Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2018

Exercise Sheet 3 SOLUTION: Mechanics for Wind Turbine

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https://goo.gl/forms/0QFRvjHgSHC7FOwH2

In this exercise sheet we'll explore the role of deflections and vibrations in wind turbine design, focusing on the blades and the tower. To accomplish this exploration, we will play briefly with simple Euler-Bernoulli beam theory, the Rayleigh energy method, and the Campbell diagram.

blade deflection **[*5 pt*]**

1. In this problem, we would like to explore the blade deflection. Let's assume that the blade is approximately straight so that it lays more-or-less in the rotor tip plane, even when deflected.

   We will (only if explicitly stated!) use the same three-bladed demonstration turbine, 'Turbine A' as used by exercise sheet 2. Turbine A is defined by the following parameters: the rotor radius $R = 50$m, and blades of constant chord $c = 5$m and constant profile shape. Turbine A is running in a freestream wind of $u_\infty = 12$m/s with air density $\rho = 1.225\text{kg/m}^3$. Remember, that $\mu = r/R$ is our non-dimensional radial location along the blade.

   You're encouraged to use the thrust distribution over the blade from the BEM problem of exercise sheet 2, but the following approximation can be used if necessary:
   $$\mathrm{d}T(\mu) \approx 30\mu^2(4-\mu)q_\infty R\mathrm{d}\mu$$

   (a) For a solid symmetric airfoil, we can[1] approximate the blade's second moment of area as $I_x \approx K_\mathrm{I} c^4 \tau^3$, where K_I approx 0.036 and $\tau = t_\mathrm{max}/c$ is the maximum airfoil thickness to chord length ratio.

      i. Let's assume that the airfoil nondimensional thickness τ =24 percent. What is the maximum thickness t_max of the airfoil? [*0.25 pt*]

      We know that $t_\mathrm{max} = \tau c = (0.24)(5)\text{m} = 1.2\text{m}$.

      ii. What is the second moment of area I_x in terms of the defined parameters? [*0.25 pt*]

      From the above expression:
      $$I_x = K_\mathrm{I} c^4 \tau^3 = 0.3\text{m}^4$$

      iii. As mentioned, this approximation assumes a solid airfoil. Do you think a more accurate approximation of I_x would be larger or smaller than this 'solid' approximation? [*0.25 pt*]

[1] https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/systems-labs-06/spl10b.pdf

The second moment of area I_x is the integral over the cross-section of the squared distance from the x-axis (y):

$$I_x := \int\int_A y^2 dxdy$$

where A is the area of the cross-section, aka the domain of the integral.

The important thing is to realize that - because we're integrating over areas - we can divide the total integral into two (or more) sub-integrals so that the domain of all of the sub-integrals together makes the full domain A. Notice, that because y^2 is always positive, this summation always consists of positive components.

So, the I_x corresponding to a solid airfoil would be equal to the sum of two second moments of area corresponding to (1) the cross-sectional area of the airfoil as it actually is, and (2) the cross-sectional area that is either filled with 'filler' material like polystyrene or 'empty'.

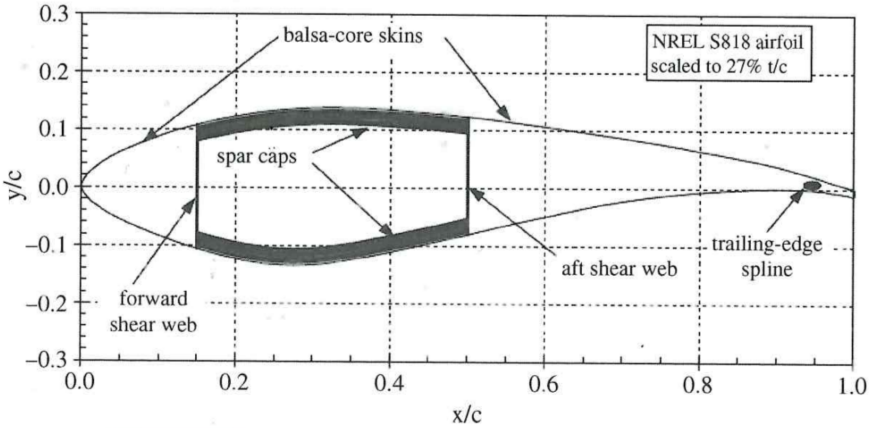


figure from Burton et al, *Wind Energy Explained*, pg. 280, Wiley: 2010, West Sussex, UK.

