Modelling and System Identification – Microexam 2

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Surname: Name: Matriculation number:

Study: Studiengang: Bachelor ☐ Master ☐

Please fill in your name above and tick exactly one box for the right answer of each question below.

1. Maximum Likelihood Estimator (MLE) for broken smartphones: we regard a phone thrown onto the ground that either breaks or has no damage. We want to assess the robustness of this particular phone brand, and the unknown probability that the phone breaks is θ. In an experiment, we have thrown the smartphone 15 times, and obtained 9 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the MLE estimate of θ ?

(a) ☐ $9\theta + 6(1-\theta)$	(b) ☐ $-6\theta - 9(1-\theta)$	(c) ☐ $6\log\theta + 9\log(1-\theta)$	(d) ☐ $-9\log\theta - 6\log(1-\theta)$

2. Imagine that you have a linear in the parameters (LIP) system that outputs a continuous and infinite flow of measurement data. Which algorithm could you use to estimate the parameters θ of the system without running into memory problems or high computational costs? ☐

3. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is **NOT** time varying ?

(a) ☐ $t^{-2}\dot{y}(t)^3 = u(t)$	(b) ☐ $\cos(t) - \ddot{y}(t) = u(t)$	(c) ☐ $\sqrt{\dot{y}(t)} = u(t)$	(d) ☐ $\dot{y}(t) = u(t) + t$

4. Given the probability density function of the exponential distribution, $p_X(x) = \theta e^{-\theta x}$, with an unknown parameter θ, and a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, what is the right minimisation problem you need to solve for a maximum likelihood estimate of θ? The problem is: $\min\limits_{\theta} \dots$?

(a) ☐ $\|\theta e^{-\theta y(k)}\|_2^2$	(b) ☐ $\sum_{k=1}^{N} \theta e^{-\theta y(k)}$	(c) ☐ $\theta^N e^{-\theta \sum_{k=1}^{N} y(k)}$	(d) ☐ $-N\log(\theta) + \theta \sum_{k=1}^{N} y(k)$

5. Suppose you are given the Fisher information matrix M of the corresponding problem, what is the relation with the covariance matrix $\Sigma_{\hat{\theta}}$ of your estimate $\hat{\theta}$? ☐

6. Give the name of the theorem that provides us with the above result.

☐

7. For the problem in Question 4, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{y_N} \nabla_\theta^2 L(\theta_0, y_N) \cdot p(y_N|\theta_0)\mathrm{d}y_N$.

(a) ☐ θ_0^2/N	(b) ☐ N/θ^2
(c) ☐ $\int_{y_N} N\theta_0^{N-2} \exp[-\theta \sum_k y_k]\mathrm{d}y_N$	(d) ☐ $\left(\int_{y_N} N\theta^{N-2} \exp[-\theta \sum_k y_k]\mathrm{d}y_N\right)^{-1}$

8. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \cdot \theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after $N+1$ measurements? $\hat{\theta}(N+1) = \arg\min\limits_{\theta} \frac{1}{2}(\dots)$

(a) ☐ $\|\theta - \hat{\theta}(N)\|_{Q_N}^2 + \|y(N+1) - \varphi(N+1)^\top\theta\|_2^2$	(b) ☐ $\|y_{N+1} - \Phi_{N+1} \cdot \theta\|_{Q_N}^2$
(c) ☐ $\|y_N - \Phi_N \cdot \theta\|_2^2$	(d) ☐ $\|\theta - \hat{\theta}(N)\|_{Q_N}^2 + \|y(N) - \varphi(N)^\top\theta\|_2^2$

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9. Which of the following statements is **NOT** correct. Recursive Least Squares (RLS):

(a) ☐ computes an estimation with a computational cost independent of the number of past measurements	(b) ☐ implicitly assumes that there is only i.i.d. and Gaussian measurement noise
(c) ☐ can use prior knowledge on the estimated parameter θ	(d) ☐ can be used as an alternative to Maximum Likelihood Estimation

10. In L_1 estimation the measurement errors are assumed to follow a ... distribution and it is generally speaking more ... to outliers compared to L_2 estimation.

(a) ☐ Gaussian, sensitive	(b) ☐ Laplace,sensitive	(c) ☐ Laplace,robust	(d) ☐ Gaussian,robust

11. The PDF of a random variable Y is given by $p(y) = \frac{1}{2\sqrt{2\pi}} \exp(-\frac{1}{2}\frac{\|y-\theta\|_2^2}{4})$, with unknown $\theta \in \mathbb{R}$. We obtained three measurements, $y(1) = 2$, $y(2) = 7$, and $y(3) = 9$. What is the minimizer θ^* of the negative log-likelihood function ?

