Exercises for Course on State-Space Control Systems (SSC)
Albert-Ludwigs-Universität Freiburg – Summer Term 2019

Exercise 10: Moving Horizon Estimation

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In the following, we consider a discrete system of the form

$$\begin{aligned} x_{k+1} &= f(x_k) + w_k \\ y_k &= h(x_k) + v_k. \end{aligned}$$

Linear System We consider the following linear state and output functions:

$$f(x) = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} x, \qquad h(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x.$$

We assume that $w_k \sim \mathcal{N}(0, 0.001 \cdot \mathbb{I})$ and $v_k \sim \mathcal{N}(0, 0.2 \cdot \mathbb{I})$. In addition, we assume a random initial state following a Gaussian distribution with mean $(2, 1)^\top$ and covariance $0.1 \cdot \mathbb{I}$.

1. Implement Matlab functions that compute f, h as well as their Jacobians $J_f = \frac{\partial f}{\partial x}$, $J_h = \frac{\partial h}{\partial x}$.
2. Plot the provided dataset.
3. Implement two Matlab functions `predict` and `update` that compute the predict and update/innovation step of the Extended Kalman Filter. The functions should have the following signatures:

```
function [x_predict, P_predict]   = predict_ekf(x_estimate, P_estimate, f, J_f, Q)
function [x_estimate, P_estimate] = update_ekf(y, x_predict, P_predict, h, J_h, R)
```

   Note that we would only need the linear Kalman Filter here, for the next part, however, the system is nonlinear.

4. Run the Kalman Filter on the provided dataset. How do you have to choose Q and R? Which quantities do you have to initialize? Which values do you use?
5. Implement a Matlab function that computes the MHE cost function. Use the results of the Kalman Filter predict step for the arrival cost.
6. Implement MHE with horizon length $N = 5$ using the function `fmincon`. Make sure you use the current solution to initialize the next MHE problem.
7. Plot the estimates obtained from Kalman Fitlering and MHE in one figure and compare the results.

Nonlinear Constrained System We consider the following state and output functions:

$$f(x) = \begin{bmatrix} 0.99 \cdot x_1 + 0.3 \cdot x_2 \\ -0.05 \cdot x_1 + 0.9 \cdot \sin(x_2) \end{bmatrix}, \qquad h(x) = x_1 - 3x_2$$

We assume a random initial state following a Gaussian distribution with mean $(0.8, 0.8)^\top$ and covariance $\mathbb{I}$. The measurement noise v_k follows Gaussian distribution with $v_k \sim \mathcal{N}(0, 0.05 \cdot \mathbb{I})$. We assume that the state noise $w_k = \text{abs}(z_k)$ and $z_k \sim \mathcal{N}(0, 0.05 \cdot \mathbb{I})$, i.e. the disturbance will always be positive.

1. Implement Matlab functions that compute f, h as well as their Jacobians $J_f = \frac{\partial f}{\partial x}$, $J_h = \frac{\partial h}{\partial x}$.
2. Run the Kalman Filter on the provided dataset and assume $w_k = z_k$, i.e. Gaussian state noise.

3. Run MHE on the provided dataset using zero arrival cost and a horizon of length $N = 15$. Again assume $w_k = z_k$.
4. We can incorporate the fact that the state disturbance will always be positive into our MHE formulation by adding constraints. Write a function $c(x)$ that you can pass to `fmincon` such that $w_k \geq 0$ is enforced. Check the documentation of `fmincon` to make sure your function has the correct format.
5. Plot the estimates obtained from Extended Kalman Filtering as well as constrained and unconstrained MHE in one figure. Compare the results.