Since, (1) is typically less than the full airoil (the structural elements of the blade consist of a box beam of two spar caps and two shear webs and a skin) - see the above figure - a 'more realistic' airfoil structural model would have a smaller I_x than the solid approximation of I_x.

(b) We might make the assumption that the blade behaves as a slender beam. In that case, the downwind-direction blade deflection x could be found with Euler-Bernoulli beam theory from the distributed load q, the Young's modulus E and the cross-section second moment of area I_x:

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(EI_x \frac{\mathrm{d}^2 x}{\mathrm{d}r^2}\right) = q$$

There are some boundary conditions to this integral:

$$x(0) = 0; \quad x'(0) = 0; \quad x''(R) = 0; \quad x'''(R) = 0$$

Briefly, what do these boundary conditions mean? [*0.25 pt*]

The B.C. $x(0) = 0$ means that there is no deflection at the blade root because the blade root is pinned to the nacelle;

$x'(0) = 0$ means that the cantilevered beam lays perpendicular to the boundary;

$x''(R) = 0$ says that there is no bending at the tip of the blade;

and $x'''(R) = 0$ says that there is no shear force at the tip of the blade.

(c) What is the relationship between the downwind-direction blade deflection at the tip and the rotor radius R? [*1 pt*]

If we integrate the differential equation four times, including constants of integration, we get:

$$x(r) = \frac{125r^2\left(14R^2(c_0 r + 3c_1) + q_\infty r^4(28R - 3r)\right)}{378cER^2t^3} + c_2 r + c_3$$

Then, we can differentiate three times and plug in the boundary conditions above.

$$x'''(r) = \frac{125\left(2c_0R^2 + 5q_\infty r^3(16R - 3r)\right)}{9cER^2t^3} \Rightarrow x'''(R) = \frac{125\left(2c_0 + 65q_\infty R^2\right)}{9cEt^3} = 0 \Rightarrow c_0 = -\frac{65q_\infty R^2}{2}$$

Plugging this in to the second derivative gives:

$$x''(r) = \frac{125\left(2c_1R^2 + q_\infty\left(-3r^5 + 20r^4R - 65rR^4\right)\right)}{9cER^2t^3} \Rightarrow x''(R) = \frac{250\left(c_1 - 24q_\infty R^3\right)}{9cEt^3} = 0 \Rightarrow c_1 = 24q_\infty R^3$$

Again,

$$x'(r) = c_2 - \frac{125q_\infty r\left(r^5 - 8r^4R + 65rR^4 - 96R^5\right)}{18cER^2t^3} \Rightarrow x'(0) = c_2 = 0 \Rightarrow c_2 = 0$$

Again,

$$x(r) = c_3 + \frac{125q_\infty r^2\left(-3r^5 + 28r^4R - 455rR^4 + 1008R^5\right)}{378cER^2t^3} \Rightarrow x(0) = c_3 = 0 \Rightarrow c_3 = 0$$

So, we've found the deflection relationship as:

$$x(r) = \frac{125q_\infty r^2\left(-3r^5 + 28r^4R - 455rR^4 + 1008R^5\right)}{378cER^2t^3}$$

(d) For 'Turbine A', what is the ratio between the tip blade deflection and the rotor radius, if the blade is made of the following materials?

i. carbon-fiber composite ($E \approx 150$GPa), [*0.25 pt*]

At the tip, $r = R$. We can also plug in our given Turbine A parameters, to get:

$$x_{\text{carbon}} = 4\text{m}$$

ii. fiberglass aka. glass-reinforced plastic ($E \approx 17$GPa), [*0.25 pt*]

At the tip, $r = R$. We can also plug in our given Turbine A parameters, to get:

$$x_{\text{GRP}} = 36\text{m}$$

iii. polystyrene ($E \approx 3$GPa)? [*0.25 pt*]

At the tip, $r = R$. We can also plug in our given Turbine A parameters, to get:

$$x_{\text{polystyrene}} = 203\text{m}$$

(e) What trade-offs might be relevant when selecting blade material? [*0.75 pt*]

There are certain features of blade materials that we are likely to care about:

- how much a material costs over the total amount of that material that is needed to make a 'safe' design.

