Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2018

Exercise Sheet 1: The Power Harvesting Factor, Wind Resource, and Momentum Theory

Prof. Dr. Moritz Diehl und Rachel Leuthold

Deadline: midnight before May 16th, 2018
`https://goo.gl/forms/wjVjknhrsrUl5PyO2`

In this exercise sheet we want to explore a concept mentioned during class - the power harvesting factor. Next, we want to take a quick look at the wind resource. Further, we want to learn about the classic momentum theory for actuator disks. In this exercise sheet, we want to see where this theory comes from, and what it means.

Power harvesting factors along a blade **[*10 pt*]**

1. Let's explore the power harvesting factors along the blades of a turbine.

   Consider a symmetrical, three-bladed ($B = 3$) wind turbine with rotor radius R. Assume a constant angular velocity Ω of the rotor and a uniform wind field with velocity $\boldsymbol{u}_\infty$ so that the dominant wind direction $\hat{\boldsymbol{x}}$ is along the turbine axis or rotation. We will also use a nondimensional spanwise position $\mu = r/R$ that is 0 at the blade root/rotor hub, and 1 at the blade tips.

   (a) What is the tip speed ratio λ of the turbine? [*0.5 pt*]

   (b) What is the local speed ratio λ_r at some spanwise location μ? [*0.5 pt*]

   (c) What is the effective wind (also called apparent velocity) $\boldsymbol{u}_\mathrm{a}$ at the position μ? [*0.5 pt*]

   (d) Sketch the velocity triangles for the following positions:

      i. $\mu = 0.1$ [*0.5 pt*]

      ii. $\mu = 0.9$ [*0.5 pt*]

   (e) You've heard that the lift c_ℓ and drag c_d coefficients are related to the angle of attack α. Assume that the blades are uniformly pitched with an angle ϕ, but have a 'perfect' twist distribution $\theta(\mu)$ so that α always takes its design value of 6 degrees. What is $\theta(\mu)$? [*0.5 pt*]

   (f) If the blade were unpitched and untwisted, where (on the blade) would the angle of attack be greatest? Smallest? [*0.5 pt*]

   (g) For arbitrary lift c_ℓ and drag c_d coefficients, what is the aerodynamic force $\mathrm{d}\boldsymbol{F}_\mathrm{aero}$ for an infinitesimal segment of area $\mathrm{d}A$ around a position μ? Assume that the blades point straight, radially outwards. [*1 pt*]

   (h) What is the mechanical power production $\mathrm{d}P(\mu)$ of that segment around position μ? [*1 pt*]

   (i) If the lift c_ℓ and drag c_d coefficients can be found with the following relations, what is the power harvested by the blade segment around position μ? [*0.25 pt*]

   $$c_\ell(\mu) = 1.2\mu, \qquad \frac{c_\ell}{c_\mathrm{d}}(\mu) = 100\mu$$

   (j) What is the relationship between the power harvesting factor ζ and μ? [*1 pt*]

   (k) The chord c of the blade is a smooth function of the spanwise distance μ: $c = c(\mu)$. What is a reasonable approximation of the blade area $\mathrm{d}A(\mu)$ between two points μ_- and μ^+ that are infinitessimally close? [*0.5 pt*]

   (l) What is $\mathrm{d}A(\mu)$ if the chord is a linear interpolation between the chord c_1 at the tip and the chord c_0 at the root? [*0.25 pt*]

   (m) How would you go about finding the total power P harvested by the entire turbine? (*Hint: just give the procedure; don't follow it yet.*) [*0.5 pt*]

   (n) How would you go about finding the power coefficient C_P of the entire turbine? [*0.5 pt*]

   (o) If we use the above model that we've described to this point, for some given parameter values ($\lambda = 7$, rad, $c_0 = 0.15R$, $c_1 = 0.05R$, $u_\infty = 10$ m/s, $\rho = 1.225$ kg/m^3, $R = 50$ m and $B = 3$), can you...

      i. plot the power harvesting factor ζ vs. μ? [*0.25 pt*]

      ii. find how much power the full turbine will extract? [*0.25 pt*]

   (p) We should then mention that the dimensions of this exercise-turbine were approximated from the Enercon E-101, a turbine rated for 3 MW. Brainstorm some possible reasons why the estimated power P is so different... [*1 pt*]

The Wind Resource **[*10 pt + 3 bonus pt*]**

2. Let's study the wind resource!

   (a) First, let's consider some real, local wind speeds, to get an idea of its probability distribution.

      i. To do this, download the hourly windspeed data at the weather station 'Feldberg/Schwartzwald' and take a moment to familiarize yourself with the data:
      `https://www.dwd.de/DE/leistungen/klimadatendeutschland/klarchivstunden.html`. [*0.25 pt*]

      ii. Extract the wind speed measurements from the dataset, and remove the entries without valid datapoints. [*0.25 pt*]

      iii. Where were the wind measurements taken: At what altitude above ground level z? At what latitude λ? [*0.5 pt*]

      iv. Consider some basic statistical properties of the wind speed dataset. What are:

         A. the average wind speed $\overline{U}$? [*0.25 pt*]

         B. the standard deviation σ of the wind speed measurements? [*0.25 pt*]

      v. Plot a histogram of the wind speeds, normalized so as to represent the probability density function. [*0.25 pt*]

      vi. If you want to quickly get an idea of the mean wind speed corresponding to a particular Weibull distribution, which of the two parameters (shape k or scale c) should you look at? [*0.25 pt*]

      vii. Estimate the Rayleigh distribution for this dataset, and add a plot of that Rayleigh distribution to your histogram. [*1 pt*]

      viii. By comparing (visually) the histogram and Rayleigh distribution, do you think that the wind at this site is 'highly variable', 'somewhat variable' or 'not variable?' Please explain. [*1 pt*]

      ix. Where is the probability density suggested by this plot likely to be accurate, and where not? Please explain. [*1 pt*]

      x. Because wind power grows with the cube of the wind speed, the average of cubed wind speeds is important for wind turbine siting decisions.

