Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015

Exercise 5: Recursive Least Squares

(to be returned on Dec 8, 2015, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

Prof. Dr. Moritz Diehl, Robin Verschueren, Jesus Lago and Fabian Girrbach

Your MATLAB solution has to run from a main script called `main.m`, which can call other functions/scripts, but when running this script all the necessary results and plots should be clearly visible. In order to submit your code, include all the necessary files in a single folder using a folder format name `Ex03_Surname1` up to the four possible surnames of the group members. Compress the folder in a `.zip` file and send it to `robin.verschueren@gmail.com`. Please state also your name and the names of your team members in the e-mail.

Exercise Tasks

1. **Extra: Variance Estimation of average wind power** (7 bonus points)

   In the last exercise sheet the parameters of the Weibull distribution $\theta = [\lambda, k]$ were estimated. With them, the expected value of the wind energy power was computed by a map $E_{\text{Power}}(\theta)$ which approximated $\mathbb{E}\{P_{\text{power}}\}$. In real life applications, not only the expected value of the power, but also the variance of this estimate is an important quantity in order to assess the quality of the location of the wind farm.

   (a) The covariance Σ_θ of the parameter guess for the Weibull distribution can not be easily estimated. Propose one approximation for it that could be obtained with the measurement data you have. (4 bonus points)

   (b) Assume now that the covariance Σ_θ is known. Propose an approximation of the variance of the power $\sigma^2_{E_{\text{Power}}(\theta)}$ and compute this value. State the power expectation obtained in the last sheet in the form 'X[kW] $\pm$ Y[kW]'.
   Hint 1: approximate the map $E_{\text{Power}}(\theta)$ by its first order Taylor expansion:

   $$E_{\text{Power}}(\theta) = E_{\text{Power}}(\hat{\theta}) + \frac{dE_{\text{Power}}}{d\theta}(\hat{\theta})(\theta - \hat{\theta}). \qquad (1)$$

   Hint 2: any derivative can be computed by finite differences. (3 bonus points)

2. **Recursive Least Squares applied to position data** (8 points)

   We will apply the Recursive Least Squares (RLS) algorithm to position data of a 2-DOF robot moving in the $X - Y$ plane, measured with a sampling time of $0.0159\,\text{s}$. It is your task to implement the RLS algorithm in MATLAB and to tune it with the appropriate "forgetting factors". The position data for this exercise can be downloaded from the course website. You can assume that the noise on the X and Y measurements is independent. The experiment starts at $t = 0\,\text{s}$.

   (a) Fit a 4-th order polynomial through the data using ordinary Least Squares *Hint: you need one estimator for each coordinate*. Plot the data and the fit both in the $X - Y$ plane and separately. Does the fit look good? Why is that (not) the case? (2 points)

(b) Implement the RLS algorithm as described in the script to estimate 4-th order polynomials to fit the data. Do not use forgetting factors yet. Plot the result against the data. Compare the LS estimator from `a)` with the RLS estimator after processing N measurements. (2 points)

(c) Add forgetting factors to your algorithm. Tune them to obtain a more or less smooth curve. Did you arrive to the same results for both forgetting factors? Plot the results on the same plot as the previous question. (2 points)

(d) Plot the "one step ahead prediction" at each point (i.e. extrapolate your polynomial fit to the next time step), along with the $1 - \sigma$ confidence ellipsoid around this point, and the data. First think what the covariance Σ_p on the position is, if you know the covariance of the estimator Σ_θ. Do the confidence ellipsoids grow bigger or smaller as you take more measurements? *Hint: use the fact that for a random variable $\gamma = A\theta$, where A is a matrix,* $\mathrm{cov}(\gamma) = A\mathrm{cov}(\theta)A^T$. (2 points)

This sheet gives in total 8 points and 7 bonus points