Nonparametric and Frequency Domain Identification Methods

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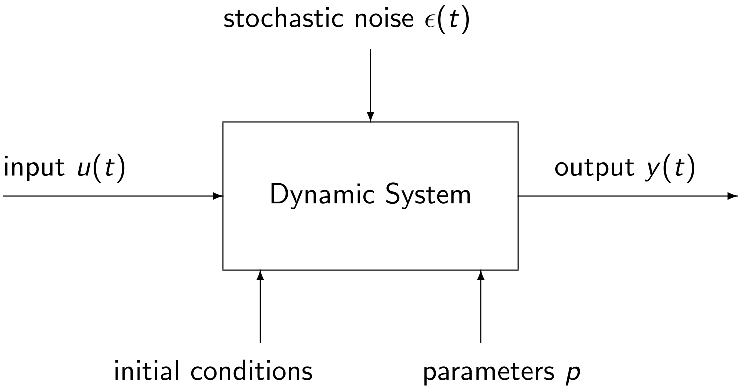
Overview

- LTI systems
- impulse response and Bode diagram
- step response experiments
- frequency sweep experiments

Recall: general identification setting

- user input $u(t)$ and output $y(t)$ can be measured
- noise $\epsilon(t)$ disturbs our experiments
- system model typically unknown



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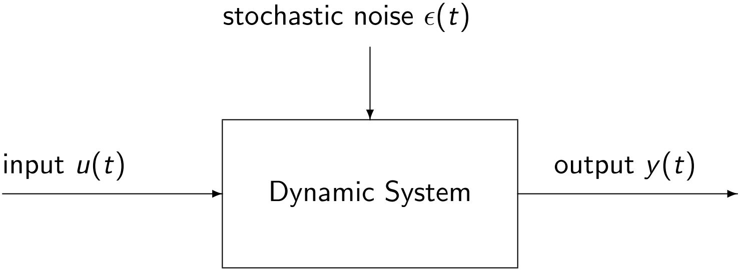


Nonparametric modelling: identify transfer function directly.

Comments

Nonparametric Modelling

- Aim of nonparametric modelling: make model predictions without real modelling work
- Approach: choose model class and identify "black-box" model
- In the special case of linear time invariant (LTI) models, it is enough to identify the impulse response function (as we will discuss in the following)

Comments

LTI models in continuous and discrete time

- A continuous time LTI system allows us to compute, for any horizon $[0, T]$ and control trajectory $u(t)$ for $t \in [0, T]$, the output trajectory $y(t)$ for $t \in [0, T]$.
- Typically, we assume the initial conditions to be zero.
- The MATLAB commands (`lsim, step, bode,...`) can be used for both discrete and continuous time models.
- We can convert one into the other with the MATLAB commands `d2c` and `c2d`.

Comments

The impulse response and transfer function

TIME DOMAIN

- If **impulse response** $g(t)$ is known, the output for any input signal $u(t)$ can be computed by a convolution

$$y(t) = \int_0^\infty g(\tau)u(t-\tau)\mathrm{d}\tau$$

CONVOL.

- In the Laplace domain, a convolution translates to a multiplication of the Laplace transforms:

$$Y(s) = G(s)U(s)$$

- The **transfer function** $G(s)$ characterizes the system completely, and is the Laplace transform of the impulse response:

$$G(s) = \int_0^\infty e^{-st}g(t)\mathrm{d}t$$

$U(s) = \dots$

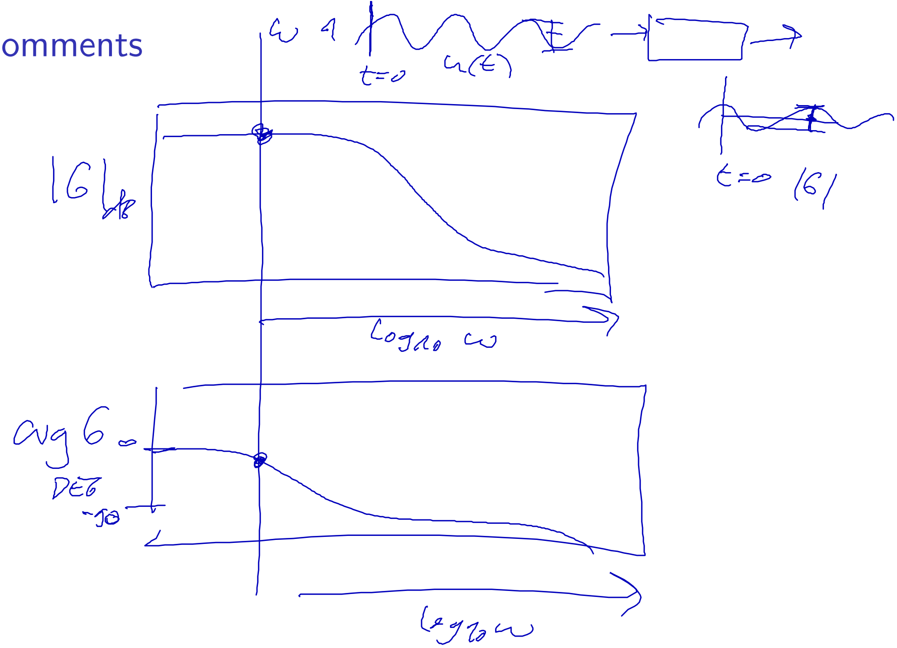
$Y(s) = \dots$

Comments

Bode diagrams

- One way to visualize the transfer function $G(s)$ is via **Bode diagrams**
- They show the values of $G(j\omega)$ for all positive values of ω (here, j is the imaginary unit, and ω is measured in rad/s)
- a Bode diagram consists of two parts, a **magnitude** and a **phase plot**, both with frequencies ω as x-axis, where the frequencies ω are logarithmically spaced.
- the magnitude plot shows the magnitudes $|G(j\omega)|$ logarithmically
- the phase plot shows the argument $\arg G(j\omega)$ of the complex number $G(j\omega)$, i.e. its angle in the complex plane.
- the MATLAB command `bode` can generate the Bode diagram of a known system.

Comments



Bode Diagrams from Frequency Sweeps

- x

Comments

Discrete time LTI systems

$t = 1, 2, \ldots,$ $\quad t = -8, -7, \ldots$

$$y(t) = \sum_{k=0}^{\infty} g(k) \cdot u(t-k)$$

INF. IMP. RESP. (IIR)
IF $g(k) \neq 0$ FOR SOME ARBITRARILY HIGH k

- x

$$= \sum_{k=0}^{n} g(k)\, u(t-k)$$

FIR

Comments

Discrete time transfer function

- If **discrete time impulse response** values $g(0), g(1), \ldots$ are known, the general output is computed by a linear combination of past inputs (again a convolution):

$$y(t) = \sum_{k=0}^{\infty} g(k)u(t-k)\mathrm{d}\tau$$

- In the so called **z-domain**, a convolution translates to a multiplication of the so called **z-transforms**:

$$Y(z) = G(z)U(z)$$

- Here, the z-transform of any signal, like g, u, y, is defined by

$$G(z) := \sum_{t=0}^{\infty} z^{-t} g(t)$$

- Note that we have a finite impulse response (FIR) model if $g(k)$ has finitely many nonzero values, otherwise it is an infinite impulse response (IIR) model

Comments

Discrete time Bode diagrams

- a discrete time Bode diagram plots the values of the complex function $G(z)$ on the unit circle, i.e. $z = e^{j\omega T}$ where T is the sampling time.
- above $\omega = \frac{2\pi}{T}$, the values of z repeat themselves. In fact, one only plots the values on the upper semi-circle, up to the Nyquist frequency $\omega_{\max} = \frac{\pi}{T}$, so the Bode diagram has a limited range of ω.
- Note that $G(e^{j\omega T})$ is given by

$$G(e^{j\omega T}) := \sum_{k=0}^{\infty} e^{-jk\omega T} g(k)$$

This looks a bit similar to the definition of the **discrete fourier transform** (DFT or FFT).