Wind Energy Systems – PRACTICE Exam

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final exam on: September 24th, 2018, 14:00 - 16:00, Freiburg, Georges-Koehler-Allee 101 Room 036

Page	0	1	2	3	4	5	6	7	8	9	sum
Points on page (max)	2	8	12	8	10	9	12	10	6	10	87
Points obtained											
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Intermediate sum (max)	2	10	22	30	40	49	61	71	77	87	

Mark: Exam inspected on: Signature of examiner:

Surname: First name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

Please fill in your name above. For the multiple choice questions tick exactly one box for the right answer. For the text questions, give a short formula or text answer just below the question in the space provided, and, if necessary, write on the **backpage of the same sheet** where the question appears, and add a comment "see backpage". Do not add extra pages (for fast correction, all pages will be separated for parallelization). The exam is a closed book exam, i.e. no books or other material are allowed besides pens, a non-programmable calculator, and one A4 page (that is, two sides) of notes. Some legal comments are found in a footnote.[1]

Please note that in some questions, you are given more information than is necessary to solve the question.

1. Approximately, what is the power density through a circular area of radius $R = 1\text{m}$, when the constant air density $\rho = 1\text{kg/m}^3$ at a constant and uniform wind speed of $u_\infty = 10\text{m/s}$. Please neglect induction effects.

(a) ☐ 160 W/m^2	(b) ☐ 500 W/m^2
(c) ☐ 610 W/m^2	(d) ☐ 1570 W/m^2

2	

[1] WITHDRAWING FROM AN EXAMINATION: In case of illness, you must supply proof of your illness by submitting a medical report to the Examinations Office. Please note that the medical examination must be done at the latest on the same day of the missed exam. In case of illness while writing the exam please contact the supervisory staff, inform them about your illness and immediately see your doctor. The medical certificate must be submitted latest 3 days after the medical examination. More informations: http://www.tf.uni-freiburg.de/studies/exams/withdrawing_exam.html

CHEATING/DISTRUBING IN EXAMINATIONS: A student who disrupts the orderly proceedings of an examination will be excluded from the remainder of the exam by the respective examiners or invigilators. In such a case, the written exam of the student in question will be graded as 'nicht bestanden' (5.0, fail) on the grounds of cheating. In severe cases, the Board of Examiners will exclude the student from further examinations.

2. Consider that there is a site at which a developer wants to put a GE 1.5MW turbine. There is a known probability distribution of wind speeds at that site, and the turbine's power curve is known. Both can be found in Figure 1.

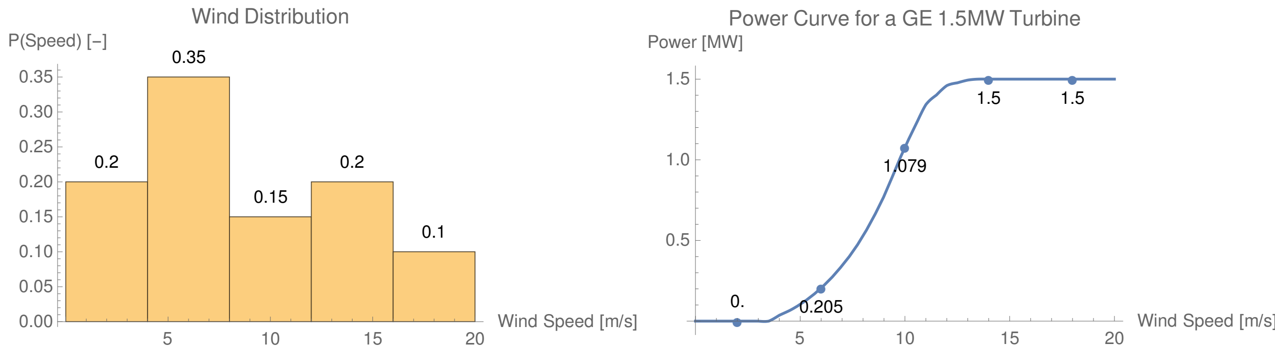

Figure 1: Wind distribution (left) and power curve (right) for problem 2

Which of the following values is closest to the expected capacity factor for this turbine?

(a) ☐ 0.20	(b) ☐ 0.33
(c) ☐ 0.46	(d) ☐ 0.59

2

3. All else being equal, at which latitude would you expect a wind farm (of turbines of height less than 100m) to produce the MOST power?

