Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg December 19, 2018, 14:10-14:55, Freiburg

Su	rname: Fi	rst Name:	Matriculation number:	
Su	bject: Program	me: Bachelor Master Leh	ramt others Signature	:
1.	We would like to know the unknow ground either breaks or has no dam	age. In an experiment we have drop	<b>OT</b> break when it is dropped. We as	sume that the phone thrown onto the 23 broken smartphones. What is the (ML) estimate of $\theta$ ?
	(a) log(77 $\theta$ ) - log(23(1 -	· θ))	(b) $23 \log \theta + 77 \log(1-\theta)$	
	(c) $\mathbf{x} - 77 \log \theta - 23 \log(1 - \theta)$	9)	(d) $\log(23\theta) + \log(77(1-\theta))$	9))
2.	You are given a pendulum which is ments. Which of the following algo			$(t + \theta_3)$ , where $y(t)$ are the measure-
	(a) Recursive Least Squares (	RLS)	(b) <b>x</b> Maximum a Posteriori Es	stimation (MAP)
	(c) Linear Least Squares (LL	S)	(d) Weighted Least Squares	(WLS)
3.		you use to estimate the parameters	$\theta$ of this linear model without run	ear in the parameters (LIP). Which ning into memory problems or high
	(a) MAP	(b) ML	(c) <b>X</b> RLS	(d) LLS
4. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question $\Sigma_{\hat{\theta}}$ model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^{\top} \Phi_N$ and $L(\theta, y_N)$ is the negative log likelihood function. The covar matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \dots$		ed in the previous question $\Sigma_{\hat{\theta}}$ . The likelihood function. The covariance		
	(a) $\Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(b) $\mathbf{x} Q_N^{-1}$	(c) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(d) $\left[ (\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N)^{-1} \right]$
5.			trameter $\theta$ , and a set of independent to solve to get a ML-estimate of $\theta$ ?	
	(a) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ $	$\ _{2}^{2}$	(b) $\square N \log(\theta) + \theta \sum_{k} y(k)$	
	(c) $\Box - \log \left( \sum_{k} \  \theta y(k) \exp(- \theta y(k)) \right)$	$- heta y(k))\ _2^2\Big)$	(d) $\mathbf{x} - N \log(\theta) + \theta \sum_{k} y(k)$	
6.	6. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$ , assuming that $\theta_0$ is the true value? The Fisher information matrix is defined as $M = \int_{y_N} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N   \theta_0) dy_N$ .			or $\hat{\theta}(y_N)$ , assuming that $\theta_0$ is the true
	(a) $N/\theta_0^2$		(b) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum$	$\left[ {{_k}{y_k}} \right]{\mathrm{d}{y_N}}$
	(c) $\mathbf{x} \theta_0^2 / N$		$ \left\  \begin{array}{c} (\mathbf{d}) \ \boxed{\int_{y_N} N \theta_0^{N-2} \left( \sum_k y(k) \right)} \end{array} \right. $	$\exp[- heta_0 \sum_k y_k] \mathrm{d}y_N$
7.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS	estimator using $L_2$ regularization.	Which of the following is <b>NOT</b> true?	,
	(a) $\theta_{\rm R}$ incorporates prior know	wledge about $\theta$ .	(b) $\theta_{\rm R}$ can be computed anal	ytically.
	(c) <b>x</b> $\theta_{\rm R}$ is asymptotically biase	ed.	(d) $\square \theta_{\rm R}$ is biased.	
8.			ion problem. Which of the following	
	(a) The inverse of the GN Hes	ssian approximates $\Sigma_{\theta}$ .	(b) The idea of GN is to line.	arize the residual function.
	(c) GN uses a Hessian approx	ximation.	(d) <b>x</b> GN finds the global mini	mizer of the objective function.

9. Given a set of measurements  $y_N = [y(1), y(2), \dots, y(N)]^T$  from the linear model  $y_N = \Phi \theta$ , where  $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter  $\hat{\theta}(N+1)$  after N+1measurements?  $\hat{\theta}(N+1) = \arg \min_{a} \frac{1}{2} (\ldots)$  [a and c are correct!]

(a) $\  \theta - \hat{\theta}(N) \ _2^2 + \  y(N+1) - \varphi(N+1)^\top \theta \ _{Q_N}^2$	(b) $\mathbf{X} \  \theta - \hat{\theta}(N) \ _{Q_N}^2 + \  y(N+1) - \varphi(N+1)^\top \theta \ _2^2$
(c) $\qquad \ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$	(d) $\qquad   y_N - \Phi_N \cdot \theta  ^2_{Q_N}$

10. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

(a) $\Box L_1$ , Gaussian	(b) <b>x</b> $L_1$ , Laplace	(c) $\Box L_2$ , Laplace	(d) $\Box L_2$ , Gaussian
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11. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct? Plot the entries of the residual vector,  $R(\theta^*)_i$ , i = 1, ..., N, as a histogram and check if it looks like a zero-mean

Gaussian. 12. For a discrete time LTI system  $x_{k+1} = Ax_k + Bu_k$ , k = 0, 1, explicitly state the forward simulation map  $f_{sim} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}$ ,  $(x_0, u_0, u_1) \mapsto (x_0, x_1, x_2)$ .

$$f(x_0, u_0, u_1) = \begin{bmatrix} x_0 \\ Ax_0 + Bu_0 \\ A^2x_0 + ABu_0 + Bu_1 \end{bmatrix}$$