- how much the material weighs over the total amount of the material that is needed to make a 'safe' design.

- how easily the material can be manufactured into the 'safe' design.

- and further lifetime concerns, such as succeptibility (of the material and design) to fatigure, etc.

(f) The fact that the blades are rotating will likley lead to a smaller deflection than predicted here. Briefly, why would that be? What is this phenomenon called? *[0.5 pt]*

This phenomenon is called 'centrifugal stiffening.' It occurs because the centrifugal force of the rotation will act to pull the blade flat into the plane of rotation.

(g) Qualitatively, what happens to the blade loading under the following conditions?

i. yawed flow *[0.25 pt]*

When the flow is symmetric, the apparent velocity is equal at all azimuthal angles. But, when the flow is asymmetric (as in the case of yaw), then as the blade travels along the azimuth, the flow will occasionally have a component that moves with the blade and occasionally a component that moves opposed to the blade.

When the flow moves opposed to the blade, the apparent velocity will be higher. This leads to higher aerodynamic forces and higher bending moments at the blade root.

When the flow moves with the blade, this happens in reverse, and the blade experiences a lower bending moment.

When the flow is yawed, then these parts of the azimuth with varying moments will be at the top and bottom pass of the blade (defined respectively as 0^o and 180^o azimuthal angle.) Which side has the higher and which side the lower apparent velocity depends on the direction of yaw, but it is typical to define yaw angles as positive when they put the high force at 180^o (or bottom dead center.)

(For completeness: there is an induction effect that tends to shift this pattern to higher azimuthal angles, but it is mainly relevant at low wind speeds where the induction factors are high.)

ii. shaft tilt *[0.25 pt]*

When the rotor is tilted, we have the same behavior as with yaw, except shifted by 90^o. Again, because there is a component of the wind that lays in the blade-tip-plane. This component will cause an increase in apparent velocity when the blade moves downwards (azimuthal angle $\psi = 90^o$) and a decrease with the blade moves upwards ($\psi = 270^o$).

iii. wind shear *[0.25 pt]*

As you saw with the logarithmic wind profile, we expect that wind speeds will increase with height. This means that apparent velocities, forces and bending moments, will all vary sinusoidally with the blade's azimuthal angle, having a maximum when $\psi = 0\,\text{deg}$ and a minimum when $\psi = 180\,\text{deg}$.

iv. tower shadow *[0.25 pt]*

Further, the tower acts as an obstacle to the flow. This means that the flow immediately ahead of the tower will be slowed, and behind the tower will be even slower and turbulent.

Then, a blade passing through $\psi = 180\,\text{deg}$ will see less wind velocity, consequently less force, and less bending moment.

preliminary tower design ***[10 pt]***

2. We would like to make a preliminary design of a wind turbine tower. This tower should support an un-yawed and un-tilted three-bladed wind turbine ('Turbine B'), with the following dimensions:

Some other information that you might find useful is as follows:

(a) **rotor thrust** ***[1 pt]***

Table 1: wind turbine dimensions and properties for Turbine B

property	symbol	value
tower height	L	84 m
nacelle + hub mass	m_{nac}	143 tonnes
rotor radius	R	12 m
design tip speed ratio	λ_{rated}	5
cut-in wind speed	$u_{\mathrm{cut-in}}$	3 m/s
rated wind speed	u_{rated}	12 m/s
cut-out wind speed	$u_{\mathrm{cut-out}}$	25 m/s

Table 2: other potentially useful information

property	symbol	value
density of A36 structural steel	ρ_{steel}	$7.8 \cdot 10^3$ kg/m^3
Young's modulus of A36 structural steel	E_{steel}	200 GPa
yield stress of A36 structural steel	U_{steel}	250 MPa
air density	ρ_{air}	1.225 kg/m^3
surface roughness length for low crops w. occasional obstacles	z_0	0.1 m
meterological mast height	z_{Ref}	10 m
approx. drag coefficient for cylinder	C_{D}	1
typical wind turbine structural safety factor	f_{safety}	1.35

i. What is the design angular velocity Ω_{rated} of the wind turbine? [*0.25 pt*]

