Exercise 3: State Feedback Control, Controllability

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MATLAB/Simulink: Buck-converter circuit

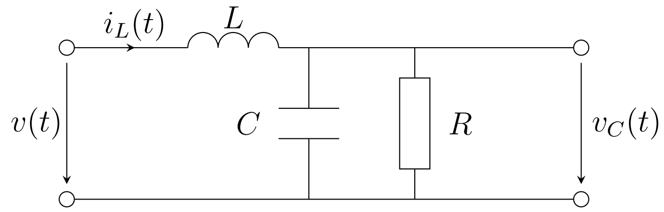
The electrical circuit sketched below shows a simplified buck-converter with a constant load at the output. The system can be described in state-space representation by

$$\dot{x} = Ax + Bu,$$

with

$$A = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}. \tag{1}$$

The state vector is defined as $x := \begin{bmatrix} i_L & v_C \end{bmatrix}^\intercal$ and the input as $u := v$.



1. Calculate the set of all possible equilibrium points x_e. What is the steady state and input for a constant capacitor voltage $v_\text{C}(t) = 1V$, if $R = 1\,\Omega$?
   Hint:
   $$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
2. Can any arbitrary point in the state-space of the system be reached in final time?
3. Transform the system into the Controllable Canonical Form.
4. Calculate the transformation matrix T.
5. Create a new MATLAB script `ex03_init.m` and define matrices A, B, $\tilde{A}$, $\tilde{B}$ and T with $L = 0.3\,H$, $C = 1\,F$ and $R = 1\,\Omega$.
6. Create and open a new Simulink model `ex03_sim` and enter
   `set_param('ex03_sim','InitFcn', 'ex03_init')`
   into the MATLAB console (script `ex03_init.m` will be executed before every simulation run).
7. Implement, simulate and compare the original to the Controllable Canonical Form system. Use a constant input of $u = 1\,V$ for both cases. Useful blocks:
   - `Gain`(different multiplication modes)
   - `Integrator`, `Constant`, `Sum`, `Scope`

Theoretical Exercises

1. Consider the double integrator
$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \tag{2}$$
Find a piecewise constant control strategy that drives the system from the origin to the state $x = (1, 1)$. Hints:

   - $x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$
   - $e^{At} = I + At + \frac{1}{2}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots = \sum_{k=0}^{\infty} \frac{1}{k!}A^k t^k$

2. Consider a system with the state x and z described by the equations
$$\frac{dx}{dt} = Ax + Bu, \quad \frac{dz}{dt} = Az + Bu,$$
If $x(0) = z(0)$ it follows that $x(t) = z(t)$ for all t regardless of the input that is applied. Show that this violates the definition of controllability and further show that the controllability matrix $\mathscr{C}$ is not full rank.

3. Show that the characteristic polynomial for a system in controllable canonical form is given by equation (7.7) and that
$$\frac{d^n z_k}{dt^n} + a_1 \frac{d^{n-1} z_k}{dt^{n-1}} + \cdots + a_{n-1} \frac{dz_k}{dt} = \frac{d^{n-k} u}{dt^{n-k}}$$
where z_k is the kth state.