13. Please identify the most general system equation that still is a Auto Regressive Model with Exogenous Inputs (ARX).

(a) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$	(b) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$
(c) $a_0y(k) + a_1y^2(k-1) + \dots + a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \dots + b_0u(k-n_b)$	(d) $\begin{bmatrix} \mathbf{X} \end{bmatrix} a_0 y(k) + a_1 y(k-1) + \ldots + a_{n_a} y(k-n_a) = b_0 u(k) + \ldots + b_{n_b} u(k-n_b)$
Which one of the following statements is <b>NOT</b> true for FIR models:	
(a) The impulse response is constant.	(b) The output does not depend on previous outputs.
(c) Output error minimization is a convex problem.	(d) They are a special class of ARX models
Which of the following model equations describes a FIR system with in	nput $u$ and output $y$ ? $y(k+1) = \dots$
(a) $u(k) + \sin(k \cdot \pi)^2$ (b) $u(k) \cdot y(k)$	(c) $x u(k) - \sqrt{\pi}u(k-2)$ (d) $u(k+1) + y(k)$
Which of the following dynamic models with inputs $u(t)$ and outputs $y$	y(t) is <b>NEITHER</b> linear <b>NOR</b> affine.
(a) $t\dot{y}(t) = u(t) + \sqrt{2\pi}$ (b) $\dot{y}(t) + \cos(t) = u(t)$	(c) $\[ \dot{y}(t) = u(t) + t \]$ (d) $\[ \mathbf{X} \dot{y}(t) = \sqrt{t \cdot u(t)} \]$
Which of the following models with input $u(k)$ and output $y(k)$ is <b>NO</b>	<b>T</b> linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$ ?
(a) $\mathbf{x} \ y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$	(b) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$
(c) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$	(d) $\qquad y(k) = \theta_1 \sqrt{u(k)}$
Which of the following models is time invariant?	
(a) $\[ \ddot{y}(t)^2 = u(t)^t + e^{u(t)} \]$ (b) $\[ \mathbf{x} \] \dot{y}(t) = \sqrt{u(t)} + 1 \]$	(c) $\begin{tabular}{ c c c c } \dot{y}(t) = -3u(t) + t^2 \end{tabular}$ (d) $\begin{tabular}{ c c } t \cdot \ddot{y}(t) = u(t)^3 \end{tabular}$
With which of the following formulas you can <b>NOT</b> compute the $(y_1, \ldots, y_N)$ given $\theta$ ? $p(y_N \theta) \neq \ldots$	conditional joint distribution of N independent measurements $y_N =$
(a) $\prod \int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$	(b) $\prod_i p(y(i) \theta)$
(c) $\mathbf{x} \int p(y_N   \theta) p(\theta) d\theta$	(d) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$
Which of the following statements about Maximum A Posteriori (MAP	P) estimation is <b>NOT</b> true
(a) The MAP estimator is biased.	(b) MAP is a generalization of ML.
(c) X MAP assumes a linear model.	(d) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$
	Image: constraint of the following model is the following model is the following model equations describes a FIR system with it is a convex problem.(c)Output error minimization is a convex problem.(c)Output error minimization is a convex problem.Which of the following model equations describes a FIR system with it is a convex problem.(a) $u(k) + \sin(k \cdot \pi)^2$ (b) $u(k) \cdot y(k)$ Which of the following dynamic models with inputs $u(t)$ and outputs $y$ (a) $t\dot{y}(t) = u(t) + \sqrt{2\pi}$ (b) $\dot{y}(t) + \cos(t) = u(t)$ Which of the following models with input $u(k)$ and output $y(k)$ is NO(a) $\mathbf{x}$ $y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$ (c) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$ Which of the following models is time invariant?(a) $\ddot{y}(t)^2 = u(t)^t + e^{u(t)}$ (b) $\mathbf{x}$ $\dot{y}(t) = \sqrt{u(t)} + 1$ With which of the following formulas you can NOT compute the $(y_1, \dots, y_N)$ given $\theta$ ? $p(y_N   \theta) \neq \dots$ (a) $\int p(y_N   x_N, \theta) p(x_N) dx_N$ (c) $\mathbf{x}$ $\int p(y_N   \theta) p(\theta) d\theta$ Which of the following statements about Maximum A Posteriori (MAF(a)The MAP estimator is biased.