The design angular velocity Ω_{rated} can be found from the design tip speed ratio λ_{rated}, the rated wind speed u_{rated} and the rotor radius R. (All of these parameters are given in Table 1.)

$$\Omega_{\mathrm{rated}} = \frac{\lambda_{\mathrm{rated}} u_{\mathrm{rated}}}{R}.$$

ii. Suppose that the angular velocity $\Omega(u_\infty)$ of the wind turbine is piecewise linear with the free-stream velocity at hub height u_∞. That is: $\Omega(u \leq u_{\mathrm{cut-in}}) = 0\mathrm{rad/s}$, $\Omega(u_{\mathrm{rated}}) = \Omega_{\mathrm{rated}}$, $\Omega(u_{\mathrm{cut-out}}) = \Omega_{\mathrm{rated}}$, and $\Omega(u > u_{\mathrm{cut-out}}) = 0\mathrm{rad/s}$.

Considering a logarithmic wind profile, where a met. mast of height z_{Ref} measures a reference wind speed of u_{Ref}, above a landscape of low crops with occasional larger obstacles, What is the angular velocity $\Omega(u_{\mathrm{Ref}})$ as a function of the reference wind speed? [*0.25 pt*]

Let's define u_∞ as the wind speed at hubheight, assuming that the rotor aligns with the dominant wind direction. From the logarithmic wind profile, we know that:

$$u_\infty = u_{\mathrm{Ref}} \frac{\log \frac{z}{z_0}}{\log \frac{z_{\mathrm{Ref}}}{z_0}}$$

Then, we can write the piecewise linear angular velocity function:

$$\Omega(u_\infty) = \frac{u_\infty - u_{\mathrm{cut-in}}}{u_{\mathrm{rated}} - u_{\mathrm{cut-in}}} \Omega_{\mathrm{rated}} \left(\mathrm{U}\left(u_\infty - u_{\mathrm{cut-in}}\right) - \mathrm{U}\left(u_\infty - u_{\mathrm{rated}}\right)\right) + \Omega_{\mathrm{rated}} \left(\mathrm{U}\left(u_\infty - u_{\mathrm{rated}}\right) - \mathrm{U}\left(u_\infty - u_{\mathrm{cut-out}}\right)\right)$$

Here, $\mathrm{U}(\cdot)$ is the Heaviside step function.

When we combine these two expressions, we have $\Omega(u_{\mathrm{Ref}})$.

iii. You happen to learn that the thrust coefficient C_{T} of this wind turbine can roughly be approximated with the following function:

$$C_{\mathrm{T}}(\lambda) \approx \frac{0.8}{\pi} \left(\arctan\left(0.5\lambda - 2\right)\right) + 0.3$$

.

What is the relationship between C_{T} and u_{Ref}? [*0.25 pt*]

We know the tip speed ratio $\lambda = \frac{\Omega(u_{\text{Ref}})R}{u_\infty(u_{\text{Ref}})}$.

Then, we can plug that tip speed ratio into the C_{T} expression above to give:

$$C_{\text{T}}(u_{\text{Ref}}) \approx \frac{0.8}{\pi}\left(\arctan\left(0.5\lambda\left(u_{\text{Ref}}\right)-2\right)\right)+0.3$$

.

iv. What is the magnitude of the thrust force F on the rotor as a function of u_{Ref}? [*0.25 pt*]

The thrust force F can be found based on the thrust coefficient:

$$F(u_{\text{Ref}}) = C_{\text{T}}(u_{\text{Ref}})\left(\frac{1}{2}\rho u_\infty(u_{\text{Ref}})^2\right)(\pi R^2)$$

where: $u_\infty = u_{\text{Ref}}\frac{\log(L/z_0)}{\log z_{\text{Ref}}/z_0}$.

