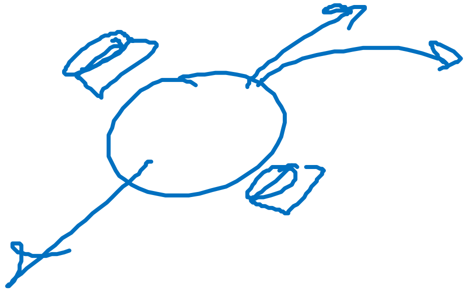
• PART III : DYNAMIC SYSTEM MODELLING AND IDENTIFICATION

- SYSTEMS AND CONTROL I (DISCR. & CONT.)
- MOD. & SIM. (HOT AIR BALLOON)
- → EL. MAG. WAVES (MAXWELL & PARTIAL DIFF. EQ.)
- EXP. PHYSICS I (NEWTON'S MECHANICS, $\boxed{m\ddot{p} = F}$)
  II (E)
- ESE LAB : SPARC IGNITION ENGINE (PROF. SCHOLL)



AIM: GIVEN $\boxed{\text{INPUT \& OUTPUT DATA}}$ OF DYN. SYSTEM, FIND MODEL EQUATIONS AND PARAMETERS THAT ALLOW YOU TO <u>PREDICT</u> THE OUTPUTS GIVEN <u>ANY</u> INPUTS

6.1

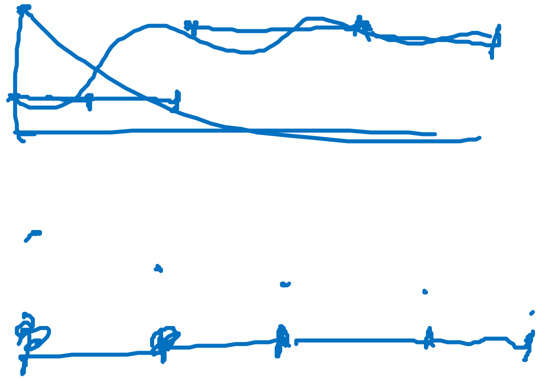
CONT. vs. DISCRETE TIME

" " STATE SPACE

FINITE vs. INF. DIM. STATE "

LIN. vs. NONLINEAR SYSTEMS

DETERMINISTIC vs STOCHASTIC SYSTEM



STATE SPACE FORM

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$u(t)$: INPUT TRAJECTORY

$y(t)$: OUTPUT "

$x(t)$: STATE "

$$\boxed{\begin{aligned} x(k+1) &= A_d\, x(k) + B_d u(k) \\ y(k) &= C_d x(k) + D_d u(k) \end{aligned}}$$

"d" DISCRETE TIME

$$y^{(n)} := \frac{d^n y}{dt^n}$$

$$y^{(1)} = \dot{y} \qquad y^{(2)} = \ddot{y} \qquad y^{(0)} := y$$

INPUT-OUTPUT FORM (COEFF. UNIQUE)

$$\boxed{y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_0 y = b_q u^{(q)} + \dots + b_0 u}$$

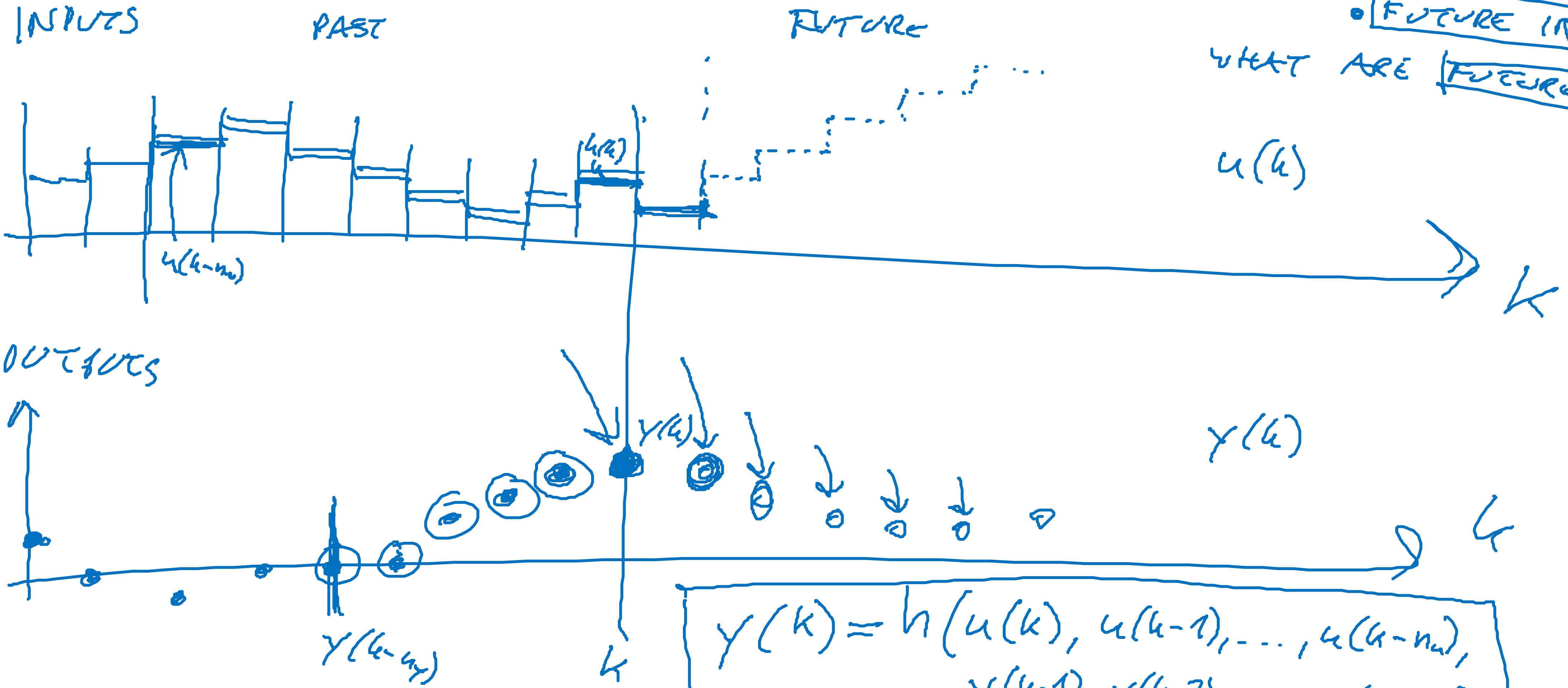
$q \leq n$, n = STATE DIM.