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Su	rname: Fi	rst Name:	Matriculation number:	
Su	bject: Program	me: Bachelor Master Leh	ramt others Signature:	
1.	Given the probability density functi		answer of each question below. rameter $\theta$ , and a set of independent r I to solve to get a ML-estimate of $\theta$ ?	
	(a) $\mathbf{x} - N \log(\theta) + \theta \sum_{k} y(k)$		(b) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ $	0
	(c) $N \log(\theta) + \theta \sum_{k} y(k)$		(d) $\Box - \log \left( \sum_{k} \  \theta y(k) \exp(- \theta y(k)) \right)$	$- heta y(k))\ _2^2 \Big)$
2.	For the problem in the previous que value? The Fisher information matr		covariance for any unbiased estimato $y_N$ ) $\cdot p(y_N   \theta_0) dy_N$ .	r $\hat{\theta}(y_N)$ , assuming that $\theta_0$ is the true
	(a) $N/\theta_0^2$		(b) $\mathbf{x} \theta_0^2/N$	
	(c) $\int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum$	$_{k}y_{k}]\mathrm{d}y_{N}$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right)$	$\exp[- heta_0 \sum_k y_k] \mathrm{d}y_N$
3.	You are given a pendulum which is ments. Which of the following algor		an be modeled by $y(t) = \theta_1 \cos(\theta_2 t)$ ne parameters $\theta$ ?	$(+ \theta_3)$ , where $y(t)$ are the measure-
	(a) Linear Least Squares (LLS	5)	(b) Weighted Least Squares (	WLS)
	(c) Recursive Least Squares	(RLS)	(d) <b>x</b> Maximum a Posteriori Es	timation (MAP)
4.		you use to estimate the parameters	approximated by a model that is lin $\theta$ of this linear model without runn data?	
	(a) LLS	(b) X RLS	(c) ML	(d) MAP
5.		with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^\top \Phi_N$	e covariance of the estimate compute $\Phi_N$ and $L(\theta, y_N)$ is the negative log	
	(a) $\left[ \left( \Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	(b) $\square \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(c) $\mathbf{X} Q_N^{-1}$	(d) $\Box \nabla^2_{\theta} L(\theta, y_N)$
	<ul> <li>6. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct? Plot the entries of the residual vector, R(θ*)<sub>i</sub>, i = 1,,N, as a histogram and check if it looks like a zero-mean Gaussian.</li> <li>7. Let θ<sub>R</sub> denote the <i>regularized</i> LLS estimator using L<sub>2</sub> regularization. Which of the following is <b>NOT</b> true?</li> </ul>			
7.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS (a) $\theta_{\rm R}$ is biased.	estimator using $L_2$ regularization.	Which of the following is <b>NOT</b> true? (b) $\mathbf{x} \ \theta_{\rm R}$ is asymptotically biase	
	(c) $\theta_{\rm R}$ can be computed analy	vtically	(d) $\theta_{\rm R}$ incorporates prior kno	
8.		-	ion problem. Which of the following	
	(a) The idea of GN is to linea		(b) <b>x</b> GN finds the global minin	_
	(c) GN uses a Hessian approx	ximation.	(d) The inverse of the GN He	essian approximates $\Sigma_{\theta}$ .

9.	Which of the following models is time invariant?		
	(a) $x \dot{y}(t) = \sqrt{u(t)} + 1$ (b) $\dot{y}(t) = -3u(t) + t^2$	(c) $\begin{tabular}{ c c c c } \ddot{y}(t)^2 = u(t)^t + e^{u(t)} \end{tabular}$ (d) $\begin{tabular}{ c c c c } t \cdot \ddot{y}(t) = u(t)^3 \end{tabular}$	
10. With which of the following formulas you can <b>NOT</b> compute the conditional joint distribution of N independent measure $(y_1, \ldots, y_N)$ given $\theta$ ? $p(y_N \theta) \neq \ldots$		conditional joint distribution of N independent measurements $y_N =$	
	(a) $\prod \int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$	(b) $\prod_i p(y(i) \theta)$	
	(c) $\mathbf{x} \int p(y_N   \theta) p(\theta) d\theta$	(d) $\square \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$	
11.	Which of the following statements about Maximum A Posteriori (MAP	) estimation is <b>NOT</b> true	
	(a) The MAP estimator is biased.	(b) MAP is a generalization of ML.	
	(c) $\square \hat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$	(d) X MAP assumes a linear model.	
12.	Which of the following model equations describes a FIR system with in	put $u$ and output $y$ ? $y(k+1) = \dots$	
	(a) $u(k) + \sin(k \cdot \pi)^2$ (b) $u(k) \cdot y(k)$	(c) $u(k+1) + y(k)$ (d) $x u(k) - \sqrt{\pi}u(k-2)$	
13.	Which of the following dynamic models with inputs $u(t)$ and outputs $y$	(t) is <b>NEITHER</b> linear <b>NOR</b> affine.	
	(a) $\begin{tabular}{ll} \dot{y}(t) = u(t) + t \end{tabular}$ (b) $\begin{tabular}{ll} t\dot{y}(t) = u(t) + \sqrt{2\pi} \end{tabular}$	(c) $\mathbf{x}\dot{y}(t) = \sqrt{t \cdot u(t)}$ (d) $\mathbf{y}(t) + \cos(t) = u(t)$	
	14. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$ , where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , who of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after $N$ measurements? $\hat{\theta}(N+1) = \arg \min_{\theta} \frac{1}{2} (\dots)$ [a and c are correct!]		
	(a) $\ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$	(b) $\  y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$	
	(c) $\mathbf{x} \  \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(N) \ _{Q_N}^2 + \  \boldsymbol{y}(N+1) - \boldsymbol{\varphi}(N+1)^\top \boldsymbol{\theta} \ _2^2$	(d) $\  \theta - \hat{\theta}(N) \ _2^2 + \  y(N+1) - \varphi(N+1)^\top \theta \ _{Q_N}^2$	
15.	Please identify the most general system equation that still is a Auto Reg	ressive Model with Exogenous Inputs (ARX).	
	(a) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$	(b) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$	
	(c) $\mathbf{X}$ $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-n_a) = b_0u(k) + \ldots + b_{n_b}u(k-n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \dots + a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \dots + b_0u(k-n_b)$	
16. We would like to know the unknown probability $\theta$ that a phone does <b>NOT</b> break when it is dropped. We assume that the p ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smar negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maximum likelihood (ML) estimates of $f(\theta)$ that we need to minimize the maxi		ped 100 smartphones and obtained 23 broken smartphones. What is the	
	(a) $23\log\theta + 77\log(1-\theta)$	(b) $\boxed{\mathbf{x}} -77\log\theta - 23\log(1-\theta)$	
	(c) $\log(23\theta) + \log(77(1-\theta))$	(d) $-\log(77\theta) - \log(23(1-\theta))$	
17. For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k, k = 0, 1$ , explicitly state the forward simulation map $f_{\text{sim}} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}, (x_0, u_0, u_1) \mapsto (x_0, x_1, x_2).$		7	
	$f(x_0, u_0, u_1) = \begin{bmatrix} Ax \\ A^2 x_0 + A \end{bmatrix}$	· · · · · · · · · · · · · · · · · · ·	
18.	Which of the following models with input $u(k)$ and output $y(k)$ is <b>NO</b>	$\Gamma$ linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$ ?	
	(a) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$	(b) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	
	(c) $y(k) = \theta_1 \sqrt{u(k)}$	(d) $x y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$	
19.	You suspect your data to contain some outliers, thus you would use bution.	estimation which assumes your measurement errors follow a distri-	
	(a) $\mathbf{x}$ $L_1$ , Laplace (b) $\Box$ $L_1$ , Gaussian	(c) $\Box L_2$ , Gaussian (d) $\Box L_2$ , Laplace	
20.	Which one of the following statements is <b>NOT</b> true for FIR models:		
	(a) They are a special class of ARX models	(b) <b>x</b> The impulse response is constant.	
	(c) Output error minimization is a convex problem.	(d) The output does not depend on previous outputs.	

