Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg January 13, 2017, 10:15-11:45, Freiburg

Su	rname:	First N	Jame:			Matriculation r	umber:	
Su	bject: Prog	ramme:	Bachelor	Master	Lehramt	others	Signature:	
1.	Please fill in your name above a We want to assess the robustness the phone breaks when it is dro experiment we have dropped 10 $f(\theta)$ that we need to minimize i	and tick as of a pa pped. W 0 smartr n order t	exactly ONE articular sma We assume the phones and of to obtain the	box for th rtphone bra at the phor btained 42 MLE estin	e right answ and hence, whe thrown of broken sma mate of θ ?	ver of each quest we would like to nto the ground o rtphones. What	ion below. know the unknown probability either breaks or has no damage. is the negative log likelihood fu	θ that In an nction
	(a) $-42\log\theta - 58\log(1 -$	- <i>θ</i>)			(b)	$-\log(42\theta) - \log(42\theta)$	$g(58(1-\theta))$	
	(c) $58 \log \theta + 42 \log(1 - \theta)$	$\theta)$			(d) 🗌 le	$\log(58\theta) + \log(4\theta)$	$42(1-\theta))$	
2.	You are given a pendulum whice $y(t)$ are the measurements. Which	ch is by i ich of the	nature a NO e following a	N-LINEAF llgorithms (R system and could you us	d can be modele se to estimate th	ed by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, e parameters θ ?	where
	(a) Maximum a Posteriori	Estimati	ion (MAP)		(b) 🗌 F	Recursive Least	Squares (RLS)	
	(c) Weighted Least Square	es (WLS	5)		(d) [] I	Linear Least Squ	uares (LLS)	
3.	Suppose now that the system giv . Which of the following algorit problems or high computational	ven in th hms cou costs fo	e previous qu ld you use to r a coninuou	uestion can estimate the sand infin	be approxir he paramete ite flow of n	mated by a mode ars θ of this linear neasurement dat	el that is linear in the parameters r model without running into ma a?	(LIP) emory
	(a) LLS	(b)	MAP		$(c) \square F$	RLS	(d) WLS	
4.	You are asked to give a compu- question $\Sigma_{\hat{\theta}}$. The model is give likelihood function. The covaria	utational en as y_N unce mat	ly efficient a $T_{V} = \Phi_{N}\theta + \theta_{N}$ rix can be ap	approximat ϵ_N with ϵ_J	ion of the $\alpha_N \sim \mathcal{N}(0, \Sigma_{\hat{\theta}} \approx .$	covariance of the Σ_{ϵ} , $Q_N = \Phi_N^{\top}$	e estimate computed in the pro- Φ_N and $L(\theta, y_N)$ is the negative	evious ve log
	(a) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(b)	$]\Phi_N^+\Sigma_{\epsilon_N}\Phi_N^+$	+⊤ V	(c) [] (Q_N^{-1}	(d) $\square (\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N)^{-1}$	
5.	Given the probability density fu of independent measurements <i>i</i> maximum likelihood estimate o	$ \begin{array}{l} \text{Inction } \mathbf{c} \\ y_N = [y] \\ \text{f } \theta? \\ \text{The} \end{array} $	of the expone $y(1), y(2), \dots$ problem is:	ential distril ., $y(N)]^T$, $\min_{\theta} \dots$?	bution, $p_X(x)$ what is the	$f(x) = \theta e^{-\theta x}$, wi e right minimisa	th an unknown parameter θ , and tion problem you need to solve	1 a set for a
	(a) $\ y(k) - \theta e^{-\theta}\ _2^2$				(b)	$-N\log(\theta) + \theta$	$\sum_{k=1}^{N} y(k)$	
	(c) $\ \theta e^{-\theta y(k)}\ _2^2$				(d)	$-\log\sum_{k=1}^{N}\theta e^{-1}$	heta y(k)	
6.	For the problem in the previous that θ_0 is the true value? The Fig.	questio sher info	n, what is a lormation mat	lower boun rix is defin	Id on the co ed as $M = \frac{1}{2}$	variance for any $\int_{yN} \nabla^2_{\theta} L(\theta_0, y_N)$	v unbiased estimator $\hat{\theta}(y_N)$, asso $y_N \cdot p(y_N \theta_0) dy_N$.	uming
	(a) $\square N/\theta^2$				(b) [] ($\left(\int_{y_N} N\theta^{N-2} \exp\right)$	$p\left[-\theta \sum_{k} y_{k}\right] dy_{N} \right)^{-1}$	
	(c) θ_0^2/N				(d)	$\int_{y_N} N\theta_0^{N-2} \exp$	$[-\theta \sum_k y_k] \mathrm{d}y_N$	
7.	Suppose you are given the Fish matrix $\Sigma_{\hat{\theta}}$ of your estimate $\hat{\theta}$?	er inforr	nation matrix	$\mathbf{X} M$ of the	correspond	ling problem, w	hat is the relation with the cova	riance

- 8. Give the name of the theorem that provides us with the above result.
- 9. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after N+1 measurements? $\hat{\theta}(N+1) = \arg \min_{\alpha} \frac{1}{2} (\ldots)$

(a) $\ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$	(b) $\ \theta - \hat{\theta}(N) \ _2^2 + \ y(N+1) - \varphi(N+1)^\top \theta \ _2^2$
(c) $ \ \theta - \hat{\theta}(N) \ _{Q_N}^2 + \ y(N) - \varphi(N)^\top \theta \ _2^2 $	(d) $\qquad \ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$

10. In L_2 estimation the measurement errors are assumed to follow a ... distribution and it is generally speaking more ... to outliers compared to L_1 estimation.