(a) ☐ 0 degrees North	(b) ☐ 15 degrees North
(c) ☐ 30 degrees North	(d) ☐ 60 degrees North

2

4. Considering the logarithmic wind profile: $u(z) = u(z_1)\frac{\log \frac{z}{z_0}}{\log \frac{z_1}{z_0}}$. Which of the following statements is FALSE?

(a) ☐ The logarithmic wind profile represents the wind speed distribution at any specific instant.	(b) ☐ This wind profile is not valid at altitudes above (approximately) 500m.
(c) ☐ This wind profile is not valid at altitudes below the roughness length.	(d) ☐ The logarithmic wind profile describes the atmospheric boundary layer.

2

5. Consider the flow travelling through the actuator disk (AD) of a turbine. Which of the following statements is FALSE?

(a) ☐ The axial-direction flow is slower at the AD than it is upstream of the AD.	(b) ☐ The axial-direction flow is faster at the AD than it is downstream of the AD.
(c) ☐ The cross-section of the streamtube is wider downstream of the AD than it is upstream of the AD.	(d) ☐ Air can cross the boundaries of the streamtube.

2

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6. Let's consider the wake rotation behind an rotor disk (RD), as described by rotor disk theory. Which of the following statements is FALSE?

(a) ☐ Upstream of the RD, there is no wake rotation.	(b) ☐ Half of the rotation is added exactly at the RD; half is added immediately downstream of the RD.
(c) ☐ For horizontal axis wind turbines, who generate power primarily through torque, wake rotation is significant.	(d) ☐ For airborne wind energy systems, who generate power primarily through torque, wake rotation is significant.

2	

7. For which of the following simplified wake models are the 2D lift and drag coefficients of the blade's cross-sectional airfoil needed?

(a) ☐ classic actuator disk model	(b) ☐ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ they are not need for any of these models

2	

8. For which of the following simplified wake models can you find the tangential momentum analytically? (That is, not numerically).

(a) ☐ classic actuator disk model	(b) ☐ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ it cannot be found analytically in any of these models

2	

9. For which of the following simplified wake models can you predict the radial component of the induced velocity?

(a) ☐ classic actuator disk model	(b) ☐ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ you cannot predict it in any of these models

2	

10. For which of the following simplified wake models do you need to know the number of blades?

(a) ☐ classic actuator disk model	(b) ☐ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ you do not need to know for any of these models

2	

11. Consider the wind turbine tower as a vertically cantilevered beam, that is only loaded by a normal point-load F that is at three-quarters of the tower height. The second moment of area of the tower cross-section I is constant over the whole tower, which has constant outer radius r. The tower height is L. Which formula will best describe the maximum bending stress on the tower?

(a) ☐ $\frac{4F}{3Lr}$	(b) ☐ $\frac{4^4 FI}{(3^4)\pi L^4 r^2}$
(c) ☐ $\frac{3^3 FL^3}{4^3 \pi r I}$	(d) ☐ $\frac{3FLr}{4I}$

2	

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12. A cantilevered beam under a concentrated end load P has a deflection $y(x,t)$. This function reads as:

$$y(x,t) = \frac{Px^2}{6EI}(3L - x)\cos(\omega t),$$

with a beam of length L, Young's modulus E and second moment of area I. The distance from from the fixed end of the beam is x, and ω is the vibration frequency. The strain energy $V(t)$ and kinetic energy $T(t)$ in the bending beam reads as:

$$V(t) = \frac{EI}{2}\int_0^L \left(\frac{\partial^2 y}{\partial x^2}\right)^2 \mathrm{d}x = \frac{P^2}{2k}\cos^2(\omega t) \qquad T(t) = \frac{m}{2L}\int_0^L \left(\frac{\partial y}{\partial t}\right)^2 \mathrm{d}x = \frac{33}{280}\frac{m\omega^2 P^2}{k^2}\sin^2(\omega t),$$

where $k = 3EI/L^3$ and m is the beam's mass.

Using the Rayleigh method, which of the following values most closely approximates the beam's natural frequency?

(a) ☐ $1.41\sqrt{\frac{k}{m}}$	(b) ☐ $2.01\sqrt{\frac{k}{m}}$
(c) ☐ $2.06\sqrt{\frac{k}{m}}$	(d) ☐ $4.24\sqrt{\frac{k}{m}}$

2	

13. Suppose the blade tip displacement in the rotation plane $\boldsymbol{w}(t) = \boldsymbol{w}_0 e^{j\omega t}$ can be described with the differential equation:

$$\boldsymbol{M}\ddot{\boldsymbol{w}} + \boldsymbol{K}\boldsymbol{w} = 0$$

where the mass matrix $\boldsymbol{M}$ and stiffness matrix $\boldsymbol{K}$ are known as follows:

$$\boldsymbol{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \boldsymbol{K} = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{pmatrix}.$$

What are (most closely) the natural frequencies of the blade deformation in radians per second?