Points on page (max. 12)

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Su	name: Fi	rst Name:	Matriculation number:	
Sul	oject: Program	me: Bachelor Master Leh	ramt others Signat	ure:
1	-	tick exactly ONE box for the right	-	
1.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS (a) $\theta_{\rm R}$ is biased.	estimator using $L_2$ regularization.	(b) $\theta_{\rm R}$ incorporates prior	
	(c) $\theta_{\rm R}$ can be computed analy	-	(d) <b>x</b> $\theta_{\rm R}$ is asymptotically b	
2.				ing statements is <b>NOT</b> true <i>in general</i> ?
	(a) GN uses a Hessian approx	.imation.	(b) The idea of GN is to b	inearize the residual function.
	(c) $\mathbf{x}$ GN finds the global minim	nizer of the objective function.	(d) The inverse of the GN	Hessian approximates $\Sigma_{\theta}$ .
3.	Please identify the most general sys	stem equation that still is a Auto Re	gressive Model with Exogenous I	nputs (ARX).
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k)$	$k(k-n_b)$	(b) $y(k) = -a_1y(k-1) - $	$\ldots - a_{n_a} y(k - n_a)$
	(c) <b>x</b> $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-1)$	$(k - n_a) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \ldots +$	$a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \ldots + b_0u(k-n_b)$
4.	Which of the following model equa	tions describes a FIR system with i	nput $u$ and output $y$ ? $y(k+1) =$	
	(a) $\mathbf{x} u(k) - \sqrt{\pi}u(k-2)$	(b) $\Box u(k) \cdot y(k)$	(c) $u(k) + \sin(k \cdot \pi)^2$	(d) $u(k+1) + y(k)$
	You suspect your data to contain so bution.	ome outliers, thus you would use	. estimation which assumes your	measurement errors follow a distri-
	(a) $\Box L_1$ , Gaussian	(b) $\Box L_2$ , Laplace	(c) $\mathbf{X}$ $L_1$ , Laplace	(d) $\Box L_2$ , Gaussian
6.	Which one of the following stateme	ents is NOT true for FIR models:		
	(a) Output error minimization	is a convex problem.	(b) They are a special class	ss of ARX models
	(c) The output does not dependent	nd on previous outputs.	(d) <b>x</b> The impulse response	is constant.
	7. We would like to know the unknown probability $\theta$ that a phone does <b>NOT</b> break when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $\theta$ ?			
	(a) $\boxed{\mathbf{x}} -77\log\theta - 23\log(1-\theta)$	)	(b) $\log(23\theta) + \log(77(1$	(- heta))
	(c) $-\log(77\theta) - \log(23(1 - 1)))$	- θ))	(d) $23 \log \theta + 77 \log(1 - \theta)$	θ)
8. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$ , where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , whi of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after $N$ + measurements? $\hat{\theta}(N+1) = \arg \min_{\theta} \frac{1}{2} (\dots)$ [a and c are correct!]			$\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , which be the parameter $\hat{\theta}(N+1)$ after $N+1$	
	(a) <b>x</b> $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N +$	$1) - \varphi(N+1)^{\top} \theta \ _2^2$	(b) $\qquad \ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$	
	(c) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N+1)\ _2^2$	$ -\varphi(N+1)^{\top}\theta  _{Q_N}^2$	(d) $\  y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$	
	Assume you solved a nonlinear para the noise distribution are correct?	ameter estimation problem using the	e Gauss-Newton algorithm. How	can you check that your assumptions on

Plot the entries of the residual vector,  $R(\theta^*)_i$ , i = 1, ..., N, as a histogram and check if it looks like a zero-mean Gaussian.

