AWESCO Winter School on Numerical Optimal Control with DAE - University of Freiburg

Exercise 13: Periodic Optimal Control with DAEs

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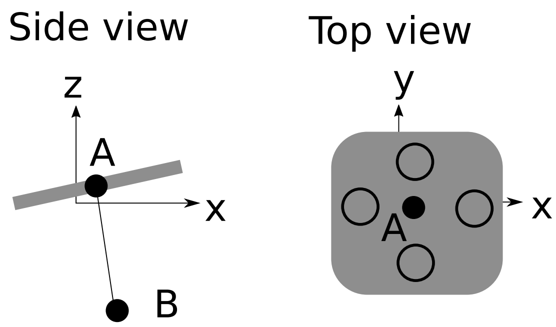
Monodromy matrix

Modeling



The model we will work with in this excercise and the previous represents a quadcopter with a 3D pendulum attached at the centre of mass. In the template you will find a direct collocation method for this system. We will analyze a periodic optimal control problem here instead.

1.1 Adapt the template such that the problem becomes periodic, and you end up with n_x less decision variables. For the null-space basis Z from the slides, use `invariants.jacobian()` and the CasADi `nullspace` command. The solution should be trivial: static hovering at $(0, 0, 0)$.

1.2 There is a parameter `r0` in the file that sets the distance between two waypoints that the quadcopter should move to. It was set to 0 previously. Increase it to 0.1 m. Does the problem converge? Set it to 0.0001 m. Does the problem converge now?

1.3 Use a homotopy strategy to solve the $r_0 = 0.1$ m problem: build a loop over the NLP construction and solution code where r_0 takes the values `[0.001 0.01 0.05 0.1]` and where the solution of one iteration serves as a starting value for the next iteration's NLP.

1.4 Set up a collocation based integrator to plot the solution trajectory with 10 times more samples:

```
options = struct;
options.tf = T/N/10;
options.implicit_solver = 'newton';
options.number_of_finite_elements = 1;
options.interpolation_order = 4;
options.collocation_scheme='radau';
options.implicit_solver_options = struct('abstol',1e-9);

intg = integrator('intg','collocation',dae,options);
```

Use `x = full(res_X{1});` as a starting point, and integrate all the way to the end time. Plot the top-side view of the end effector and quadcopter position trajectory and make sure it looks like a closed orbit.

1.5 Extra. For periodic systems, the monodromy matrix $\frac{\partial x(t_f)}{\partial x(0)}$ provides information about the stability of the system. Compute the monodromy matrix using symbolic calls to the integrator function with T/N interval, and using `jacobian`. Is the system stable?