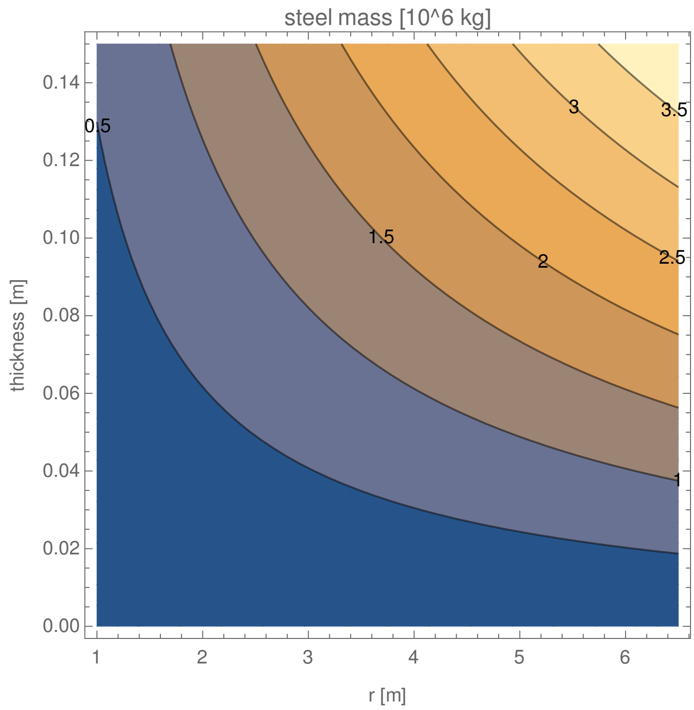
(b) **tower bending stress** **[*3 pt*]**

Let's consider the tower as a simple cantilevered beam, where an aerodynamic drag force is acting continuously along the tower length, and the rotor thrust is acting at the top of the tower.

Let's assume that the tower is a thin walled tube with a constant cross-section along its length. This constant cross-section is an annulus, with an outer radius of r and a thickness τ.

i. Make a contour plot of the total mass of steel in the tower, based on $r \in [1\text{m}, 6\text{m}]$ and $\tau \in [0\text{m}, 0.15\text{m}]$. [*0.25 pt*]

The steel mass is $m_{\text{steel}} = \rho_{\text{steel}} L\pi(r^2 - (r-\tau)^2)$:



ii. You happen to know that the second moment of area of a filled circlular area with radius a is $\pi a^4/4$. What is the second moment of area I_x of the tower cross-section? [*0.25 pt*]

The tower will be bent along one of the symmetric axes of the annulus. Since the second moment of area is an integral over the area, we can construct I_{annulus} from I_{circle}. That is, the second moment of area:

$$I_x = I_{\text{outercircle}} - I_{\text{innercircle}} = \frac{\pi}{4}\left(r^4 - (r-\tau)^4\right)$$

iii. What is the distance d between the beam's neutral axis and the outer radius? [*0.25 pt*]

The beam's neutral axis is the centroid of the beam's cross-section, which - since an annulus is symmetric - is the center of the annulus. Then, this distance is just r.

iv. Considering the logarithmic wind profile, what is the bending moment of the tower at the ground due only to the drag along the tower length M_{D}? [*0.25 pt*]

We know that the force per unit length D' is:

$$D' = C_{\mathrm{D}}(2r)q_{\infty}(z)$$

where $q_{\infty} = \frac{1}{2}\rho_{\mathrm{air}}u_{\infty}(z, u_{\mathrm{Ref}})^2$, defining $u_{\infty}(z, u_{\mathrm{Ref}}) = u_{\mathrm{Ref}}\frac{\log(z/z_0)}{\log z_{\mathrm{Ref}}/z_0}$.

Then, the bending moment per unit length M'_{D} is:

$$M'_{\mathrm{D}} = D'z$$

We can integrate this over the tower length to get the total bending moment due to the drag:

$$M_{\mathrm{D}} = \int_0^L M'_{\mathrm{D}} dz \approx \left[0.06ru_{\mathrm{Ref}}^2\left(1.75z^2 + 0.5z^2\log^2(z) + 1.80z^2\log(z)\right)\right]_0^L \approx 8\cdot 10^3 ru_{\mathrm{Ref}}^2$$

v. What is the bending moment of the tower at the ground due only to the thrust on the rotor M_{T}? [*0.25 pt*]