10. For a discrete time LTI system  $x_{k+1} = Ax_k + Bu_k$ , k = 0, 1, explicitly state the forward simulation map  $f_{sim} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}$ ,  $(x_0, u_0, u_1) \mapsto (x_0, x_1, x_2)$ .

	$x_0$
$f(x_0, u_0, u_1) =$	$Ax_0 + Bu_0$
	$A^2x_0 + ABu_0 + Bu_1$

11. Given the probability density function  $p_X(x) = \theta x \exp(-\theta x)$ , with parameter  $\theta$ , and a set of independent measurements  $y_N = [y(1), y(2), \dots, y(N)]^T$ , which minimisation problem you need to solve to get a ML-estimate of  $\theta$ ? The problem is: min...?

(a) $\square N \log(\theta) + \theta \sum_{k} y(k)$	(b) $\Box - \log \left( \sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2} \right)$
(c) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2}$	(d) $\mathbf{x} - N \log(\theta) + \theta \sum_{k} y(k)$

12. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator  $\hat{\theta}(y_N)$ , assuming that  $\theta_0$  is the true value? The Fisher information matrix is defined as  $M = \int_{y_N} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$ .

(a) $N/\theta_0^2$	(b) $\prod \int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right) \exp\left[-\theta_0 \sum_k y_k\right] dy_N$
(c) $\mathbf{x} \ \theta_0^2 / N$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k y_k] dy_N$

13. Which of the following dynamic models with inputs u(t) and outputs y(t) is **NEITHER** linear **NOR** affine.

|--|

14. You are given a pendulum which is by nature a nonlinear system and can be modeled by  $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$ , where y(t) are the measurements. Which of the following algorithms should you use to estimate the parameters  $\theta$ ?

(a) Linear Least Squares (LLS)	(b) Recursive Least Squares (RLS)
(c) Weighted Least Squares (WLS)	(d) <b>x</b> Maximum a Posteriori Estimation (MAP)

15. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP). Which of the following algorithms could you use to estimate the parameters  $\theta$  of this linear model without running into memory problems or high computational costs for a continuous and infinite flow of measurement data?

(a) **X** RLS ML MAP LLS (b) (c) (d) 16. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question  $\Sigma_{\hat{\alpha}}$ . The

The are asked to give a computationary efficient approximation of the covariance of the estimate computed in the previous question  $\Sigma_{\hat{\theta}}$ . The model is given as  $y_N = \Phi_N \theta + \epsilon_N$  with  $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^{\top} \Phi_N$  and  $L(\theta, y_N)$  is the negative log likelihood function. The covariance matrix can be approximated by  $\Sigma_{\hat{\theta}} \approx \dots$ 

(b)  $\Box \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$ (a)  $\mathbf{X} Q_N^{-1}$ (c)  $\left[ \left( \Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$  $\nabla^2_{\theta} L(\theta, y_N)$ (d)

(c)

 $\int t \cdot \ddot{y}(t) = u(t)^3$ 

(d)

 $] \dot{y}(t) = -3u(t) + t^2$ 

17. Which of the following models is time invariant? (a)  $\underline{x} \ \dot{y}(t) = \sqrt{u(t)} + 1$  (b)  $\square \ \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$ 

1

2

18. With which of the following formulas you can **NOT** compute the conditional joint distribution of N independent measurements  $y_N = (y_1, \ldots, y_N)$  given  $\theta$ ?  $p(y_N | \theta) \neq \ldots$ 

	(a) $\int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$	(b) $\prod_i p(y(i) \theta)$
	(c) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$	(d) $\mathbf{x} \int p(y_N   \theta) p(\theta) d\theta$
9.	Which of the following statements about Maximum A Posteriori (MAP)	) estimation is <b>NOT</b> true
	(a) MAP is a generalization of ML.	(b) X MAP assumes a linear model.
	(c) The MAP estimator is biased.	(d) $\square \hat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N   \theta)) - \log(p(\theta))]$
0. Which of the following models with input $u(k)$ and output $y(k)$ is <b>NO</b>		$\Gamma$ linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$ ?
	(a) $\mathbf{x} y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$	(b) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$
	(c) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	(d) $y(k) = \theta_1 \sqrt{u(k)}$

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg December 19, 2018, 14:10-14:55, Freiburg