(a) Laplace, sensitive	(b) Laplace, robust	(c) Gaussian, robust	(d) Gaussian, sensitive

11. The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2}\frac{\|y-\theta\|_2^2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measurements, y(1) = 3, y(2) = 6, and y(3) = 12. What is the minimizer θ^* of the negative log-likelihood function ?

(a) 7	(b) <u>6</u>	(c) <u>9</u>	(d) 4

- 12. Please give the ODE of a linear time invariant (LTI) system, with state vector x and input vector $u. \dot{x} = \dots$
- 13. Please identify the most general transfer function that still is a Auto Regressive Model with Exogenous Inputs (ARX) where $n = \max(n_a, n_b)$. $G(z) = \dots$

	(a) $\boxed{\frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}}$	(b) $\boxed{\frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(c) $\square \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\qquad \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}$
14.	Identify most general transfer fu	nction that still is a Finite Impul	se Response (FIR) model with n	$= \max(n_a, n_b). \ G(z) = \dots$
	(a) $ \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n} $	(b) $\boxed{\frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}}$	(c) $\boxed{\frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(d) $\qquad \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}$
15.	Which of the following model e	quations describes a FIR system	with input u and output y ? $y(k - \frac{1}{2})$	$(+1) = \dots$
	(a) $\Box u(k+1) + y(k)$	(b) $\square u(k) \cdot y(k)$	(c) $\Box u(k) + \sin(k \cdot \pi)$	(d) $\Box u(k) - 5 \cdot u(k-1)$
16.	Which of the following dynamic	c models with inputs $u(t)$ and ou	tputs $y(t)$ is NEITHER linear N	OR affine.
	(a) $\[\dot{y}(t) + \sin(t) = u(t) \]$	(b) $\qquad \dot{y}(t) = u(t) + t$	(c) $\Box \dot{y}(t) = \sqrt{t \cdot u(t)}$	(d) $\Box t\dot{y}(t) = u(t) + 2$
17.	Which of the following models	with input $u(k)$ and output $y(k)$	is NOT linear-in-the-parameters	w.r.t. $\theta \in \mathbb{R}^2$?
	(a) $\qquad y(k) = \theta_1 u(k)^2 + \theta_2$ ex	$\exp(u(k))$	(b) $\qquad y(k) = \theta_1 \sqrt{u(k)} + \theta_2$	u(k)
	(c) $y(k) = y(k-1) \cdot (\theta_1)$	$+ \theta_2 u(k))$	(d) $\Box y(k) = \theta_1 \exp(\theta_2 u(k))$)
18.	Which of the following models	is time invariant?		
	(a) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(b) $\[\dot{y}(t) = 5u(t) + t \]$	(c) $\[\dot{y}(t) = \sqrt{u(t)} \]$	(d) $\Box t \cdot \ddot{y}(t) = u(t)^3$
19.	By which of the following form	ulas is the joint distribution for <i>N</i>	V independent measurements y_N	$\in \mathbb{R}^N$ given? $p(y_N \theta) = \dots$
	(a) $\prod \int_{y_N} p(y \theta) dy$	(b) $\prod \sum_{i=0}^{N} p(y(i) \theta)^2$	(c) $\prod \prod_{i=0}^{N} p(y(i) \theta)$	(d) $\prod \sum_{i=0}^{N} p(y(i) \theta)$
20.	Which of the following statemen	nts about Maximum A Posteriori	(MAP) estimation is not true	
	(a) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$		(b) MAP assumes a linear model	
	(c) MAP is a generalization	on of ML	(d) MAP requires a-priori	knowledge on θ

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Surname:	First Name:	Matriculation number:
Subject:	Programme: Bachelor Master Lehra	amt others Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. Given the probability density function of the exponential distribution, $p_X(x) = \theta e^{-\theta x}$, with an unknown parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, what is the right minimisation problem you need to solve for a maximum likelihood estimate of θ ? The problem is: $\min_{\theta} \dots$?

