Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015

Exercise 2: Optimization and Liner Least Squares

(to be returned on Nov 10, 2015, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

Prof. Dr. Moritz Diehl, Robin Verschueren, Jesus Lago and Fabian Girrbach

The aim of this sheet is to give an overview of numerical optimization as well as introduce least square estimation.
Your MATLAB solution has to run from a main script called `main.m`, which can call other functions/scripts, but when running this script all the necessary results and plots should be clearly visible. In order to submit your code, include all the necessary files in a single folder using a folder format name `Ex02_Surname1.Name1_Surname2.Name2` up to the four possible surnames and names of the group members. If submitting the report by mail, keep this report within the same folder. Compress the folder in a `.zip` file and send it to `robin.verschueren@gmail.com`. Please state also your name and the names of your team members in the e-mail for better clarification.

Exercise Tasks

1. Which of the following functions $f : \mathbb{R}^n \to \mathbb{R}$ are convex? Justify your answer. Here, A, B are fixed matrices, b and c fixed vectors of appropriate dimensions and $c^\top x > 0$. (4 points)

   (a) $f(x) = c^\top x + x^\top A^\top A x$

   (b) $f(x) = -c^\top x - x^\top A^\top A x$

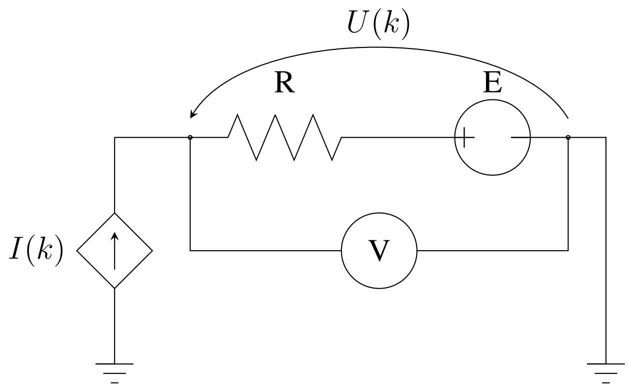
   (c) $f(x) = \log(c^\top x) + \exp(b^\top x)$

   (d) $f(x) = -\log(c^\top x) + \exp(b^\top x)$

2. (a) Give a formula for the minimizer x^* of the function $f : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto f(x) = \|Ax - b\|_2^2$, where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$ are given. You can assume that A has rank n. Justify your answer. (2 points)

   (b) Assume now that b is a random variable with mean μ_b and covariance matrix Σ_b, while A remains fixed. This makes x^* also a random variable. What is the mean and what is the covariance matrix of x^*? (2 points)

3. Consider the following experimental set up to estimates the values of E and R.



Every experiment consists of N measurements of the voltage $U(k)$ for different values of $I(k)$. The measurements $U(k)$ are affected by additive Gaussian noise with mean μ and standard deviation σ:

$$U(k) = E + RI(k) + n_u(k)$$

Here we assume that the input variable $I(k)$ is not affected by noise.

Tasks:

(a) Import the data available on the course website to MATLAB and plot the $U(k)$, $I(k)$ relation using 'x' markers. (1 points)

(b) Use a least squares estimator in matrix form to find the experimental values of R and E and plot the linear fit through the $U(k)$, $I(k)$ data. (2 points)

(c) A thermistor is a resistor which resistance varies with a change of the resistor temperature. A basic model of such a effect is $R = R_0(1 + k_1(T(t) - T_0))$, where R_0 is the resistance at ambient temperature T_0, and where $k_1[\frac{\Omega}{K}]$ is positive for PTC (positive temperature coefficient) thermistors and negative for NTC (negative temperature coefficient) thermistors. On the other hand, resistor self-heating due to power dissipation increases the resistor temperature, being this power dissipation also a function of the temperature difference between the ambient and the resistor that can me modelled as $P = k_2(T(t) - T_0))$, where $k_2[\frac{W}{K}] > 0$. Modelling $R_0^2 k_1/k_2$ as a single constant k_3, and assuming that the power dissipated can be approximated by $P \approx I^2 R_0$, obtain the new equation model of U and compute the least squares estimator of R_0, k_3 and E. (3 points)
Optional: Finally plot the nonlinear fit into the same figure as before (use `legend` and different colors to clearly show the correspondence of each plot).

(d) Using the estimation values of part c, give an approximation of $\frac{k_1}{k_2}$. Is it a NTC or PTC type of thermistor? Justify your answer. (1 points)

This sheet gives in total 15 points points