We know the bending moment due to the rotor thrust is:

$$M_{\mathrm{T}} = LF(u_{\mathrm{Ref}}) = LC_{\mathrm{T}}(u_{\mathrm{Ref}})\left(\frac{1}{2}\rho u_{\infty}(L, u_{\mathrm{Ref}})^2\right)(\pi R^2).$$

vi. What is the total bending moment of the tower at the ground M? [*0.25 pt*]

Then, the total bending moment of the tower at the ground is the sum of the contributing bending moments:

$$M = M_{\mathrm{T}} + M_{\mathrm{D}}$$

vii. What is the maximum stress $\sigma_{\max}$ due to bending on the tower? [*0.25 pt*]

The bending stress can be found from the bending moment, the distance to the neutral axis, and the moment of inertia:

$$\sigma_{\max} = \frac{Mr}{I_x}$$

viii. Considering the safety factor f_{safety}, please devise a ratio ϕ which indicates whether the tower can safely support the maximum bending stress. Let's define $\phi < 1$ as safe, and $\phi > 1$ as unsafe. [*0.25 pt*]

Let's define this 'safe' ratio ϕ as:

$$\phi = \frac{f_{\mathrm{safety}}\sigma_{\max}}{U_{\mathrm{steel}}}$$

Notice that the safety factor has to 'inflate' the actually increased stress, because it is defined positive.

ix. Over all reference velocities u_{Ref} that the wind turbine is likley to experience in its lifetime, when will the bending stress criteria be the strictest? [*0.25 pt*]

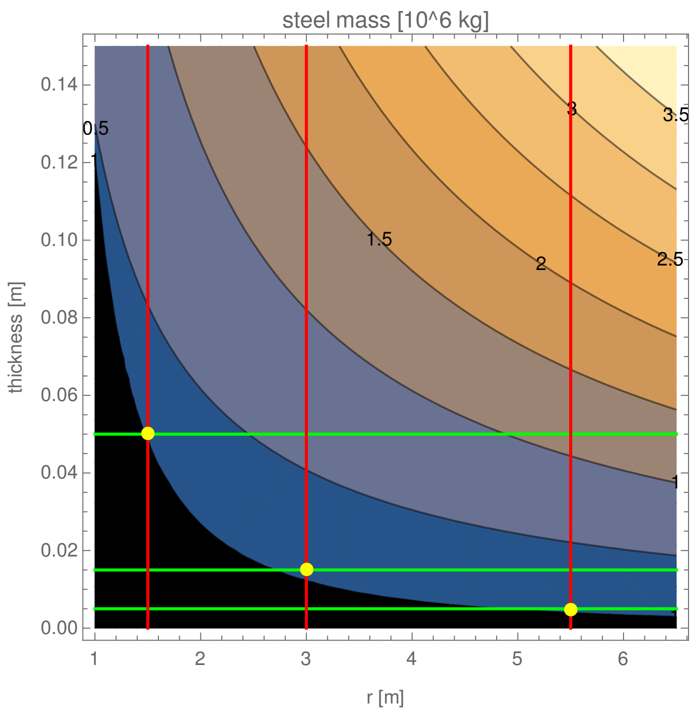
The drag on the tower will increase for all wind speeds, but the thrust on the rotor will drop sharply after the cut-out wind speed. So, we would expect the bending stress to be greatest 'just' before the cut-out speed. We can find the u_{Ref} at which the cut-out wind speed occurs at hub-height:

$$u_{\text{Ref}} \frac{\log \frac{L}{z_0}}{\log \frac{z_{\text{Ref}}}{z_0}} - u_{\text{cut-out}} = 0, \quad \Rightarrow \quad u_{\text{Ref,cut-out}} \approx 17.09\text{m/s}$$

x. For the following proposed tower outer diameters r, what thickness τ would you propose? Please motivate your choices. Also, please round thicknesses to the nearest 5mm.

What we can do, for the following diameters, is to plot the line of $\phi = 1$ on a plot of thickness vs. tower radius. We know that ϕ has to be smaller than one for the tower to be 'safe', so we can shade out the region of the plot where $\phi > 1$. (Here, I've done that over the same steel mass plot from before.

