AWESCO Winter School on Numerical Optimal Control with DAE - University of Freiburg

Exercise 11: High Index DAEs and Index Reduction

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A first toy example

Let us consider the following system of Differential Algebraic Equations (DAE):

$$\begin{aligned} \dot{x}_1(t) - x_3(t) &= 0 \\ x_2(t)(1 - x_2(t)) &= 0 \\ x_1(t)x_2(t) + x_3(t)(1 - x_2(t)) &= t \end{aligned} \tag{1}$$

The following tasks should be carried out on pen and paper, so without using CasADi:

1.1 What is the differential index of the DAE system above? Does the index depend on the initial condition $[x_1(0), x_2(0), x_3(0)]$?

1.2 If yes, how does the DAE index change exactly with respect to that initial condition?

1.3 In case the system has an index higher than one, derive the equivalent index-1 DAE by differentiating with respect to time.

1.4 Additionally, write down the corresponding consistency conditions (if there are any) which are necessary to keep your DAE model equivalent to the original system.

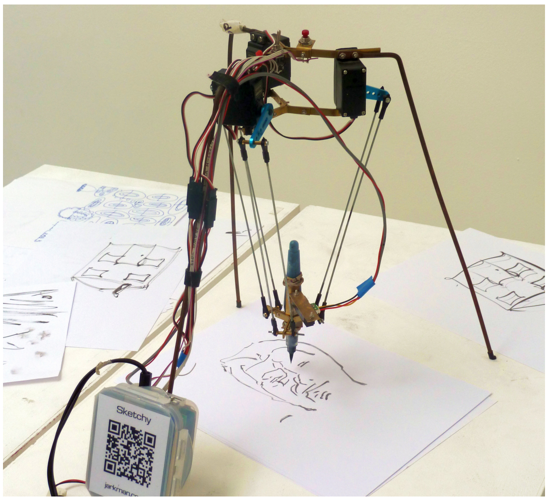
The delta robot model

As a more illustrative example, we consider the dynamic model for a delta robot as in Figure . You can check again slide 18 of Lecture 10 for the DAE, using the Lagrange formalism.

Tasks:

2.1 Starting from the template file "*delta_robot.m*" on the `www.syscop.de` website, try to complete the DAE model as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = F_{\mathrm{g}}. \tag{2}$$



2.2 What is the differential index of this DAE system? If necessary, you should reformulate it into an equivalent index-1 DAE model.
TIP: Use the information that the above DAE is semi-explicit.

2.3 Now, we will look for an initial value $q(0) = q_0$ which satisfies both the constraints $c(q_0) = 0$ and $\dot{c}(q_0) = 0$. Given this initial value q_0, we want to compute the corresponding motor torques u_0 to keep the delta robot in a steady state condition, i.e. $\dot{q} = 0$ and $\ddot{q} = 0$. For this purpose, you should complete the template code implementing Newton's method to solve these systems of nonlinear equations.

2.4 We are ready to simulate the DAE system using e.g. the IDAS solver. Try to run the code both with the flag `steady_state` set to 1 and 0. What happens to our consistency conditions $c(q(t)) = 0$ and $\dot{c}(q(t), \dot{q}(t)) = 0$ in both simulations?

```
dae = struct('x',[q;dq],'z',lam,'p',u,'ode',[dq;ddq],'alg',ddc);
opts = struct('tf',h,'abstol',1e-10,'reltol',1e-10);
F = integrator('F', 'idas', dae, opts);
```

2.5 To avoid such a potential drift of constraints when simulating the DAE system for a longer time, implement the Baumgarte stabilization as explained in the slides of Lecture 10. Note that satisfying these constraints depends both on the consistency of the initial value $[q(0), \dot{q}(0)]$ and the numerical accuracy of the integrator (`abs_tol`, `rel_tol`).