Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2018

Exercise Sheet 5: Wind Turbine Control and Airborne Wind

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Deadline: midnight before July 18th, 2018
`https://goo.gl/forms/LEHDDEhJJg8OygL52`

In this exercise sheet we'll study one of the typical control laws for wind turbine operation, as well as take a look at drag-mode airborne wind energy systems.

Control **[*5 + 2 bonus pt*]**

1. In this problem, we want to control the torque on the electrical machine of the wind turbine, in order to find and track the optimal rotation speed. We'll do this using a very handy physical equality.

   The following information describes the wind turbine (Turbine C) we'll use in this problem, and may end up being useful.

Table 1: some useful values

property	symbol	value	units
turbine radius	R	40	m
air density	ρ	1.225	kg / m^3
rotor inertia	I	$8.6 \cdot 10^6$	kg m^2
blade pitch angle	β	0	deg

The ordinary differential equation (ODE) that describes the rotation speed control is:

$$I\dot{\Omega} = Q_{\mathrm{aero}}(\Omega, u_\infty) - Q_{\mathrm{m}}(\Omega).$$

Using the tip-speed ratio $\lambda = \Omega R/u_\infty$, these various functions are defined as:

$$Q_{\mathrm{aero}}(\Omega, u_\infty) = \frac{1}{2}\rho\pi R^3 \frac{C_{\mathrm{P}}(\lambda)}{\lambda} u_\infty^2, \quad (1)$$

$$Q_{\mathrm{m}}(\Omega) = \frac{1}{2}\rho\pi R^5 \frac{C_{\mathrm{P}}^*}{\lambda^{*3}} \Omega^2. \quad (2)$$

(a) First, consider the power coefficient $C_{\mathrm{P}}(\lambda, \beta)$, where λ is the tip-speed-ratio and β is the blade pitch angle in degrees. A typical form for this expression is:

$$C_{\mathrm{P}}(\lambda,\beta) = \max\left(c_1(c_2 h(\lambda,\beta) - c_3\beta - c_4\beta^{c_7} - c_5)\exp(-c_6 h(\lambda,\beta)), 0\right)$$

where:

$$h(\lambda,\beta) = \frac{1}{\lambda - 0.02} - \frac{0.003}{\beta^3 + 1}.$$

Here, we will use the constants: $c_1 = 0.73$, $c_2 = 151$, $c_3 = 0.58$, $c_4 = 0.002$, $c_5 = 13.2$, $c_6 = 18.4$, and $c_7 = 2.14$.

i. Please make a contour plot of the power coefficient vs the tip speed ratio $\lambda \in [1, 20]$ vs the blade pitch angle $\beta \in [0\,\mathrm{deg}, 50\,\mathrm{deg}]$. [*0.25 pt*]

ii. For $\beta = 0\,\mathrm{deg}$, at what tip speed ratio λ^* does the maximum value of C_{P} occur? [*0.5 pt*]

iii. What is the value of C_{P}^*, the maximum value of C_{P} when $\beta = 0\,\mathrm{deg}$? [*0.25 pt*]

(b) BONUS! Now, let's see where our control ODE came from!

i. BONUS! What is the aerodynamic torque on the turbine, depending on the freestream wind speed u_∞? [*bonus 0.25 pt*]

ii. BONUS! What would the optimal generator torque be? (*Hint: consider that we don't know the wind speed, but we do know the optimal power coefficient C_{P}^* and optimal tip speed ratio λ^**) [*bonus 0.75 pt*]

iii. BONUS! Please derive the control ODE. [*bonus 1 pt*]

(c) In your favorite programming language, write a function that will allow you to simulate Ω based on your ODE. [*2 pt*]

(d) Let's simulate!

Simulate Ω for 100 seconds, under the given conditions. For each set of conditions, make a plot of Ω vs time and λ vs time.

Note that here, $\mathrm{U}(\cdot)$ is the heaviside step function.

i. $\Omega(0) = 0.7$ [rad/s], $u_\infty(t) = 4$ [m/s] *[0.5 pt]*

ii. $\Omega(0) = 0.7$ [rad/s], $u_\infty(t) = 4 + \mathrm{U}(t - 5\mathrm{s})$ [m/s] *[0.5 pt]*

iii. $\Omega(0) = 1.7$ [rad/s], $u_\infty(t) = 10 + \mathrm{U}(t - 5\mathrm{s})$ [m/s] *[0.5 pt]*

(e) Estimate the rise time in your solution of 1(d)ii and 1(d)iii *[0.5 pt]*

Drag-mode Airbone Wind Energy System **[*11 pt*]**

2. In this problem, we will explore one type of airborne wind energy system. That is, we will concern ourselves with a system that is heavily inspired by the Makani M600 system, a large drag-mode system with a rated power of 600 kW. Here, we will try to figure out roughly where this number comes from.