Su	name: Fi	rst Name:	Matriculation number:		
Sul	oject: Program	me: Bachelor Master Lehr	ramt others Signature:		
Please fill in your name above and tick exactly ONE box for the right answer of each question below. 1. Which of the following models is time invariant?					
	(a) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(b) $\[ \dot{y}(t) = -3u(t) + t^2 \]$	(c) $\mathbf{x} \dot{y}(t) = \sqrt{u(t)} + 1$	(d) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	
2.	With which of the following form $(y_1, \ldots, y_N)$ given $\theta$ ? $p(y_N \theta) \neq$		conditional joint distribution of N	independent measurements $y_N =$	
	(a) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$	)	(b) $\mathbf{x} \int p(y_N   \theta) p(\theta) \mathrm{d}\theta$		
	(c) $\prod_i p(y(i) \theta)$		(d) $\int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$	۲ ۲	
3.	Which of the following statements	about Maximum A Posteriori (MAP	) estimation is <b>NOT</b> true		
	(a) MAP is a generalization o	f ML.	(b) The MAP estimator is bia	sed.	
	(c) X MAP assumes a linear mo	odel.	(d) $\hat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-h]$	$\log(p(y_N \theta)) - \log(p(\theta))]$	
		by nature a nonlinear system and ca orithms should you use to estimate th		In be modeled by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$ , where $y(t)$ are the measure- e parameters $\theta$ ?	
	(a) <b>x</b> Maximum a Posteriori Est	timation (MAP)	(b) Linear Least Squares (LL	S)	
	(c) Recursive Least Squares	(RLS)	(d) Weighted Least Squares (	WLS)	
5. Suppose now that the system given in the previous question can be approximated of the following algorithms could you use to estimate the parameters $\theta$ of this line computational costs for a continuous and infinite flow of measurement data?		$\theta$ of this linear model without runr			
	(a) LLS	(b) MAP	(c) <b>X</b> RLS	(d) ML	
6. You are asked to give a computationally efficient approximation of the covariance of the model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^{\top} \Phi_N$ and $L(\theta, y_N)$ matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \dots$					
	(a) $\mathbf{x} Q_N^{-1}$	(b) $\square \Phi_N^+ \Sigma_{\epsilon_N} {\Phi_N^+}^\top$	(c) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(d) $\left[ \left( \Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	
7. We would like to know the unknown probability $\theta$ that a phone does <b>NOT</b> break when it is dropped. We assume that the phone ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphone likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of $\theta$		23 broken smartphones. What is the			
	(a) $23 \log \theta + 77 \log(1-\theta)$		(b) $\log(23\theta) + \log(77(1-\theta))$	)))	
	(c) $-\log(77\theta) - \log(23(1 - 1)))$	- θ))	(d) $\boxed{\mathbf{x}} - 77\log\theta - 23\log(1-\theta)$	))	
	Given a set of measurements $y_N$ = of the following minimisation prob measurements? $\hat{\theta}(N+1) = \arg m$	lems is solved at each iteration step	e linear model $y_N = \Phi \theta$ , where $\Phi$ = of the RLS algorithm to estimate the	= $[\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , which the parameter $\hat{\theta}(N+1)$ after $N+1$	
	(a) $\ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$		(b) $\  y_N - \Phi_N \cdot \theta \ _{Q_N}^2$		
	(c) <b>x</b> $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N + \theta)\ _{Q_N}^2$	$(-1) - \varphi(N+1)^\top \theta \ _2^2$	(d) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N+1)\ _2^2$	$\ -\varphi(N+1)^{\top}\theta\ _{Q_N}^2$	

9. Which of the following dynamic models with inputs u(t) and outputs y(t) is **NEITHER** linear **NOR** affine.

	(a) $\Box t\dot{y}(t) = u(t) + \sqrt{2\pi}$	(b) $\[ \dot{y}(t) + \cos(t) = u(t) \]$	(c) $\[\dot{y}(t) = u(t) + t\]$	(d) $\mathbf{x}\dot{y}(t) = \sqrt{t \cdot u(t)}$	
10.	Which of the following models with	input $u(k)$ and output $y(k)$ is <b>NO</b>	<b>T</b> linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$ ?		
	(a) $\mathbf{x} y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$		(b) $\qquad y(k) = \theta_1 \sqrt{u(k)}$		
	(c) $y(k) = \exp(y(k-1)) \cdot (k-1)$	$( heta_1+ heta_2u(k))$	(d) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(k)$	p(u(k))	
11.	Which of the following model equa	tions describes a FIR system with in	put $u$ and output $y$ ? $y(k+1) = \dots$		
	(a) $\boxed{\mathbf{x}} u(k) - \sqrt{\pi}u(k-2)$	(b) $u(k+1) + y(k)$	(c) $\[ u(k) \cdot y(k) \]$	(d) $\qquad u(k) + \sin(k \cdot \pi)^2$	
12.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS	estimator using $L_2$ regularization. V	Which of the following is <b>NOT</b> true?		
	(a) $\square \theta_{\rm R}$ can be computed analy	tically.	(b) $\square \theta_{\rm R}$ is biased.		
	(c) $\theta_{\rm R}$ incorporates prior known	wledge about $\theta$ .	(d) <b>x</b> $\theta_{\rm R}$ is asymptotically biase	ed.	
13.	We use the Gauss-Newton (GN) alg	orithm to solve a nonlinear estimati	on problem. Which of the following	statements is NOT true in general?	
	(a) The idea of GN is to linear	rize the residual function.	(b) The inverse of the GN He	ssian approximates $\Sigma_{\theta}$ .	
	(c) GN uses a Hessian approx	timation.	(d) $\mathbf{x}$ GN finds the global minim	nizer of the objective function.	
14.	Which one of the following stateme	nts is <b>NOT</b> true for FIR models:			
	(a) They are a special class of	ARX models	(b) Output error minimization is a convex problem.		
	(c) The output does not dependent	nd on previous outputs.	(d) $\mathbf{x}$ The impulse response is c	onstant.	
15.	5. For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k, k = 0, 1$ , explicitly state the forward simulation map $f_{\text{sim}} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}, (x_0, u_0, u_1) \mapsto (x_0, x_1, x_2).$				
		$f(x_0, u_0, u_1) = \begin{bmatrix} Ax \\ A^2 x_0 + \end{bmatrix}$	$\begin{bmatrix} x_0 \\ 0 + Bu_0 \\ ABu_0 + Bu_1 \end{bmatrix}$		
	You suspect your data to contain so bution.	me outliers, thus you would use	estimation which assumes your mea	asurement errors follow a distri-	
	(a) $\Box L_2$ , Gaussian	(b) $\mathbf{X}$ $L_1$ , Laplace	(c) $\Box L_2$ , Laplace	(d) $\Box L_1$ , Gaussian	
17.	Please identify the most general sys	tem equation that still is a Auto Reg	ressive Model with Exogenous Inpu	ts (ARX).	
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u$	$(k - n_b)$	(b) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$		
	(c) <b>X</b> $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-1)$	$k - n_a) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \ldots + a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \ldots + b_0u(k-n_b)$		
	8. Given the probability density function $p_X(x) = \theta x \exp(-\theta x)$ , with parameter $\theta$ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ , which minimisation problem you need to solve to get a ML-estimate of $\theta$ ? The problem is: $\min_{\theta} \dots$ ?				
	(a) $\Box - \log \left( \sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2} \right)$		(b) $\square N \log(\theta) + \theta \sum_k y(k)$		
	(c) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))$	$\ _{2}^{2}$	(d) $\mathbf{x} - N \log(\theta) + \theta \sum_{k} y(k)$		
19.	For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$ , assuming that $\theta_0$ is the tr value? The Fisher information matrix is defined as $M = \int_{y_N} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N   \theta_0) dy_N$ .		$\hat{\theta}(y_N)$ , assuming that $\theta_0$ is the true		
	(a) $\int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k$	$y_k]\mathrm{d}y_N$	(b) $\mathbf{x} \theta_0^2/N$		
	(c) $\boxed{N/\theta_0^2}$		(d) $\int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right) \epsilon$	$\exp[-\theta_0 \sum_k y_k] \mathrm{d}y_N$	

20. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct? Plot the entries of the residual vector,  $R(\theta^*)_i$ , i = 1, ..., N, as a histogram and check if it looks like a zero-mean

Gaussian.

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg December 19, 2018, 14:10-14:55, Freiburg

Su	rname: First Name:		Matriculation number:	
				nature:
	5			
	Please fill in your name above and tick exact	ly ONE box for the right	answer of each question below	λ.
1.	For a discrete time LTI system $x_{k+1} = Ax_k + f_{sim}$ : $\mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}$ , $(x_0, u_0, u_1) \mapsto (x_0, u_0, u_1)$	$+ Bu_k, k = 0, 1, $ explicit $_0, x_1, x_2).$	ly state the forward simulation	і тар
	f(x	$x_0, u_0, u_1) = \begin{bmatrix} Ax \\ A^2 x_0 + \end{bmatrix}$	$\begin{bmatrix} x_0 \\ x_0 + Bu_0 \\ ABu_0 + Bu_1 \end{bmatrix}$	
2.	Please identify the most general system equat	ion that still is a Auto Reg	gressive Model with Exogenou	as Inputs (ARX).
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$		(b) $a_0y(k) + a_1y^2(k-1) + .$	$\dots + a_{n_a} y^{n_a + 1} (k - n_a) = b_0 u(k) + \dots + b_0 u(k - n_b)$
	(c) $y(k) = -a_1 y(k-1) - \ldots - a_{n_a} y(k-1)$	$(-n_a)$	(d) <b>X</b> $a_0y(k) + a_1y(k-1) +$	$(1 + a_{n_a}y(k - n_a)) = b_0u(k) + \ldots + b_{n_b}u(k - n_b)$
3.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS estimator	using $L_2$ regularization.	Which of the following is <b>NO</b>	۲ true?
	(a) <b>x</b> $\theta_{\rm R}$ is asymptotically biased.		(b) $\square \theta_{\rm R}$ incorporates prior	ior knowledge about $\theta$ .
	(c) $\square \theta_{\rm R}$ is biased.		(d) $\square \theta_{\rm R}$ can be compute	ed analytically.
4.	We use the Gauss-Newton (GN) algorithm to solve a nonlinear estimation problem. Which of the following statements is <b>NOT</b> true <i>in general</i> ?			
	(a) $\mathbf{X}$ GN finds the global minimizer of the	e objective function.	(b) GN uses a Hessian	approximation.
	(c) The inverse of the GN Hessian appr	coximates $\Sigma_{\theta}$ .	(d) The idea of GN is	to linearize the residual function.
5. Which of the following models is time invariant?				
	(a) $\[ \dot{y}(t) = -3u(t) + t^2 \]$ (b) $\[ \]$	$t \cdot \ddot{y}(t) = u(t)^3$	(c) $\mathbf{x} \ \dot{y}(t) = \sqrt{u(t)} + 1$	(d) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$
6.	With which of the following formulas you $(y_1, \ldots, y_N)$ given $\theta$ ? $p(y_N \theta) \neq \ldots$	can NOT compute the	conditional joint distribution	of N independent measurements $y_N =$
	(a) $\mathbf{x} \int p(y_N \theta) p(\theta) \mathrm{d}\theta$		(b) $\prod_i p(y(i) \theta)$	
	(c) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$		(d) $\int p(y_N x_N,\theta)p(x)$	$(x_N) \mathrm{d}x_N$
7.	Which of the following statements about Max	imum A Posteriori (MAF	P) estimation is <b>NOT</b> true	
	(a) The MAP estimator is biased.		(b) $\square \hat{\theta}_{MAP} = \arg \min_{\theta}$	$\in \mathbb{R}[-\log(p(y_N \theta)) - \log(p(\theta))]$
	(c) $\mathbf{x}$ MAP assumes a linear model.		(d) MAP is a generalized	zation of ML.
8.	Which of the following model equations desc	ribes a FIR system with in	nput $u$ and output $y$ ? $y(k+1)$	) =
	(a) $u(k) + \sin(k \cdot \pi)^2$ (b) $\Box$	u(k+1) + y(k)	(c) $\mathbf{x} u(k) - \sqrt{\pi}u(k-1)$	2) (d) $\Box u(k) \cdot y(k)$
9.	Which of the following dynamic models with	inputs $u(t)$ and outputs $y$	y(t) is <b>NEITHER</b> linear <b>NOR</b>	affine.
	(a) $t\dot{y}(t) = u(t) + \sqrt{2\pi}$ (b) <b>x</b>	$\dot{y}(t) = \sqrt{t \cdot u(t)}$	(c) $\boxed{\dot{y}(t) + \cos(t) = u(t)}$	t) (d) $\[ \dot{y}(t) = u(t) + t \]$
	We would like to know the unknown probabil ground either breaks or has no damage. In an negative log likelihood function $f(\theta)$ that we	experiment we have drop	oped 100 smartphones and obt	ained 23 broken smartphones. What is the

