Lecture Energy Systems Hardware and Control (ESHC)
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Revision Exercise

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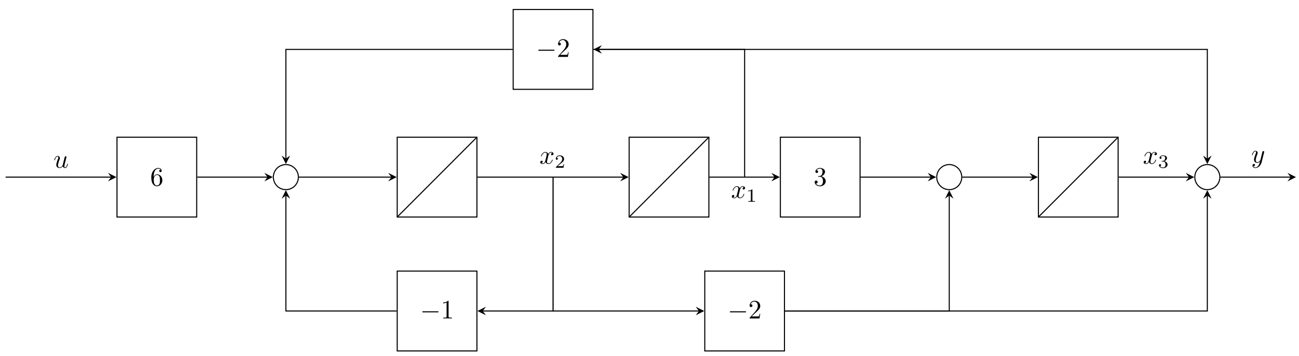
Dynamic Systems in State Space

1. Any systems dynamics can be represented by the equations

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t)$$

How can f and g be expressed with matrices for LTI systems?

2. Given the following block diagram what are the equations of the state space representation?



3. Consider the following system

$$\dot{x} = \begin{bmatrix} 0 & 2 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 6 \end{bmatrix} u$$
$$y = \begin{bmatrix} 3 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u$$

(a) Find the characteristic polynomial.

(b) Calculate the eigenvalues.

(c) Is the system stable and/or oscillating?

4. How many state, input and output variables does the following system have?

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

5. What are the dimensions of A, B, C and D if a SISO-system has n state variables?

6. Formulate the state space representation in the control canonical form out of the given ODE

$$\dddot{y} + 2\ddot{y} - 5\dot{y} + 3y = 2\ddot{u} + \dot{u} - 8u$$

Structural Analysis

1. What must hold for a LTI system A, B, C, D if it is controllable?

2. Calculate the observability and controllability matrices of the following system.

$$\dot{x} = \begin{bmatrix} 0 & 2 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 \\ -2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

3. Is the following system observable?

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 \\ -1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

4. Is the following system observable?

$$\dot{x} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x$$

State Feedback Control

1. Calculate $\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -5 & \pi \end{bmatrix}$
2. What does the closed loop matrix A_{cl} look like if the state feedback K is applied to the system A, B, C?
3. Sketch the general block diagram for state feedback of an LTI System.
4. Calculate $-BK$ with $B = \begin{bmatrix} 10 & -2 & 3 \end{bmatrix}$ and $K = \begin{bmatrix} -3 \\ 12 \\ 4 \end{bmatrix}$
5. What is the dimension of the state feedback K if the system has 231 state, 153 input and 42 output variables?
6. Describe the idea of pole-placement for state feedback.
7. The characteristic polynomial of the closed loop is $p(\lambda) = \lambda^2 + (k_1 + k_2)\lambda - 2 + 2k_1$. What $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ is needed the place the closed loop eigenvalues to $\lambda_1 = -4$ and $\lambda_2 = -2$?

Linear Quadratic Regulator

1. What kind of feedback is implemented by using LQR?
2. What is the effect of the weighting matrices Q and R on the controller dynamics?

Luenberger Observer

1. How is the matrix A_{obs}, that defines the error dynamics $\dot{e}(t) = A_{obs}e(t)$, calculated?
2. Which input signals enter the observer algorithm?
3. Calculate $-LC$ with $L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -2 & 5 \end{bmatrix}$
4. Sketch the general block diagram for a LTI system with observer and LQR feedback on the estimated states using as blocks only integrators, sums and matrix multiplications. Bonus points if the paths are not intersecting.