(a) $\ \theta e^{-\theta y(k)}\ _2^2$	(b) $\Box -N\log(\theta) + \theta \sum_{k=1}^{N} y(k)$
(c) $\ y(k) - \theta e^{-\theta} \ _2^2$	(d) $\Box - \log \sum_{k=1}^{N} \theta e^{-\theta y(k)}$

2. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{uN} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\square \theta_0^2/N$	(b) $\square \left(\int_{y_N} N \theta^{N-2} \exp[-\theta \sum_k y_k] \mathrm{d}y_N \right)^{-1}$
(c) $\square N/\theta^2$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta \sum_k y_k] dy_N$

3. You are given a pendulum which is by nature a NON-LINEAR system and can be modeled by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, where y(t) are the measurements. Which of the following algorithms could you use to estimate the parameters θ ?

(a) Maximum a Posteriori Estimation (MAP)	(b) Linear Least Squares (LLS)	
(c) Recursive Least Squares (RLS)	(d) Weighted Least Squares (WLS)	

4. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP) . Which of the following algorithms could you use to estimate the parameters θ of this linear model without running into memory problems or high computational costs for a coninuous and infinite flow of measurement data?

(a) MAP	(b) RLS	(c) LLS	(d) WLS
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5. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question $\Sigma_{\hat{\theta}}$. The model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon})$, $Q_N = \Phi_N^{\top} \Phi_N$ and $L(\theta, y_N)$ is the negative log likelihood function. The covariance matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \ldots$

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6. The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2}\frac{\|y-\theta\|_2^2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measurements, y(1) = 3, y(2) = 6, and y(3) = 12. What is the minimizer θ^* of the negative log-likelihood function ?

	(a) 4	(b) 6	(c) 9	(d) 7
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- 7. Suppose you are given the Fisher information matrix M of the corresponding problem, what is the relation with the covariance matrix $\Sigma_{\hat{\theta}}$ of your estimate $\hat{\theta}$?
- 8. Give the name of the theorem that provides us with the above result.

9.	Which of the following models	is time invariant?		
	(a) $\begin{subarray}{c} \dot{y}(t) = 5u(t) + t \end{subarray}$	(b) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(c) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(d) $\qquad \dot{y}(t) = \sqrt{u(t)}$
10.	By which of the following form	ulas is the joint distribution for I	V independent measurements y_N	$\in \mathbb{R}^N$ given? $p(y_N \theta) = \dots$
	(a) $\prod \prod_{i=0}^{N} p(y(i) \theta)$	(b) $\prod \sum_{i=0}^{N} p(y(i) \theta)^2$	(c) $\prod \sum_{i=0}^{N} p(y(i) \theta)$	(d) $\prod \int_{y_N} p(y \theta) dy$
11.	Which of the following statement	nts about Maximum A Posteriori	(MAP) estimation is not true	
	(a) MAP is a generalizatio	n of ML	(b) MAP assumes a linear	model
	(c) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-$	$-\log(p(y_N \theta)) - \log(p(\theta))]$	(d) MAP requires a-priori	knowledge on θ
12.	Which of the following model e	quations describes a FIR system	with input u and output y ? $y(k - y)$	$(+1) = \dots$
	(a) $u(k) - 5 \cdot u(k-1)$	(b) $\Box u(k+1) + y(k)$	(c) $\Box u(k) \cdot y(k)$	(d) $\Box u(k) + \sin(k \cdot \pi)$
13.	Which of the following dynamic	c models with inputs $u(t)$ and ou	tputs $y(t)$ is NEITHER linear N	OR affine.
	(a) $\[\dot{y}(t) = \sqrt{t \cdot u(t)}\]$	(b) $\qquad \dot{y}(t) + \sin(t) = u(t)$	(c) $\Box t\dot{y}(t) = u(t) + 2$	(d) $\Box \dot{y}(t) = u(t) + t$
14.	Given a set of measurements y_N which of the following minimis $\hat{\theta}(N+1)$ after $N+1$ measurements	$= [y(1), y(2), \dots, y(N)]^T \text{ from}$ sation problems is solved at eac ments? $\hat{\theta}(N+1) = \arg\min_{\theta} \frac{1}{2} (.$	the linear model $y_N = \Phi \theta$, when h iteration step of the RLS algo)	The $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, rithm to estimate the parameter
	(a) $\ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$		(b) $\ y_N - \Phi_N \cdot \theta \ _{Q_N}^2$	
	(c) $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(n)\ _{Q_N}^2$	$N) - \varphi(N)^{\top} \theta \ _2^2$	(d) $\qquad \ \theta - \hat{\theta}(N)\ _2^2 + \ y(N - \hat{\theta}(N)\ _2^2)$	$+1) - \varphi(N+1)^\top \theta \ _2^2$
15.	Please identify the most generative $n = \max(n_a, n_b)$. $G(z) = \dots$	ll transfer function that still is a	Auto Regressive Model with E	Exogenous Inputs (ARX) where
	(a) $1 \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$	(b) $\boxed{\frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(c) $\qquad \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\boxed{\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}}$
16. We want to assess the robustness of a particular smartphone brand hence, we would like to know the unknown probability the phone breaks when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage experiment we have dropped 100 smartphones and obtained 42 broken smartphones. What is the negative log likelihood fut $f(\theta)$ that we need to minimize in order to obtain the MLE estimate of θ ?				the unknown probability θ that breaks or has no damage. In an negative log likelihood function
	(a) $58 \log \theta + 42 \log(1 - \theta)$	9)	(b) $-\log(42\theta) - \log(58(2\theta)))$	(1- heta))
	(c) $-42\log\theta - 58\log(1 - 6)$	- <i>θ</i>)	(d) $\log(58\theta) + \log(42(1 - \theta))$	- <i>θ</i>))
17.	Please give the ODE of a linear	time invariant (LTI) system, with	n state vector x and input vector x	$u. \dot{x} = \dots$
18.	Which of the following models	with input $u(k)$ and output $y(k)$	is NOT linear-in-the-parameters	w.r.t. $\theta \in \mathbb{R}^2$?
	(a) $y(k) = \theta_1 \exp(\theta_2 u(k))$		(b) $y(k) = y(k-1) \cdot (\theta_1 + \theta_2 u(k))$	
	(c) $\Box y(k) = \theta_1 u(k)^2 + \theta_2 e^{-2k}$	$\exp(u(k))$	(d) $\qquad y(k) = \theta_1 \sqrt{u(k)} + \theta_2$	u(k)
19.	In L_2 estimation the measuremet compared to L_1 estimation.	ent errors are assumed to follow	a distribution and it is genera	lly speaking more to outliers
	(a) Gaussian, sensitive	(b) Gaussian, robust	(c) Laplace, sensitive	(d) Laplace, robust
20.	Identify most general transfer fu	nction that still is a Finite Impul	se Response (FIR) model with n	$= \max(n_a, n_b). \ G(z) = \dots$