(a) ☐ 6	(b) ☐ 4	(c) ☐ 9	(d) ☐ 12

12. We want to estimate the resistance R of a new metal and we found in the only existing previous article that an estimate of R is given by $10[\Omega]$ with standard deviation 0.5Ω. Our own measurement apparatus sets a current I as a noise-free input, and measures the output voltage V which has Gaussian errors with a standard deviation of $0.1[V]$. Given a set of N measurements, $[V(1),\ldots,V(N)]$ obtained from a set $[I(1),\ldots,I(N)][A]$, what function is minimized by the Bayesian Maximum-A-Posteriori (MAP) estimator in this context? To simplify notation we assume that all variables are made unitless.

(a) ☐ $\frac{(R-10)^2}{0.5} + \sum_{i=1}^N \frac{(V(i)/I(i)-R)^2}{0.1}$	(b) ☐ $\frac{(R-10)^2}{0.5} + \sum_{i=1}^N \frac{(V(i)-I(i)R)^2}{0.1}$
(c) ☐ $4(R-10)^2 + \sum_{i=1}^N 100(V(i)-I(i)R)^2$	(d) ☐ $0.5(R-10)^2 + \sum_{i=1}^N 0.1(V(i)-I(i)R)^2$

13. Given a one step ahead prediction model $\boxed{y(k) = \theta_2 u(k)^3 + \theta_1 y(k-1) + \epsilon(k)}$ with unknown parameter vector $\theta = (\theta_1,\theta_2)^T$, and assuming i.i.d. noise $\epsilon(k)$ with zero mean, and given a sequence of N scalar input and output measurements $u(1),\ldots,u(N)$ and $y(1),\ldots,y(N)$, we want to compute the linear least squares (LLS) estimate $\hat{\theta}$ by minimizing a function $f(\theta) = \|y_N - \Phi_N\theta\|_2^2$. How do we need to choose the matrix Φ_N and vector y_N?

(a) ☐ $\Phi_N = \begin{bmatrix} y(1) & u(1)^3 \\ \vdots & \vdots \\ y(N) & u(N)^3 \end{bmatrix}, \quad y_N = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix}$	(b) ☐ $\Phi_N = \begin{bmatrix} y(1) & u(2)^3 \\ \vdots & \vdots \\ y(N-1) & u(N)^3 \end{bmatrix}, \quad y_N = \begin{bmatrix} y(2) \\ \vdots \\ y(N) \end{bmatrix}$
(c) ☐ $\Phi_N = \begin{bmatrix} u(2)^3 & y(1) \\ \vdots & \vdots \\ u(N)^3 & y(N-1) \end{bmatrix}, \quad y_N = \begin{bmatrix} y(2) \\ \vdots \\ y(N) \end{bmatrix}$	(d) ☐ $\Phi_N = \begin{bmatrix} y(2) & u(2)^3 \\ \vdots & \vdots \\ y(N) & u(N)^3 \end{bmatrix}, \quad y_N = \begin{bmatrix} y(2) \\ \vdots \\ y(N) \end{bmatrix}$

14. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is **NEITHER** linear **NOR** affine.

(a) ☐ $t\dot{y}(t) = u(t)$	(b) ☐ $\dot{y}(t) = \sqrt{t}u(t)$	(c) ☐ $\dot{y}(t)+\sin(t) = u(t)$	(d) ☐ $\dot{y}(t)^2 = u(t)+t$

15. Which of the following models with input $u(k)$ and output $y(k)$ is **NOT** linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?

(a) ☐ $y(k) = \theta_1 y(k-1) - \sin(\theta_2 u(k))$	(b) ☐ $y(k) = \sin(y(k-1)) \cdot (\theta_1 + \theta_2 \sin u(k))$
(c) ☐ $y(k) = \theta_1\sqrt{u(k)} + \theta_2 u(k)$	(d) ☐ $y(k) = \theta_1 u(k)^2 + \theta_2 \sin(u(k))$

16. Which one of the following statements is **NOT** true for FIR models:

(a) ☐ The output of the system does not depend on previous outputs	(b) ☐ Their dimension is usually big when used in Output Error Minimization
(c) ☐ They can be used for Model Error Minimization	(d) ☐ They are a special class of ARX models

17. Write a general expression for an Auto Regressive with Exogenous Inputs(ARX) Model:

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