(a) ☐ 1 and 2	(b) ☐ 1 and 4
(c) ☐ 2 and 3	(d) ☐ 4 and 9

2	

14. Consider different wind energy conversion systems in normal operation at their rated wind speeds. For which system are heavy components mounted at the highest altitude?

(a) ☐ horizontal axis wind turbine	(b) ☐ darrieus wind turbine
(c) ☐ savonius wind turbine	(d) ☐ lift-mode airborne wind energy system

2	

15. Consider different wind energy conversion systems in normal operation at their rated wind speeds. For which system is the ratio between the produced power and the blade surface area the smallest?

(a) ☐ horizontal axis wind turbine	(b) ☐ darrieus wind turbine
(c) ☐ savonius wind turbine	(d) ☐ lift-mode airborne wind energy system

2	

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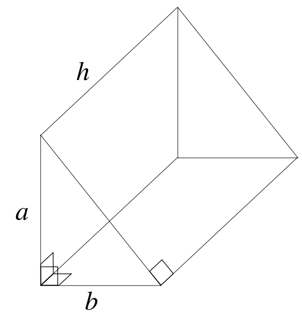
16. Consider Loyd's limit for the maximum power-harvesting factor of an airborne wind energy system. Which of the following assumptions was NOT made during its derivation?

(a) ☐ steady crosswind flight	(b) ☐ lift force parallel to the tether
(c) ☐ reel-out direction parallel to freestream wind	(d) ☐ known lift and drag coefficients

2	

17. Consider a control volume of air, through which wind is flowing. This control volume has the shape of a right triangular prism. This prism has height h and triangle dimensions, as given by the the triangular face's altitude a perpendicular to a base of length b. The height is parallel to the uniform and steady wind velocity $\boldsymbol{u}$.



(a) Given a uniform and constant air density ρ, how much kinetic energy is present in the air within the control volume?

2	

(b) Please use the above expression to derive the power density passing through the triangular cross-section.

2	

18. Consider the power harvesting factor ζ and the power coefficient C_P.

(a) Which value is likely to be more useful for horizontal axis wind turbines? Briefly, why?

2	

(b) Which value is likely to be more useful for an airborne wind energy system? Briefly, why?

2	

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19. Regard a high-pressure region in the northern hemisphere at a latitude of $\phi = 50\,\text{deg}$, with an air density of approximately 1kg/m^3. We have learnt that the geostrophic wind - as well as its refinement, the gradient wind - is parallel to the isobars, and grows with the gradient of the pressure.

    (a) In what direction (as seen from above) does the air flow around the high pressure region described: clockwise or counterclockwise?

1	

    (b) The pressure gradient at a specific location A on the boundary of the high-pressure region is 5 Pa/km. What would be the geostrophic wind at this location?

2	

    (c) Would the gradient wind be faster or slower than the geostrophic wind at this location?

2	

20. In the context of wind energy, please explain (briefly, in words, and without equations) the main purpose (or use) of the 'Weibull distribution.'

2	

21. A turbine design engineer is able to represent the equations of motion of a wind turbine blade, such that the displacement of the blade tip is given by a function $x(t) = x_0 \mathrm{e}^{j\omega_k t}$. It is further found that the natural frequency of the blade is ω_n. Upon finding the relevant values, the design engineer determines that a dynamic analysis of the blade behavior is necessary, and a static analysis is not sufficient. What inequality must the design engineer have noticed?

2	

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22. Please describe briefly, in words and without equations, what the difference is between ”supervisory control” and ”dynamic control”.

2	

23. Please describe briefly, in words and without equations, what the comparative advantages and disadvantages are between lift-mode and drag-mode airborne wind energy systems.

2	

24. Please describe briefly, in words and without equations, for what purpose a turbine designer would need to estimate the maximum bending stress of a structural component.

2	

25. Consider actuator disk theory for wind turbines, with an induction factor a.
    (a) Please fill in the blanks:
    ’Actuator disk theory is valid for wind turbines for values of a that satisfy the relationship: ...... $\leq a \leq$ ....’