We will assume that the tethers are straight and rigid, though they only are assumed to only support tensile loads and not compressive ones. We will assume a 3 degree of freedom kite, to which roll-control is applied. The kite - with span b is assumed to fly a uniform, circular flight path, of radius R and angular velocity Ω. The freestream wind field is assumed to be uniform, with speed u_∞. Describing this flight path are two nondimensional values, the flight path relative radius ϱ and the kite speed ratio λ:

$$\varrho := \frac{R}{b}, \qquad \lambda := \frac{\Omega R}{u_\infty}.$$

Some properties that might be useful to you can be found in the table below:

Table 2: some other useful information

property	symbol	value	units
kite span	b	28	m
kite aspect ratio	$\mathcal{R}$	20	-
kite mass	m_K	1050	kg
tether mass	m_T	250	kg
tether length	L	440	m
tether diameter	d_T	0.01	m
average elevation angle	θ	30	deg
flight path relative radius	ϱ	4.8	-
kite speed ratio	λ	5	-
air density	ρ	1.225	kg/m^3
freestream wind speed (assumed uniform in height)	u_∞	8	m/s
kite lift coefficient	C_L	1.2	-
kite drag coefficient	$C_{\mathrm{D,K}}$	0.04	-
tether drag coefficient	$C_{\mathrm{D,T}}$	1	-

We will here define three reference frames, as shown in figure 1. First, there is an earth fixed reference frame where $\hat{\boldsymbol{x}}$ points along the dominant wind direction, $\hat{\boldsymbol{y}}$ points across the wind window, and $\hat{\boldsymbol{z}}$ points upwards. Second, there is a tether reference frame where $\hat{\boldsymbol{e}}_1$ points upwards along the tether, $\hat{\boldsymbol{e}}_2$ points along $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{e}}_3$ points according to a right-handed coordinate system. Last, there is a rotating reference frame where $\hat{\boldsymbol{r}}$ points radially outwards from the center of a circular flight path to the kite, $\hat{\boldsymbol{t}}$ points tangential to the circular flight path in the direction of motion, and $\hat{\boldsymbol{e}}_1$ points as previously defined.

(a) First, let's try to define these coordinate frames, and then place the kite.

i. What are $\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{e}}_2$ and $\hat{\boldsymbol{e}}_3$ in terms of $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$? *[0.25 pt]*

ii. Given an azimuthal angle ψ as shown in figure 1 ($\psi = 0$ at the horizontal position when moving upwards), what are $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{t}}$? *[0.25 pt]*

iii. Use vector addition to find the position $\boldsymbol{x}_\mathrm{K}$ of the kite, as a function of ψ? *[0.25 pt]*

iv. What is the velocity $\dot{\boldsymbol{x}}_\mathrm{K}$ of the kite, as a function of ψ? *[0.25 pt]*

(b) Let's see what Loyd predicted the power output of this system to be...

i. What is the power harvesting factor ζ_Loyd that Loyd predicted for the AWE system? *[0.25 pt]*

ii. What is the 'best case' power output P_{Loyd} that Loyd predicts for the AWE system? (*Hint: remember that the aspect ratio* Æ *of a wing is defined as* $\text{Æ} := b/c_{\text{Ref}} = b^2/S_{\text{K}}$*, where* c_{Ref} *is the mean aerodynamic chord, and* S_{K} *is the planform area of the wing.*) [*0.25 pt*]

iii. How does the Loyd power output compare to the system specifications? What might be one way to improve the model? [*0.5 pt*]

(c) The kite is acted on, at any moment in time by a number of forces: a kite lift force $\boldsymbol{L}_{\text{K}}$, a kite drag force $\boldsymbol{D}_{\text{K}}$, a tether drag force $\boldsymbol{D}_{\text{T}}$, a centrifugal force $\boldsymbol{C}$, a gravitational force $\boldsymbol{G}$, a tether tension $\boldsymbol{T} = -\kappa \boldsymbol{x}_{\text{K}}$ and a propeller force $\boldsymbol{P} = -f \dot{\boldsymbol{x}}_{\text{K}}$. Here, both κ and f are scaling factors where $\kappa, f \in \mathbb{R}^+$.

