Exercises for Lecture Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2017

Exercise 11: Kalman Filter

(to be returned on Feb 6th, 2017, 8:15 in SR 00-010/014,
or before in building 102, 1st floor, 'Anbau')

Prof. Dr. Moritz Diehl, Robin Verschueren, Rachel Leuthold, Tobias Schöls, Mara Vaihinger

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In this exercise, you will implement your own Kalman Filter from scratch. This will give you a better understanding of this commonly used algorithm. Create a MATLAB script called `main.m` with your code, possibly calling other functions/scripts. From running this script, all the necessary results and plots should be clearly visible. Compress all the files necessary to run your code in a `.zip` file and send it to `msi.syscop@gmail.com`. Please state your name and the names of your team members in the e-mail.

Exercise Task

Consider the following measurement setup, consisting of $N+1$ connected compartments with permeable walls as in the figure below.

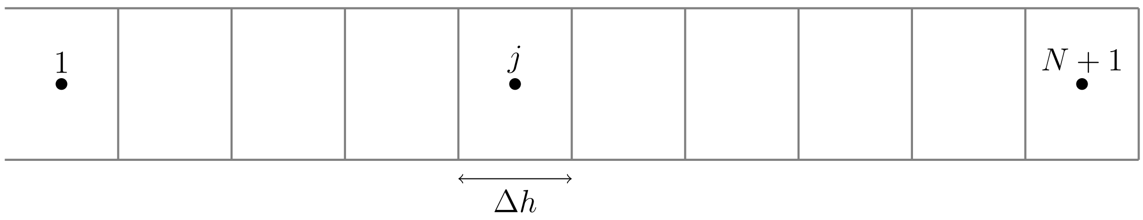

Figure 1: The measurement setup: Compartment j is connected to compartments $j-1$ and $j+1$.

Each compartment has a temperature $T_j[°\mathrm{C}],\ j = 1,\dots,N+1$, where $T_{N+1} = 20\,°\mathrm{C}$. The continuous-time dynamics of the temperature in a compartment is given by

$$\dot{T}_j = \frac{k(T_{j+1} + T_{j-1} - 2T_j)}{(\Delta h)^2}, \quad j = 2,\dots,N, \tag{1}$$

with $k = 0.025$ being some constant coefficient. The only temperature we measure is the one of compartment $N/2$:

$$y = T_{N/2} + v, \tag{2}$$

with $v \sim \mathcal{N}(0, 0.1)$ being the unknown measurement error. Each compartment is of width $\Delta h = 0.5\,\mathrm{m}$. The temperature in the first compartment varies between 12°C and 23°C and is modelled as a random walk defined by

$$T_{1,t+1} = T_{1,t} + w,\ w \sim \mathcal{N}(0, 2.5) \tag{3}$$

1. Which are the conditions on the model and noise for using the Kalman filter for recursive state estimation purposes? Are these conditions met for this particular problem setup?

(1 point)

2. Derive on paper the complete vector and matrix expressions for this problem in a Kalman filter framework. In particular give the state vector x, the process model matrix A and the measurement model matrix C as a general expression. (2 points)

3. Choose an appropriate expression for the process noise matrix W and the measurement noise matrix V. Take into account that the model is not perfect and that the measurement error is small. Justify in one sentence your answer. (2 bonus points)

4. Implement the function for the prediction step using the following prototype:
   `[x_pre, P_pre]=predict(x_k,P_k,A_k,W_k).` (1 bonus point)

5. Implement the function for the innovation update step using the following prototype:
   `[x_upd, P_upd]=update(y_k,x_k,P_k,C_k,V_k).` (1 bonus point)

6. Download the dataset provided on the course website which contains the temperature measurements in room $N/2$. Use this dataset with a sampling time $\Delta t = 5\text{s}$ to develop a Kalman filter based estimation of the temperatures in the single compartments. Initialize x_0 and P_0 to some reasonable values corresponding to the knowledge of the setup and the average ambient temperature being $20°\text{C}$. Plot the temperature of compartment 1 and $N/2$ over time using the `subplot` command. (3 bonus points)

7. Plot how the uncertainty of each compartment's temperature estimate evolves with each filter iteration in a single plot (x-axis: compartments, y-axis uncertainty using `semilogy`). Is the result as expected? (1 bonus point)

8. If the conditions in task 1 are violated, how can you still use the Kalman filter framework for recursive state estimation problems? What are the main extensions to the original Kalman filter? (1 bonus point)

Note: It is important that you address all questions in words, ideally on paper.

This sheet gives in total 3 points and 9 bonus points