2	

    (b) Please describe (briefly, in words and without equations) what happens outside of the lower end of this range.

2	

    (c) Please describe (briefly, in words and without equations) what happens outside of the upper end of this range.

2	

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26. Consider actuator disk theory, with an induction factor a, a freestream wind velocity u_∞, and a constant air density ρ.

(a) What is the axial-direction wind speed at the actuator disk u_1, based on the definition of the induction factor?

| 2 | |

(b) Please describe (briefly, in words, and without equations) how to go about finding the relationship between u_2, u_∞ and a, where u_2 is the axial-direction wind speed far downstream of the actuator disk.

| 2 | |

(c) What is the power per unit area (P/A) that an actuator disk can extract?

| 2 | |

(d) What optimal induction factor a^* will maximize the power per unit area P/A?

| 2 | |

(e) What is the maximum power per unit area $(P/A)^*$ that corresponds to this optimal induction factor?

| 2 | |

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27. Let's say you'd like to use the blade element momentum method to predict the wind turbine blade loading at the (known, steady and uniform) rated wind speed u_{rated} and a known air density ρ. Assume that there are 3 equivalent blades, and that the freestream wind speed is parallel to the axis of rotation.

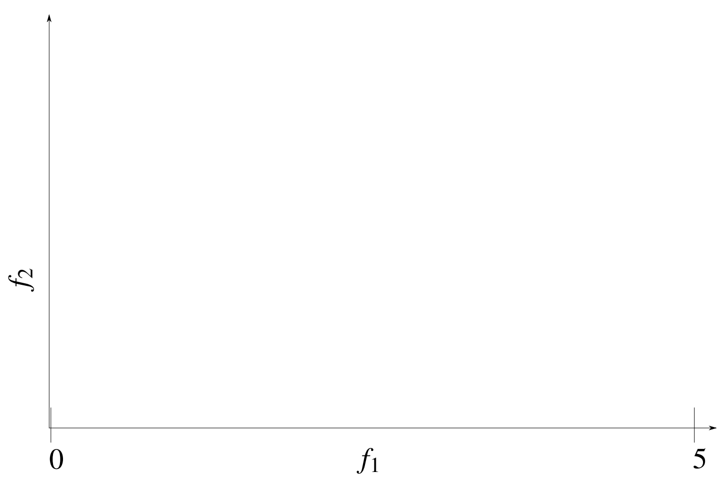
   (a) You look up various pieces of information on the turbine manufacturer's specification sheet. What pieces of information do you need to find.

2	

   (b) Please (briefly, in words and without equations) describe how you're going to use this information in the blade element momentum method?

2	

28. A wind turbine blade has a natural frequency ω_n when the blade is stationary. Consider that Ω is the angular velocity of the turbine, such that we can define a nondimensional value $f_1 := \Omega/\omega_n$. If the blade natural frequency ω depends on Ω, then we can define a nondimensional value $f_2 := \omega/\omega_n$. In the axes given below, please sketch f_2 vs. f_1.



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29. In this problem, we will consider the blade element momentum method (BEM). Some values are given in Table 1 that MAY OR MAY NOT satisfy BEM. In this problem, you need to determine if these values give an wind turbine rotor annulus that is consistent with BEM or not.

Table 1: Values for Problem 29

parameter	symbol	value	parameter	symbol	value
annulus spanwise position	μ	1	number of blades	B	6
total rotor radius	R	1 m	chord-length	c	$\frac{5\pi}{21\sqrt{3}}$ m
air density	ρ	1 kg/m^3	free-stream wind speed	u_∞	$\frac{60}{7}$ m/s
2d lift coefficient	c_ℓ	1	2d drag coefficient	c_d	$\frac{\sqrt{3}}{5}$
tip speed ratio	λ	$\frac{2}{\sqrt{3}}$	axial induction factor	a	$\frac{6}{20}$
tangential induction factor	a'	$\frac{1}{20}$			

Please use the symbols a and a' to denote the axial and tangential induction factors, respectively.

(a) Given the above values, what is the magnitude of the effective velocity W?

2

(b) Given the above values, what is the flow angle ϕ in radians?

2

(c) Please describe (briefly, in words, and without equations) how you would go about determining whether the above values ARE OR ARE NOT a solution to the blade element momentum method.

2

(d) Without filling in any of the values given, what are the blade element momentum method equation(s)?

2

(e) TRUE OR FALSE: The values given in Table 1 ARE a solution to the blade element momentum method.

2

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