(a) $ \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n} $	(b) $\sum_{b_0 z^n + b_1 z^{n-1} + \dots + b_n \atop a_0 z^n}$	(c) $\square \frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\qquad \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$
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Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg January 13, 2017, 10:15-11:45, Freiburg

Surname: First Name:		Matriculation num	ber:			
Su	bject: Prog	ramme: Bachelor Master	Lehramt others Si	gnature:		
1.	 Please fill in your name above and tick exactly ONE box for the right answer of each question below. 1. Suppose you are given the Fisher information matrix M of the corresponding problem, what is the relation with the covariance matrix Σ_{θ̂} of your estimate θ̂? 					
2.	Give the name of the theorem th	nat provides us with the above re	esult.			
3.	Please identify the most generative $n = \max(n_a, n_b)$. $G(z) = \dots$	al transfer function that still is	a Auto Regressive Model with	n Exogenous Inputs (ARX) where		
	(a) $\boxed{\frac{z^n}{a_0z^n+a_1z^{n-1}+\ldots+a_n}}$	(b) $\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(c) $\boxed{\frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}}$	(d) $\square \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}$		
4.	Which of the following model e	quations describes a FIR system (b) $\Box u(k) \cdot u(k)$	n with input u and output y ? $y($	$\frac{k+1) = \dots}{\left (d) \prod u(k+1) + u(k) \right }$		
5	$\begin{bmatrix} a \end{bmatrix} \begin{bmatrix} u(\kappa) - 5 \cdot u(\kappa - 1) \end{bmatrix}$	$(0) \sqsubseteq u(\kappa) \cdot g(\kappa)$	$(c) \bigsqcup u(\kappa) + \sin(\kappa \cdot \pi)$	$(\mathbf{u}) \sqsubseteq u(\kappa + 1) + y(\kappa)$		
5.	compared to L_1 estimation.	ent errors are assumed to ronow				
	(a) Laplace, robust	(b) Laplace, sensitive	(c) Gaussian, robust	(d) Gaussian, sensitive		
6.	Identify most general transfer fu	Inction that still is a Finite Impu	Ilse Response (FIR) model with	$n = \max(n_a, n_b). \ G(z) = \dots$		
	(a) $\boxed{\frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(b) $\boxed{\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}}$	(c) $\boxed{\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(d) $\square \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$		
7.	7. We want to assess the robustness of a particular smartphone brand hence, we would like to know the unknown probability θ that the phone breaks when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 42 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the MLE estimate of θ ?			ow the unknown probability θ that er breaks or has no damage. In an he negative log likelihood function		
	(a) $\log(58\theta) + \log(42(1 - \theta))$	(- heta))	(b) $58 \log \theta + 42 \log(1)$	$(- \theta)$		
	(c) $-42\log\theta - 58\log(1 - 6)$	(- heta)	(d) $-\log(42\theta) - \log(5)$	8(1- heta))		
8. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi\theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$ which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after $N+1$ measurements? $\hat{\theta}(N+1) = \arg\min_{\theta} \frac{1}{2} (\dots)$				here $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, gorithm to estimate the parameter		
	(a) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N)\ _2^2$	$(+1) - \varphi(N+1)^{\top} \theta \ _2^2$	(b) $\qquad y_{N+1} - \Phi_{N+1} \cdot \theta $	2		
	(c) $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(\theta)\ _{Q_N}^2$	$N) - \varphi(N)^{\top} \theta \ _2^2$	$ \ (\mathbf{d}) \Box \ y_N - \Phi_N \cdot \theta \ _{Q_N}^2 $			
9.	The PDF of a random variable T ments, $y(1) = 3$, $y(2) = 6$, and	Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp y(3) = 12$. What is the minimized	$\theta(-\frac{1}{2}\frac{\ y-\theta\ _2^2}{4})$, with unknown θ zer θ^* of the negative log-likeli	$\in \mathbb{R}$. We obtained three measure- hood function ?		
	(a) 9	(b) 6	(c) 4	(d) 7		