Let's try to use these forces to refine our power model.

i. If the kite is in a uniform circular orbit, what expression relates all of the forces given above? [*1 pt*]

ii. What is the gravitational force $\boldsymbol{G}$, in this situation? [*0.5 pt*]

iii. What is the centrifugal force $\boldsymbol{C}$ in this situation? [*0.5 pt*]

iv. Let's concern outselves with the kite drag $\boldsymbol{D}_{\text{K}}$.

A. What is the apparent velocity $\boldsymbol{u}_{\text{a}}$ of the kite? [*0.25 pt*]

B. What is the apparent dynamic pressure q_{a} of the kite? [*0.25 pt*]

C. Along what unit vector $\hat{\boldsymbol{d}}$ will the drag point? [*0.25 pt*]

D. What is the kite drag force $\boldsymbol{D}_{\text{K}}$ in this situation? [*0.25 pt*]

v. Let's try to figure out the tether drag $\boldsymbol{D}_{\text{T}}$, making some assumptions. First: assume that lambda is large enough that the apparent velocity of the tether $\boldsymbol{u}_{\text{a,T}}$ is a linear relationship with the position along the tether. That is, at the groundstation where $s = 0$, and at the top of the tether where $s = L$:

$$\boldsymbol{u}_{\text{a,T}}(s = 0) = 0, \qquad \boldsymbol{u}_{\text{a,T}}(s = L) = -\dot{\boldsymbol{x}}_{\text{K}}.$$

Second, we assume that the tether drag acts in the same direction as the kite drag, $\hat{\boldsymbol{d}}$.

A. What is the apparent velocity $\boldsymbol{u}_{\text{a,T}}$ of the tether, at a given position s? [*0.25 pt*]

B. What is the apparent dynamic pressure $q_{\text{a,T}}$ of the tether, at a given position s? [*0.25 pt*]

C. What is the magnitude of the total tether drag $D_{\Sigma,\text{T}}$ acting over the entire tether? [*0.25 pt*]

D. What is the magnitude of the moment $Q_{\Sigma,\text{T}}$ at the groundstation due (only) to the tether drag, again assuming that the tether behaves like a rigid rod? [*0.25 pt*]

E. Let's convert this total tether drag information into two 'equivalent' tether drag forces, one per endpoint of the tether. Assume both of these equivalent forces act in the $\hat{\boldsymbol{d}}$ direction. Let's call the force acting on the top endpoint $\boldsymbol{A}$ and the force on the groundstation endpoint $\boldsymbol{B}$, with respective magnitudes A and B.

What is the relationship between A, B, and $D_{\Sigma,\text{T}}$? [*0.25 pt*]

F. What is the relationship between A, B, and $Q_{\Sigma,\text{T}}$? [*0.25 pt*]

G. Please find A and B. [*0.5 pt*]

H. What is the tether drag force $\boldsymbol{D}_{\text{T}}$ in this situation? (*Hint: as relevant to the kite...*) [*0.25 pt*]

vi. Let's try to find the lift force.

A. Let's assume that the kite is under perfect roll control. Then, the spanwise direction on the kite $\hat{\boldsymbol{b}} = \frac{\hat{\boldsymbol{r}} + \gamma \hat{\boldsymbol{e}}_1}{||\hat{\boldsymbol{r}} + \gamma \hat{\boldsymbol{e}}_1||_2}$, depending on the value of $\gamma \in \mathbb{R}$. Then, along what unit vector $\hat{\boldsymbol{l}}$ will the lift point? [*0.5 pt*]

B. What is the kite lift force $\boldsymbol{L}_{\text{K}}$ in this situation? [*0.25 pt*]

vii. You happen to gain some information about the azimuthal variation in both γ and κ, as relevant to the M600 system:

$$\gamma(\psi) \approx 0.003 - 0.062 \cos\psi + 0.003 \cos 2\psi - 0.223 \sin\psi + 0.024 \sin 2\psi \ [-], \qquad \kappa(\psi) \approx 94 + 21 \cos\psi \ [\text{N/m}]$$

Please use this information, and the expression from 2(c)i to find a (plausible) approximation for $f(\psi)$ along the form $f(\psi) = f_0 + f_1 \cos\psi$.