Then, to avoid over-designing the system (which would be expensive), we might try to chose the smallest allowed thickness for a 'safe' design. The intersections between thickness (green) and radius (red) give our design points, in the following plot...



Notice that we're not allowed to round our thicknesses 'down' because then ϕ becomes 'unsafe'. We can only round the thicknesses 'up' to the nearest 5mm. (I acknowledge that this is a fairly arbitrary number, but - in real life - sheet metal cannot be ordered in continuous thicknesses, but only in discrete units of thickness.)

A. $r = 5.5$ m [*0.25 pt*]

From the plot, we find a thickness $t = 0.005$m.

B. $r = 3.0$ m [*0.25 pt*]

From the plot, we find a thickness $t = 0.015$m.

C. $r = 1.5$ m [*0.25 pt*]

From the plot, we find a thickness $t = 0.05$m.

(c) **tower natural frequency** **[*3 pt*]**

Let's use Rayleigh's energy method to estimate the natural frequency of the tower. In this method, we assume that the strain energy from bending perfectly trades off with the kinetic energy of the tower's displacement x. We will again approximate the tower as a cantilevered beam.

Let's assume that the tower's displacement is sinusoidal in time:

$$x(t) = x_0 \sin(\omega t)$$

and that the tower remains approximately straight during its displacement.

Futher, we know that the strain energy from bending can be found as:

$$V = \frac{1}{2}kx^2, \text{ where } k = 3\frac{E_{\text{steel}}I_x}{L^3}.$$

i. What is $\dot{x}(t)$? [*0.25 pt*]

ii. What is the kinetic energy due to the nacelle displacement T_{nac}? [*0.25 pt*]

iii. What is the kinetic energy due to the displacement of the tower T_{t}? (*Hint: the tower is not massless...*) (*Hint: also, you might assume that the deflection of the tower is roughly proportional to the distance to the fixed point.*) [*0.5 pt*]

iv. What is the total kinetic energy T of the swaying cantilevered beam? [*0.25 pt*]

v. What equation can you formulate, that would implicitly define the vibration frequency ω? [*0.5 pt*]

vi. Please find ω. [*0.25 pt*]

vii. What is the natural frequency f_{nat} of the cantilevered tower? [*0.25 pt*]

viii. What is the natural frequency of each of the three potential tower designs (defined by r and τ) that you determined in (2(b)x)? (*Hint: If you do not have a solution to (2(b)x), you can use the following combinations of* (r,τ)*:* $(1.5m, 0.05m)$, $(3.0m, 0.02m)$, $(5.5m, 0.01m)$.)

A. $r = 5.5$ m [*0.25 pt*]

B. $r = 3.0$ m [*0.25 pt*]

C. $r = 1.5$ m [*0.25 pt*]

(d) **Campbell diagram** [***3 pt***]

i. With what frequency (1P, 2P, 3P, ...) would you expect the tower to experience the following effects? What is this frequency (in Hertz), as a function of the wind turbine's rotor speed (in RPM)?

A. 'rotor-rotation' effects, such as having unequally dirty blades? [*0.25 pt*]

B. 'blade-passing' effects, such as tower shadow? [*0.25 pt*]

ii. Make a plot of frequency [Hz] vs rotor speed [RPM] that we will call the Campbell plot. Add the 'rotor-rotation' and 'blade-passing' frequencies into the Campbell plot. Include a 15 percent safety margin to either side of each curve. [*0.25 pt*]

iii. What is the design rotor speed (in RPM) of the wind turbine? [*0.25 pt*]

iv. Please show the design rotor speed in the Campbell plot. [*0.25 pt*]

v. Please add the tower natural frequencies corresponding to your three possible tower designs (one for each of the outer diameters 5.5m, 3.0m and 1.5m) into the Campbell plot. [*0.25 pt*]

vi. Which of the three investigated tower designs (outer diameters 5.5m, 3.0m, 1.5m) can be classified as the following? Please explain briefly.

A. soft-soft [*0.25 pt*]

B. soft-stiff [*0.25 pt*]

C. stiff-stiff [*0.25 pt*]

vii. Suggest some considerations you might have when chosing between your three proposed tower designs? [*1 pt*]