Points or	page	(max.	9)
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11. Given the probability density function of the exponential distribution, $p_X(x) = \theta e^{-\theta x}$, with an unknown parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, what is the right minimisation problem you need to solve for a maximum likelihood estimate of θ ? The problem is: min ...?

(a) $\ \theta e^{-\theta y(k)}\ _2^2$	(b) $\Box -N\log(\theta) + \theta \sum_{k=1}^{N} y(k)$
(c) $\Box - \log \sum_{k=1}^{N} \theta e^{-\theta y(k)}$	(d) $\ y(k) - \theta e^{-\theta} \ _2^2$

12. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{uN} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta \sum_k y_k] \mathrm{d}y_N$	(b) $\left[\int_{y_N} N\theta^{N-2} \exp[-\theta \sum_k y_k] dy_N \right]^{-1}$
(c) $\square N/\theta^2$	(d) $\square \theta_0^2/N$

13. Which of the following dynamic models with inputs u(t) and outputs y(t) is **NEITHER** linear **NOR** affine.

14. You are given a pendulum which is by nature a NON-LINEAR system and can be modeled by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, where y(t) are the measurements. Which of the following algorithms could you use to estimate the parameters θ ?

(a) Maximum a Posteriori Estimation (MAP)	(b) Weighted Least Squares (WLS)	
(c) Linear Least Squares (LLS)	(d) Recursive Least Squares (RLS)	

15. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP) . Which of the following algorithms could you use to estimate the parameters θ of this linear model without running into memory problems or high computational costs for a coninuous and infinite flow of measurement data?

(a) LLS	(b) MAP	(c) WLS	(d) RLS
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16. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question $\Sigma_{\hat{\theta}}$. The model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^\top \Phi_N$ and $L(\theta, y_N)$ is the negative log likelihood function. The covariance matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \dots$

(a) $\square Q_N^{-1}$ (b) $\square \nabla_{\theta}^2 L(\theta, y_N)$	(c) $\Box \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(d) $\square (\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N)^{-1}$
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17. Which of the following models is time invariant?

(a) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(b) $\qquad \dot{y}(t) = 5u(t) + t$	(c) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(d) $\qquad \dot{y}(t) = \sqrt{u(t)}$

18. By which of the following formulas is the joint distribution for N independent measurements $y_N \in \mathbb{R}^N$ given? $p(y_N | \theta) = \dots$

(a) $\sum_{i=0}^{N} p(y(i) \theta)$	(b) $\prod \sum_{i=0}^{N} p(y(i) \theta)^2$	(c) $\prod \int_{y_N} p(y \theta) dy$	(d) $\prod \prod_{i=0}^{N} p(y(i) \theta)$

19. Which of the following statements about Maximum A Posteriori (MAP) estimation is not true

(a) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$	(b) \square MAP requires a-priori knowledge on θ	
(c) MAP assumes a linear model	(d) MAP is a generalization of ML	
Which of the following models with input $u(k)$ and output $y(k)$ is NOT linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?		
(a) $\forall y(k) = y(k-1) \cdot (\theta_1 + \theta_2 y(k))$	(b) $\prod u(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	

(a) $y(k) = y(k-1) \cdot (\theta_1 + \theta_2 u(k))$	(b) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$
(c) $\qquad y(k) = \theta_1 \sqrt{u(k)} + \theta_2 u(k)$	(d) $\qquad y(k) = \theta_1 \exp(\theta_2 u(k))$

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg January 13, 2017, 10:15-11:45, Freiburg

