomotion may be seen as a periodic motion...
What about using optimization to generate openloop stable trajectories?

Robotica (2009) volume 27, pp. 321–330. © 2008 Cambridge University Press
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Using optimization to create self-stable human-like running

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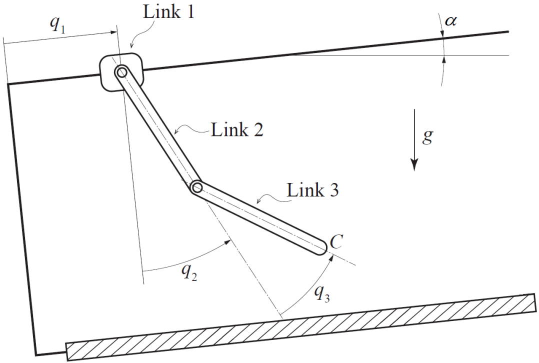
Benchmark system

3 DoF planar system



Joint limits:

$$-\frac{\pi}{2} \leq q_2 \leq \frac{\pi}{2}$$

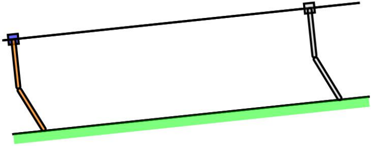
$$0 \leq q_3 \leq \frac{\pi}{2}$$

Task

Reach a specified final position (3 m away along the track), and stop there.

Discrete time optimal control problem

Time horizon $T = 6$ s discretized in $N = 200$ time intervals of duration h



Decision variables

$$w = (q_0, \dot{q}_0, u_0, f_{C_0}, \dots, q_k, \dot{q}_k, u_k, f_{C_k}, \dots, q_N, \dot{q}_N)$$

Contact parameters

Normal contact $f_0 = 20$ N

$\kappa = 10^4$ N/m

Tangential contact $\widehat{V}_t = 5 \cdot 10^{-3}$ m/s

Discrete time optimal control problem

NLP

$$\begin{aligned} \min_{w} \quad & f(w) \\ \text{s.t.} \quad & c_{\min} \leq c(w) \leq c_{\max} \\ & w_{\min} \leq w \leq w_{\max} \end{aligned}$$

Minimize:

- input torques
- accelerations
- work dissipated by friction
- input torque variations
- contact force variations

Discrete time optimal control problem

NLP

$$\begin{aligned} \min_{w} \quad & f(w) \\ \text{s.t.} \quad & c_{\min} \leq c(w) \leq c_{\max} \\ & w_{\min} \leq w \leq w_{\max} \end{aligned}$$

Constraints:

- Discrete system dynamics

$$\frac{\dot{q}_{k+1} - \dot{q}_k}{h} = g\left(\frac{q_{k+1} + q_k}{2}, \frac{\dot{q}_{k+1} + \dot{q}_k}{2}, f_{C_k}, u_k\right)$$

- Kinematic reconstruction

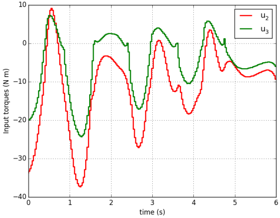
$$q_{k+1} - q_k - h\frac{\dot{q}_{k+1} + \dot{q}_k}{2} = 0$$

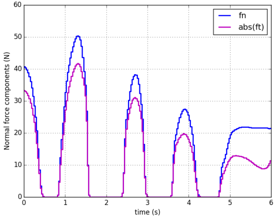
- Discrete contact force model

$$f_{n_k} = f_n\left(g_n\left(\frac{q_{k+1} + q_k}{2}\right)\right) \quad f_{t_k} = f_t\left(g_n\left(\frac{q_{k+1} + q_k}{2}\right), V_t\left(\frac{q_{k+1} + q_k}{2}, \frac{\dot{q}_{k+1} + \dot{q}_k}{2}\right)\right)$$

Results

Solution obtained after 300 s CPU time (671 iterations)





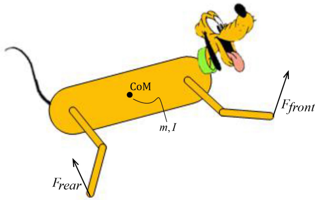
Multi stage model

Two different dynamics describing when the contact holds and when it does not, separated by discontinuous transitions. The numerical effort to solve the problem is lower (approximately 100 iterations to convergence).
How to choose the optimal number of transitions in this case?

3 strides

4 strides

Gait optimization



Dog model

- Legs dynamics is neglected
- Periodicity constraints
- Homotopy on contact model parameters

Task

Find *a priori* unscheduled interactions with the ground in order to run at a given mean velocity.

Within a time horizon $T = 1$ s, the dog has to go a prescribed distance.

Discrete time optimal control problem

NLP

$$\begin{aligned} \min_{w} \quad & f(w) \\ \text{s.t.} \quad & c_{\min} \leq c(w) \leq c_{\max} \\ & w_{\min} \leq w \leq w_{\max} \end{aligned}$$

Minimize:

- velocities
- accelerations
- contact forces
- contact forces variations

Discrete time optimal control problem

NLP

$$\min_{w} \quad f(w)$$
$$\text{s.t.} \quad c_{\min} \leq c(w) \leq c_{\max}$$
$$w_{\min} \leq w \leq w_{\max}$$

Constraints:

- Dynamics
- Algebraic constraints on the contact force variables
- Periodicity on some states
- Final horizontal position
- Direct collocation: 2 Radau collocation points per time interval

Results

Multi stage model

Different equations of motion depending on whether each foot is in contact with the ground or not.
The numerical effort to solve the problem is considerably lower.

Results

Solution obtained after 26 iterations, the CPU time required for computation is less than 1 second!

However, the obtained behaviour will be affected by our *a priori* choice...

How to proceed?

Conclusions

- Scheduled contacts are much easier to treat, but in some context the contact sequence is difficult (or not convenient) to be *a priori* specified. Additional contacts may also help to balance!
- Penalty based formulation of contact forces leads to significant numerical issues.

Further work

- Tackle the complexity of the dynamics... Robotran
- We have not considered a key feature of our problem so far: model uncertainties!

Thank you for your attention!

Any questions?