Exercises for Lecture Course on Numerical Optimization (NUMOPT)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015-2016

Exercise 9: Christmas Sheet

(to be sent by email to dimitris.kouzoupis@imtek.uni-freiburg.de before Jan 12, 2015)

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Goal of this Christmas sheet is to exchange some information regarding the projects and offer some exposure to exam-type questions.

Exercise Tasks

1. **Commitment on project content:** For this task, you are asked to give us some information regarding the project of your choice. Let us first introduce some rules and guidelines:

   (a) As already announced, the project can be done by one or two people.

   (b) Projects can be either application- or algorithm-oriented. For application-based projects, the focus should be on the mathematical description of the chosen model, its numerical solution and the interpretation of the results. For algorithm-based projects, aim is to investigate the performance of the scheme using several simple examples.

   (c) The report should contain at least one sketch of the modeled system or implemented algorithm.

   (d) The main result is a written report in LaTeX (6 pages, two-column format) submitted as a PDF file. Please use the official IEEE template for conferences that can be downloaded here:

   `www.ieee.org/conferences_events/conferences/publishing/templates.html`

   (e) The report must be a new and self-written document and may not contain any copy of other text or figures. Not a single one. The report must be solely written by the author(s).

   (f) The report must include a short, interesting title, the name(s) of the author(s) and an abstract. The content should be clearly structured in sections. It should start with an introduction and conclude with a short summary and critical discussion of the results.

   (g) Figures and tables should have a short caption and be referenced in the text properly, e.g., "the results are shown in Fig. 1". Use the latex commands `\caption`, `\label` and `\ref`.

   (h) Plots must contain physical units and axis descriptions.

   (i) The report must cite all external sources as references at the end and other people's contributions must be acknowledged. Using other people's ideas and help is allowed, even encouraged. But not citing or acknowledging them properly is a crime.

   (j) Mathematical or physical variables shall consist of one letter only and be printed in italics. This is automatic in Latex, e.g., a_i as `$a_i$`. Physical units and sub- or superscripts that refer to words are in normal roman letters (use `mathrm` when in Latex mathmode, e.g. x_{initial} as `$x_{\mathrm{initial}}$` or $\frac{\mathrm{kg}}{\mathrm{m}^3}$ as `$\frac{\mathrm{kg}}{\mathrm{m}^3}$`. Write, e.g., $m = 5$ kg (and not $m = 5kg$ or m= 5kg).

   (k) On February 12, 2016, during the lecture, a short presentation of 15 minutes (with maximum 10 slides) shall be given by the author(s) to the teacher and the class. The slides can be based on material taken from the report and may contain additional content, e.g., videos, if required.

   (l) The project grade is based on the form and content of the report, the originality and quality of the results, the quality of the slides, the oral presentation and the answers to any questions.

(m) **Deadline** for submission of the written report is:

February 11, 2016, at 13:00, sent by email to dimitris.kouzoupis@imtek.uni-freiburg.de

Your first task for 2016 is to send us an email that contains the title of your project, the corresponding authors and a short description of your concept. For application-oriented projects the latter should comprise a description (in words) of your objective, constraints and decision variables. For algorithm-oriented projects the idea of the algorithmic scheme should be briefly discussed.

(0 points)

2. **A sample exam question:** Regard the following minimization problem.

$$\boxed{\min_{x \in \mathbb{R}^2} \quad x_2^4 + (x_1+2)^4 \quad \text{subject to} \quad \begin{cases} x_1^2 + x_2^2 & \leq & 8 \\ x_1 - x_2 & = & 0 \end{cases}}$$

(a) How many variables, how many equality, and how many inequality constraints does this problem have?

2	

(b) Sketch the feasible set $\Omega \in \mathbb{R}^2$ of this problem.

3	

(c) Bring this problem into the NLP standard form:

$$\min_{x \in \mathbb{R}^n} f(x) \quad \text{subject to} \quad \begin{cases} g(x) & = & 0 \\ h(x) & \geq & 0 \end{cases}$$

by defining the functions f, g, h appropriately.

3	

FROM NOW ON UNTIL THE END TREAT THE PROBLEM IN THIS STANDARD FORM.

(d) Is this optimization problem convex? Justify.

2	

(e) Write down the Lagrangian function of this optimization problem.

2	

(f) A feasible solution of the problem is $\bar{x} = (2, 2)^T$. What is the active set $\mathcal{A}(\bar{x})$ at this point?

2	

(g) Is the *linear independence constraint qualification (LICQ)* satisfied at $\bar{x}$? Justify.

3	

(h) An optimal solution of the problem is $x^* = (-1, -1)^T$. What is the active set $\mathcal{A}(x^*)$ at this point?

1	

(i) Is the *linear independence constraint qualification (LICQ)* satisfied at x^* ? Justify.

2	

(j) Describe the tangent cone $T_\Omega(x^*)$ (the set of feasible directions) to the feasible set at this point x^*, <u>by a set definition formula with explicitly computed numbers.</u>

2	

(k) Compute the Lagrange gradient and find the multiplier vectors λ^*, μ^* so that the above point x^* satisfies the KKT conditions.

4	

(l) Describe the critical cone $C(x^*, \mu^*)$ at the point (x^*, λ^*, μ^*) <u>in a set definition using explicitly computed numbers</u>

3	

(m) Formulate the second order necessary conditions for optimality (SONC) for this problem and test if they are satisfied at (x^*, λ^*, μ^*). Can you prove that x^* is a local or global minimizer?

4	

This sheet gives in total 11 points.
NOTE: your score in the exam-type question will be divided by 3.