Exercises for Lecture Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2016

Exercise 3: Linear Least Squares

(to be returned on Nov 14th, 2016, 8:15 in SR 00-010/014, or before in building 102, 1st floor, 'Anbau')

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In this exercise you deepen your knowledge of linear least squares and covariance matrices.
For the the MATLAB exercises, create a MATLAB script called `main.m` with your code, possibly calling other functions/scripts. From running this script, all the necessary results and plots should be clearly visible. Compress all the files/functions/scripts necessary to run your code in a `.zip` file and send it to `msi.syscop@gmail.com`. Please state your name and the names of your team members in the e-mail.

Exercise Tasks

1. Given a sequence of i.i.d. scalar random variables $x(1), \dots, x(N)$, each with mean μ and variance σ^2, what is the expected value and variance of their sample mean, i.e. of the random variable y_N defined by $y_N = \frac{1}{N}\sum_{k=1}^{N} x(k)$. (2 point)

2. Implement a function called `myVar(vec)` that computes the sample variance for a given vector `vec`. The formula to compute the sample variance of a series of measurements $\mathbf{v}$ is $S_{\mathrm{v}}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(v_i - \mu_{\mathrm{v}})^2$, where μ_{v} is the sample mean of $\mathbf{v}$ (script section 2.4). (*Hint:* you can use `MATLAB`'s `var` command to check your implementation.) (1 point)

3. Recall the experimental setup to estimate the value of a resistor from exercise 1. We assumed that the measurements were perturbed by additive noise $n_i(k)$ and $n_u(k)$: $i(k) = i_0 + n_i(k)$ and $u(k) = u_0 + n_u(k)$. Given these assumptions, derive why the estimator for the resistance given by

$$\hat{R}_{\mathrm{LS}}(N) = \frac{\dfrac{1}{N}\sum_{k=1}^{N} u(k)i(k)}{\dfrac{1}{N}\sum_{k=1}^{n} i(k)^2} \tag{1}$$

is a least squares estimator. Please give the full derivation, pointing to the script is not sufficient. (2 points)

4. As announced in the lecture task 3 of exercise 2 was extended to this week, therefore the points for this task will be accredited in this exercise. Please hand in your code and solution again if you handed in already. We apologize for the inconvenience. (5 points)

   Additional hints for this question:

   (a) please assume a polynomial of the form $\sum_{i=0}^{n} a_i p^i$ and use at least a 3rd order polynomial ($n \geq 3$).

   (e) We are only looking for *one* fit (using `MATLAB`'s backslash operator) and your analysis of it. Please have a closer look at the coefficients, do you notice something?

   (f) Use the backslash operator, don't invert matrices and have a look at the coefficients.

This sheet gives in total 10 points