Su	name:	First Name:		М	[atriculation num]	ber:
Sul	oject: Progr	ramme: Bachelor	Master	Lehramt of	thers Si	gnature:
1.	Please fill in your name above a Which of the following models	and tick exactly ONI	E box for th	e right answer	of each question	below.
	(a) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(b) $\qquad \dot{y}(t) = 5u$	t(t) + t	(c) $\qquad \dot{y}(t)$	$) = \sqrt{u(t)}$	(d) $\Box t \cdot \ddot{y}(t) = u(t)^3$
2.	By which of the following form	ulas is the joint distr	ibution for	N independen	t measurements y	$y_N \in \mathbb{R}^N$ given? $p(y_N \theta) = \dots$
	(a) $\prod_{i=0}^{N} p(y(i) \theta)$	(b) $\prod \int_{y_N} p(y \theta)$) dy	(c) $\sum_{i=1}^{N}$	$\sum_{i=0}^{N} p(y(i) \theta)$	(d) $\Box \sum_{i=0}^{N} p(y(i) \theta)^2$
3.	Which of the following statement	nts about Maximum	A Posterior	ri (MAP) estim	ation is not true	
	(a) MAP is a generalizatio	n of ML		(b) MA	P requires a-prio	ri knowledge on θ
	(c) MAP assumes a linear	model		$ (\mathbf{d}) \square \hat{\theta}_{\mathrm{M}} $	$_{\rm AP} = \arg\min_{\theta\in\mathbb{F}}$	$\sum_{n} \left[-\log(p(y_N \theta)) - \log(p(\theta))) \right]$
4.	You are given a pendulum whic $y(t)$ are the measurements. Whi	h is by nature a NO	N-LINEAF algorithms	R system and c could you use	an be modeled b to estimate the pa	y $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, where rameters θ ?
	(a) Weighted Least Square	s (WLS)		(b) Ma	ximum a Posterio	ori Estimation (MAP)
	(c) Linear Least Squares ((LLS)		(d) Rec	cursive Least Squ	ares (RLS)
5.	Suppose now that the system giv . Which of the following algorith problems or high computational	ven in the previous q hms could you use to costs for a coninuor	uestion can o estimate t us and infin	be approximation be parameters of the flow of mean the second sec	ted by a model th θ of this linear masurement data?	at is linear in the parameters (LIP) odel without running into memory
	(a) WLS	(b) MAP		(c) LLS	5	(d) RLS
6.	6. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previou question $\Sigma_{\hat{\theta}}$. The model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon})$, $Q_N = \Phi_N^{\top} \Phi_N$ and $L(\theta, y_N)$ is the negative log-likelihood function. The covariance matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \dots$				stimate computed in the previous and $L(\theta, y_N)$ is the negative log	
	(a) $\square Q_N^{-1}$	(b) $\Box \Phi_N^+ \Sigma_{\epsilon_N} \Phi$	$_{N}^{+ T}$	(c) $\square (\Phi_{I}^{\neg})$	$ \sum_{\kappa_N} \sum_{\epsilon_N} \Phi_N)^{-1} $	(d) $\Box \nabla^2_{\theta} L(\theta, y_N)$
7.	We want to assess the robustness the phone breaks when it is dro experiment we have dropped 10 $f(\theta)$ that we need to minimize it	ss of a particular sma pped. We assume th 0 smartphones and c n order to obtain the	artphone brand the phono botained 42 MLE estin	and hence, we he thrown onto broken smartp hate of θ ?	would like to know the ground eithe hones. What is th	by the unknown probability θ that or breaks or has no damage. In an ne negative log likelihood function
	(a) $\log(58\theta) + \log(42(1 - \theta))$	$(\theta))$		(b)	$2\log\theta - 58\log(1)$	$1 - \theta$)
	(c) $-\log(42\theta) - \log(58(2\theta)))$	(1- heta))		(d) 581	$\log \theta + 42 \log(1 - $	- θ)
8.	Given a set of measurements y_N which of the following minimis $\hat{\theta}(N+1)$ after $N+1$ measurer	$= [y(1), y(2), \dots, y]$ sation problems is s ments? $\hat{\theta}(N+1) =$	$(N)^T$ from olved at early $\min_{\theta} \frac{1}{2}$ (n the linear mo ch iteration ste)	del $y_N = \Phi \theta$, where $y_N = \Phi \theta$, where $y_N = \Phi \theta$, where $\Phi \theta$ is the RLS and θ	ere $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, gorithm to estimate the parameter
	(a) $\ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$			(b) $\square \ \theta \ $	$-\hat{\theta}(N)\ _{Q_N}^2 + \ \mathfrak{g}\ _{Q_N}^2$	$y(N) - \varphi(N)^{\top} \theta \ _2^2$
	(c) $\ \theta - \hat{\theta}(N)\ _{2}^{2} + \ y(N)\ _{2}^{2}$	$(+1) - \varphi(N+1)^\top$	$\theta \ _2^2$	$ \left\ (\mathbf{d}) \bigsqcup \ y_N \right\ $	$\ - \Phi_{N+1} \cdot \theta \ $	22