         A. Compute the average $\overline{U^3}$ of U_i^3, and the cubed average speed $\overline{U}^3$. Which number is higher? [*0.5 pt*]

         B. Would this relationship between $\overline{U^3}$ and $\overline{U}^3$ be the case for any arbitrary time series, i.e., is there a mathematical reason for it? [*1 pt*]

         C. **Bonus!** Assume you lost the time series data, but you still have available the average wind speed $\overline{U}$ and the standard deviation σ. Can you make an informed estimate of $\overline{U^3}$ [*1 bonus pt*]

   (b) Based on a long wind measurement campaign at a prospective site for a medium scale wind turbine with 60m hub height, you have average wind speed values available at altitudes of 30m and of 60m, namely $u_{30} = 6$m/s and $u_{60} = 7$m/s. The site developer got excited by these high values and wants you to investigate if it makes sense to place a larger turbine with a hub height of 100m at this site.

      i. For a quick assessment, you assume a logarithmic wind profile and using the two existing values you can use it to estimate the wind speed at 100m altitude. What is your estimate for u_{100}? [*1 pt*]

      ii. **Bonus!** In addition, if you like, list pros and cons of the larger turbine with higher hub height, and decide on your recommendation to the developer. [*1 bonus pt*]

   (c) Regard a high-pressure region in the northern hemisphere at a latitude of $\phi = 50^o$. We have learnt that geostrophic wind - as well as its refinement, the gradient wind - is parallel to the isobars, and grows with the gradient of the pressure.

      i. In what direction (as seen from above) does the air flow around the high pressure region described: clockwise or counterclockwise? [*0.5 pt*]

      ii. The pressure gradient at a specific location A on the boundary of the high-pressure region is 5 Pa/km. What would be the geostrophic wind at this location? [*1 pt*]

      iii. Would the gradient wind be faster or slower than the geostrophic wind at this location? [*1 pt*]

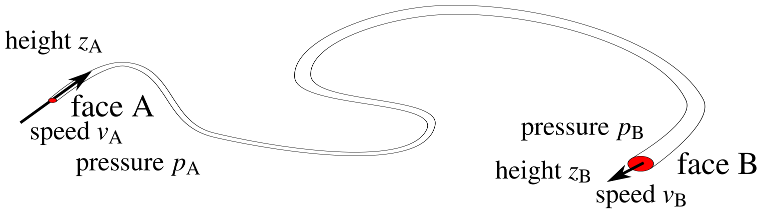
      iv. **Bonus!** If the radius of curvature of the isobars at location A is given by R =2000km[1], what size would the gradient wind have? [*1 bonus pt*]

[1] Our apologies that the sheet was uploaded with a typo of $R = 500$km.

Bernoulli's principle **[*5 pt*]**

3. Now, we want to look into Bernoulli's principle, which is a key component to the classic momentum theory.

Let's define a thin tube-like control volume (CV) in which the fluid velocity is everywhere parallel to the tubular structure. Then, fluid only enters the CV at one end of the CV and leaves the CV at the other end. Let's call the entrance end 'face A', and the exit end 'face B'. The fluid passing through each face has a speed v, pressure p and height z, as shown in the sketch.



The first law of thermodynamics says that a change in the energy E inside the CV is due to: Q the heat added to the CV, W the work done by the fluid in the CV, and E_A and E_B the energy that, respectively, enters and leaves the CV through faces A and B.

The work W can be divided into some parts: W_{shaft} the shaft work removed from the CV by machinery such as turbines and pistons; W_{flow} the flow work expended by the fluid as it moves to fill the CV; W_{viscous} the viscous work lost to fiction.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{Q} - \dot{W} + \dot{E}_A - \dot{E}_B = \dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{flow}} - \dot{W}_{\text{viscous}} + \dot{E}_A - \dot{E}_B.$$

Let's assume, for the following question, that the mass flow rate $\dot{m}$ into the CV equals the mass flow rate out of the CV, so that the mass m contained within CV is always constant. Further, assume that the fluid within the CV is incompressible, so that the fluid density ρ is always constant. Then, we know that the flow work can be found to be: $\dot{W}_{\text{flow}} = \left(\frac{\dot{m}_B}{\rho_B} p_B - \frac{\dot{m}_A}{\rho_A} p_A\right) = \frac{\dot{m}}{\rho}(p_B - p_A)$.

(a) What single-word name is given to a CV described by the picture and first paragraph? [*0.5 pt*]

(b) Let's define the rate of energy entering the CV as $\dot{E}_A := \dot{U}_A + \dot{E}_{\text{kinetic,A}} + \dot{E}_{\text{potential,A}}$, where U is the internal energy, E_{kinetic} is the kinetic energy and $E_{\text{potential}}$ is the potential energy. Define the kinetic and potential energy entering the CV through face A to express $\dot{E}_A$. [*1 pt*]

(c) Repeat the above question for face B. [*1 pt*]

(d) What assumptions have we made to this point? [*1 pt*]

(e) What remaining assumptions do we need to make in order to arrive at Bernoulli's principle: [*1.5 pt*]

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant?}$$