Exercises for Course on State-Space Control Systems (SSC)
Albert-Ludwigs-Universität Freiburg – Summer Term 2019

Exercise 6: Observability, Luenberger Observer, Kalman Decomposition

Prof. Dr. Moritz Diehl, Dr. Dang Doan, Benjamin Stickan, Katrin Baumgärtner

Exercises

1. Consider the following dynamic system

$$\dot{x} = \begin{bmatrix} -\alpha-\beta & 1 \\ -\alpha\beta & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ k \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

   - Is the system observable?
   - Consider a Luenberg observer for the given system with observer gain $L = [l_1, l_2]^\top$. What is the characteristic polynomial of the dynamic system describing the estimation error? How would you choose l_1 and l_2?
   - For $\alpha = 2$, $\beta = 1$ and $u = 0$, simulate the system using Matlab and compute the corresponding state estimates for different values of l_1, l_2.
     Use initial estimates $\hat{x}(0) \neq x(0)$.
   - Suppose we choose a different output:

$$\tilde{y} = \begin{bmatrix} 1 & \gamma \end{bmatrix} x$$

     Are there any values of γ for which the system is *not* observable?

2. Consider a linearized inverted cart system, characterized by the matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.184 & 2.822 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.48 & 32.93 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1.84 \\ 0 \\ 4.8 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

   Use the Matlab command `obsv` to calculate the observability matrix. Is this system observable?

   Suppose we modify the sensor system, with: $C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$, is the modified system observable?

3. Consider a linear system characterized by the matrices

$$A = \begin{bmatrix} -2 & 1 & -1 & 2 \\ 1 & -3 & 0 & 2 \\ 1 & 1 & -4 & 2 \\ 0 & 1 & -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}, \quad D = 0$$

   Construct a Kalman decomposition for the system. (Hint: A has distinct eigenvalues, try to diagonalize.)

4. Consider the linear system

$$\dot{x} = Ax, \quad y = Cx$$

   Assume that the observability matrix W_o is invertible. Show that

$$\hat{x} = W_o^{-1} \begin{bmatrix} y & \dot{y} & \ddot{y} & \dots & y^{(n-1)} \end{bmatrix}^\top$$

   is an observer. Why would you not use this observer in practice?

5. Construct an example that shows that the Luenberg observer for given eigenvalues is in general not unique. Note that your system needs to have at least two outputs.