9. Which of the following dynamic models with inputs u(t) and outputs y(t) is **NEITHER** linear **NOR** affine.

	(a) $\[\dot{y}(t) + \sin(t) = u(t) \]$	(b) $\Box t\dot{y}(t) = u(t) + 2$	(c) $\Box \dot{y}(t) = u(t) + t$	(d) $\qquad \dot{y}(t) = \sqrt{t \cdot u(t)}$
10.	Which of the following models	with input $u(k)$ and output $y(k)$	is NOT linear-in-the-parameters	w.r.t. $\theta \in \mathbb{R}^2$?
	(a) $y(k) = \theta_1 u(k)^2 + \theta_2$ e	$\operatorname{xp}(u(k))$	(b) $y(k) = y(k-1) \cdot (\theta_1)$	$+ \theta_2 u(k))$
	(c) $y(k) = \theta_1 \sqrt{u(k)} + \theta_2$	u(k)	(d) $\qquad y(k) = \theta_1 \exp(\theta_2 u(k))$)
11.	Which of the following model e	quations describes a FIR system	with input u and output y ? $y(k - y)$	$(+1) = \dots$
	(a) $u(k) - 5 \cdot u(k-1)$	(b) $\Box u(k) + \sin(k \cdot \pi)$	(c) $\Box u(k) \cdot y(k)$	(d) $[] u(k+1) + y(k)$

- 12. Suppose you are given the Fisher information matrix M of the corresponding problem, what is the relation with the covariance matrix $\Sigma_{\hat{\theta}}$ of your estimate $\hat{\theta}$?
- 13. Give the name of the theorem that provides us with the above result.
- 14. Identify most general transfer function that still is a Finite Impulse Response (FIR) model with $n = \max(n_a, n_b)$. $G(z) = \dots$

(a) $\boxed{\frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}}$	(b) $\square \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(c) $\prod \frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\boxed{ \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}}$
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- 15. Please give the ODE of a linear time invariant (LTI) system, with state vector x and input vector $u. \dot{x} = \dots$
- 16. In L_2 estimation the measurement errors are assumed to follow a ... distribution and it is generally speaking more ... to outliers compared to L_1 estimation.

(a) Laplace, sensitive (b) Gaussian, sensitive (c) (c)	Gaussian, robust (d) 🗌 Laplace, robust
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17. Please identify the most general transfer function that still is a Auto Regressive Model with Exogenous Inputs (ARX) where $n = \max(n_a, n_b)$. $G(z) = \dots$

(a) $\qquad \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(b) $\square \frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(c) $\qquad \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\qquad \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}$
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18. Given the probability density function of the exponential distribution, $p_X(x) = \theta e^{-\theta x}$, with an unknown parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, what is the right minimisation problem you need to solve for a maximum likelihood estimate of θ ? The problem is: $\min_{\alpha} \dots$?

(a) $\ \theta e^{-\theta y(k)}\ _2^2$	(b) $\Box - \log \sum_{k=1}^{N} \theta e^{-\theta y(k)}$
(c) $\Box -N\log(\theta) + \theta \sum_{k=1}^{N} y(k)$	(d) $\qquad \ y(k) - \theta e^{-\theta}\ _2^2$

19. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{yN} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\square N/\theta^2$	(b) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta \sum_k y_k] \mathrm{d}y_N$
(c) $\square \theta_0^2/N$	(d) $\left[\int_{y_N} N\theta^{N-2} \exp[-\theta \sum_k y_k] dy_N \right]^{-1}$

20. The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2} \frac{\|y-\theta\|_2^2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measurements, y(1) = 3, y(2) = 6, and y(3) = 12. What is the minimizer θ^* of the negative log-likelihood function ?

|--|

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg January 13, 2017, 10:15-11:45, Freiburg

Surname:	First Name:	Matriculation number:
Subject:	Programme: Bachelor Master Lehramt	others Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. Please give the ODE of a linear time invariant (LTI) system, with state vector x and input vector u. $\dot{x} = \dots$

2. Please identify the most general transfer function that still is a Auto Regressive Model with Exogenous Inputs (ARX) where $n = \max(n_a, n_b)$. G(z) = ...

(a) $\boxed{\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}}$ (b) $\boxed{\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}}$	(c) $\qquad \frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\square \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$
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- 3. Suppose you are given the Fisher information matrix M of the corresponding problem, what is the relation with the covariance matrix $\Sigma_{\hat{\theta}}$ of your estimate $\hat{\theta}$?
- 4. Give the name of the theorem that provides us with the above result.
- 5. Which of the following models is time invariant?

