Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015

Exercise 6: Linear Least Squares for ARX models

(to be returned on Dec 22, 2015, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

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Your MATLAB solution has to run from a main script called `main.m`, which can call other functions/scripts, but when running this script all the necessary results and plots should be clearly visible.Compress the folder in a `.zip` file and send it to `fabiangirrbach@gmail.com`. Please state also your name and the names of your team members in the e-mail.

Exercise Task

Consider a test setup with a LED light driven by a PWM (Pulse Width Modulation) signal with a frequency of $f_0 = 100\,\mathrm{Hz}$. The dutycycle $D \in [0, 1]$ of a PWM signal determines what portion of a cycle a signal is active, i.e. it is active for D/f_0 s, and inactive for $(1 - D)/f_0$ s within one cycle. We want to identify an ARX-type model (Auto-Regressive with eXogeneous inputs) for the heating of the LED. We take the following form for the ARX model:

$$T(k) = -\sum_{i=1}^{n_a} a_i T(k-i) + \sum_{i=1}^{n_b} b_i D(k-i) + \epsilon(k), \tag{1}$$

where $T\,[\mathrm{C}]$ is the temperature of the LED, $D\,[-]$ is the dutycycle of the PWM signal, n_a, n_b are the number of past outputs and inputs, respectively, and $\epsilon\,[\mathrm{C}]$ is the output noise.

1. **Problem formulation**

   (a) What are the outputs, what are the inputs? What assumption do we need in order to do a Linear Least Squares fit? (1 point)

   (b) Write down (on paper) the Linear Least Squares problem you need to solve in order to estimate the parameters a_i, b_i in Eq. (1). Clearly state all the vectors and matrices that are needed. (3 points)

   (c) How would you estimate the uncertainty of a one-step-ahead predictor starting from the values identified for the previous question? Cf. Chapter 4 of the script. (2 points)

2. **MATLAB implementation**

   Download from the course website the simulation routine `LEDsim.m`, which you can use to simulate the LED. It takes N values for the dutycycles and returns $N + 1$ resulting temperatures. The initial temperature is set within this simulation function.

   (a) Choose an input signal with N between 50 and 500 and generate measurements from the function `LEDsim.m`. (1 point)

   (b) Implement the above Linear Least Squares estimation in MATLAB. (2 points)

   (c) Plot the measurements along with the one-step-ahead predictors at each time step. (1 point)

(d) Calculate the variance on the one-step-ahead predictors and plot it along with the value of the predictions. *Hint:* `errorbar`. How well does your identified ARX model do? Try different combinations of n_a and n_b. Compare the residuals and look up what 'overfitting' means. (2 points)

3. **Optional: MATLAB System Identification Toolbox**

   If you have the MATLAB System Identification Toolbox installed, we remark that all of the above can be conveniently done with (among others) the `arx` function. Check if you get similar results as with your self-written routine.

This sheet gives in total 12 points