(a) $\boxed{\mathbf{x}} - 77\log\theta - 23\log(1-\theta)$	(b) $23 \log \theta + 77 \log(1-\theta)$
(c) $-\log(77\theta) - \log(23(1-\theta))$	(d) $\log(23\theta) + \log(77(1-\theta))$

Points on page (max. 0)

11. Given the probability density function  $p_X(x) = \theta x \exp(-\theta x)$ , with parameter  $\theta$ , and a set of independent measurements  $y_N = [y(1), y(2), \dots, y(N)]^T$ , which minimisation problem you need to solve to get a ML-estimate of  $\theta$ ? The problem is: min ...?

(a) $-\log\left(\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2}\right)$	(b) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _2^2$
(c) $N \log(\theta) + \theta \sum_{k} y(k)$	(d) $\mathbf{x} - N \log(\theta) + \theta \sum_{k} y(k)$

12. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator  $\hat{\theta}(y_N)$ , assuming that  $\theta_0$  is the true value? The Fisher information matrix is defined as  $M = \int_{uN} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$ .

(a) $\prod \int_{y_N} N \theta_0^{N-2} \left( \sum_k y(k) \right) \exp[-\theta_0 \sum_k y_k] dy_N$	(b) $\square N/\theta_0^2$
(c) $\mathbf{x} \theta_0^2 / N$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k y_k] dy_N$

13. Given a set of measurements  $y_N = [y(1), y(2), \dots, y(N)]^T$  from the linear model  $y_N = \Phi \theta$ , where  $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ , which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter  $\hat{\theta}(N+1)$  after N+1measurements?  $\theta(N+1) = \arg \min_{\theta} \frac{1}{2} (...)$  [a and c are correct!]

(a) $\ \theta - \hat{\theta}(N)\ _{2}^{2} + \ y(N+1) - \varphi(N+1)^{\top}\theta\ _{Q_{N}}^{2}$	(b) $\  y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$
(c) $\mathbf{x} \  \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(N) \ _{Q_N}^2 + \  \boldsymbol{y}(N+1) - \boldsymbol{\varphi}(N+1)^\top \boldsymbol{\theta} \ _2^2$	(d) $\ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$

14. Which one of the following statements is NOT true for FIR models:

(a) They are a special class of ARX models	(b) <b>x</b> The impulse response is constant.
(c) The output does not depend on previous outputs.	(d) Output error minimization is a convex problem.

15. You are given a pendulum which is by nature a nonlinear system and can be modeled by  $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$ , where y(t) are the measurements. Which of the following algorithms should you use to estimate the parameters  $\theta$ ?

(a) Weighted Least Squares (WLS)	(b) Recursive Least Squares (RLS)
(c) X Maximum a Posteriori Estimation (MAP)	(d) Linear Least Squares (LLS)

16. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP). Which of the following algorithms could you use to estimate the parameters  $\theta$  of this linear model without running into memory problems or high computational costs for a continuous and infinite flow of measurement data?

(a) ML	(b) LLS	(c) MAP	(d) <b>X</b> RLS
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17. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question  $\Sigma_{\hat{\theta}}$ . The model is given as  $y_N = \Phi_N \theta + \epsilon_N$  with  $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^\top \Phi_N$  and  $L(\theta, y_N)$  is the negative log likelihood function. The covariance matrix can be approximated by  $\Sigma_{\hat{\theta}} \approx \dots$ 

(a) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(b) $\mathbf{X} Q_N^{-1}$	(c) $\left[ \left( \Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	(d) $\square \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$
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18. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

	(a) $\mathbf{X}$ $L_1$ , Laplace	(b) $\Box L_1$ , Gaussian	(c) $\Box L_2$ , Laplace	(d) $\Box L_2$ , Gaussian
19.	19. Which of the following models with input $u(k)$ and output $y(k)$ is <b>NOT</b> linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$ ?			
	(a) $\[ y(k) = \theta_1 \sqrt{u(k)} \]$		(b) $x y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$	
	(c) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$		(d) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$	

20. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct? Plot the entries of the residual vector,  $R(\theta^*)_i$ , i = 1, ..., N, as a histogram and check if it looks like a zero-mean Gaussian.