(*Hint: you only have one unknown, but you likely have more than one distinct expression. On the other hand, due to the approximation above, each of these distinct expressions is unreliable in a region around its singularities.*) [*1 pt*]

(d) Let's put this information together:

i. How much power $P(\psi)$ does our model suggest the M600 produces, as a function of ψ? [*0.5 pt*]

ii. What is the average of this power $\overline{P}$ over the cycle? [*0.25 pt*]

iii. Can you interpret the sign of $P(\psi)$ from a physical perspective? [*0.25 pt*]

iv. What is the average power coefficient $\overline{C_{\mathrm{P}}}$ based on this model, over one full circular trajectory? [*0.25 pt*]

v. What is the average power harvesting factor $\overline{\zeta}$ based on this model, over one full circular trajectory? [*0.25 pt*]

vi. How can you interpret these two numbers? [*0.5 pt*]

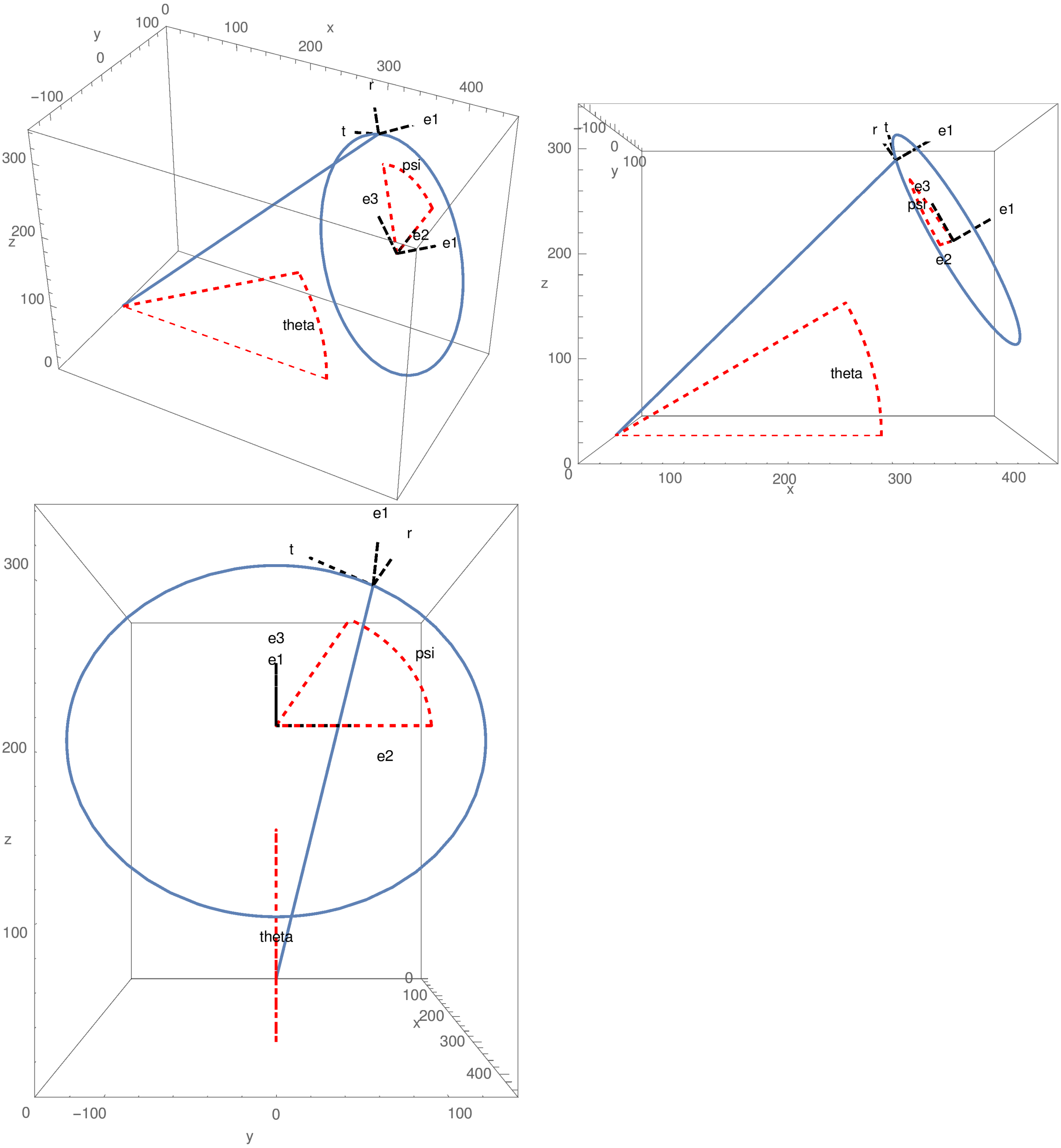

Figure 1: geometry sketch