	e			
	(a) $\qquad \dot{y}(t) = 5u(t) + t$	(b) $\qquad \dot{y}(t) = \sqrt{u(t)}$	(c) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(d) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$
6.	By which of the following form	y which of the following formulas is the joint distribution for N independent measurements $y_N \in \mathbb{R}^N$ given? $p(y_N \theta) = \dots$		
	(a) $\prod \int_{y_N} p(y \theta) dy$	(b) $\prod \sum_{i=0}^{N} p(y(i) \theta)$	(c) $\prod \sum_{i=0}^{N} p(y(i) \theta)^2$	(d) $\prod \prod_{i=0}^{N} p(y(i) \theta)$
7.	Which of the following statements about Maximum A Posteriori (MAP) estimation is not true			
	(a) MAP is a generalizatio	n of ML	(b) \square MAP requires a-priori knowledge on θ	
	(c) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-$	$-\log(p(y_N \theta)) - \log(p(\theta))]$	(d) MAP assumes a linear	model
8.	Which of the following model equations describes a FIR system with input u and output y ? $y(k + 1) =$			$(+1) = \dots$
	(a) $u(k) + \sin(k \cdot \pi)$	(b) $[] u(k) - 5 \cdot u(k-1)$	(c) $\Box u(k) \cdot y(k)$	(d) $\Box u(k+1) + y(k)$
9.	Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is NEITHER linear NOR affine.		OR affine.	
	(a) $\[\dot{y}(t) = \sqrt{t \cdot u(t)}\]$	(b) $[\dot{y}(t) + \sin(t) = u(t)$	(c) $\Box \dot{y}(t) = u(t) + t$	(d) $\Box t \dot{y}(t) = u(t) + 2$
10.	We want to assess the robustness of a particular smartphone brand hence, we would like to know the unknown probability θ that the phone breaks when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In a experiment we have dropped 100 smartphones and obtained 42 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the MLE estimate of θ ?			
	(a) $58\log\theta + 42\log(1-\theta)$)	(b) $-42 \log \theta - 58 \log(1-\theta)$	
	(c) $[-\log(42\theta) - \log(58(1-\theta))]$		(d) $\log(58\theta) + \log(42(1-\theta))$	

11. Given the probability density function of the exponential distribution, $p_X(x) = \theta e^{-\theta x}$, with an unknown parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, what is the right minimisation problem you need to solve for a maximum likelihood estimate of θ ? The problem is: $\min_{\alpha} \dots$?

(a) $\square -N \log(\theta) + \theta \sum_{k=1}^{N} y(k)$	(b) $[] y(k) - \theta e^{-\theta} _2^2$
(c) $\ \theta e^{-\theta y(k)}\ _2^2$	(d) $\Box - \log \sum_{k=1}^{N} \theta e^{-\theta y(k)}$

12. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{y_N} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\prod \left(\int_{y_N} N \theta^{N-2} \exp[-\theta \sum_k y_k] \mathrm{d}y_N \right)^{-1}$	(b) $\square N/\theta^2$
(c) $\square \theta_0^2/N$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta \sum_k y_k] dy_N$

13. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi\theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after N+1 measurements? $\hat{\theta}(N+1) = \arg \min_{0} \frac{1}{2} (\ldots)$

(a) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N+1) - \varphi(N+1)^\top \theta\ _2^2$	(b) $\ y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$
(c) $\ y_N - \Phi_N \cdot \theta \ _{Q_N}^2$	(d) $\qquad \ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N) - \varphi(N)^\top \theta\ _2^2$

14. Identify most general transfer function that still is a Finite Impulse Response (FIR) model with $n = \max(n_a, n_b)$. $G(z) = \dots$

(a) $\square \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$ (b) $\square \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{a_0 z^n}$ (c)	(c) $\prod \frac{z^n}{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}$	(d) $\qquad \frac{b_0 z^n}{z^n + a_1 z^{n-1} + \ldots + a_n}$
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15. You are given a pendulum which is by nature a NON-LINEAR system and can be modeled by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, where y(t) are the measurements. Which of the following algorithms could you use to estimate the parameters θ ?

(a) Recursive Least Squares (RLS)	(b) Maximum a Posteriori Estimation (MAP)
(c) Linear Least Squares (LLS)	(d) Weighted Least Squares (WLS)

16. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP) . Which of the following algorithms could you use to estimate the parameters θ of this linear model without running into memory problems or high computational costs for a coninuous and infinite flow of measurement data?

(a) WLS	(b) MAP	(c) RLS	(d) LLS
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17. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question $\Sigma_{\hat{\theta}}$. The model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon})$, $Q_N = \Phi_N^\top \Phi_N$ and $L(\theta, y_N)$ is the negative log likelihood function. The covariance matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \dots$

a) $\square Q_N^{-1}$	$\Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+ \Phi_N^+$
a) $\square Q_N^{-1}$	$\Box \Phi_N^+ \Sigma_{\epsilon_N} {\Phi_N^+}^\top$

18. In L_2 estimation the measurement errors are assumed to follow a ... distribution and it is generally speaking more ... to outliers compared to L_1 estimation.

	(a) Laplace, robust	(b) Gaussian, robust	(c) Gaussian, sensitive	(d) Laplace, sensitive
9.	Which of the following models	with input $u(k)$ and output $y(k)$	is NOT linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?	
	(a) $y(k) = \theta_1 \sqrt{u(k)} + \theta_2 u(k)$		(b) $\Box y(k) = y(k-1) \cdot (\theta_1 + \theta_2 u(k))$	
	(c) $y(k) = \theta_1 \exp(\theta_2 u(k))$		(d) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	
0.). The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2} \frac{\ y-\theta\ _2^2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measure-			

1

20. The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2}\frac{\|y-v\|_2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measurements, y(1) = 3, y(2) = 6, and y(3) = 12. What is the minimizer θ^* of the negative log-likelihood function ?

(a) 9 (b) 4 (c) 6 (d) | 7