$$\boxed{\begin{aligned} & y(k+n) + a^d_{n-1} y(k+n-1) + \dots + a^d_0 y(k) \\ & = b^d_q u(k+q) + \dots + b^d_0 u(k) \end{aligned}}$$

6.6

6.6 DETERMINISTIC MODELS IN DISCRETE TIME (INPUT-OUTPUT FORM)

AIM OF PREDICTION:
GIVEN • $\boxed{\text{PAST DATA}}$
• $\boxed{\text{FUTURE INPUTS}}$
WHAT ARE $\boxed{\text{FUTURE OUTPUTS}}$



$$\boxed{\begin{aligned} y(k) = h\big(&u(k), u(k-1), \ldots, u(k-n_u), \\ &y(k-1), y(k-2), \ldots, y(k-n_y)\big) \end{aligned}}$$

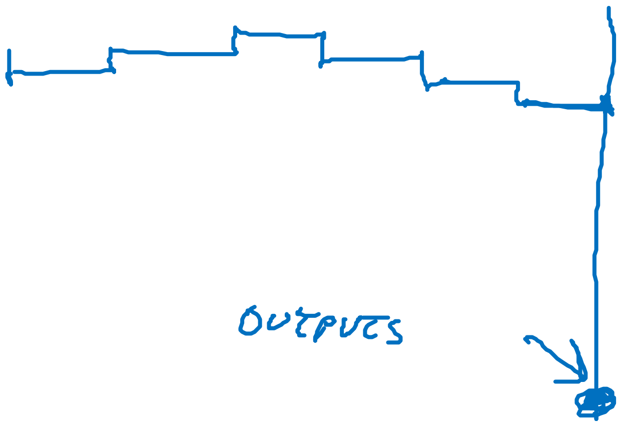
GENERAL FORM.

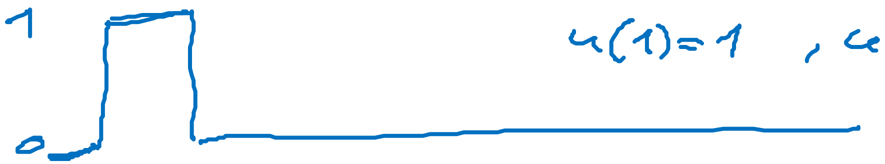
FIRST SPECIAL: FINITE IMPULSE RESPONSE (FIR) MODELS

$$y(k) = \boxed{b_0 \cdot u(k) + b_1 \cdot u(k-1) + \dots + b_{n_u} \cdot u(k-n_u)}$$

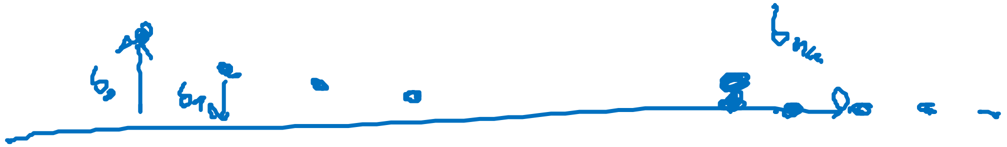
$$= \sum_{i=0}^{n_u} b_i \cdot u(k-i)$$

(IF n_u BIG ENOUGH, ALL STABLE SYSTEMS ARE COVERED (PRACTICALLY))





$u(1)=1$, $u(k)\equiv 0$ FOR $k \geq 2$
FOR $k \leq 0$



$y(k) = \sum b_i\, u(k-i)$

$y(1) = b_0 \cdot$

$y(2) = b_1\, u(2-1)$

$y(3) = b_2$

$\cancel{y(n_u+1) = (n_u+1-n_u)}$

$$Y(n_k-1) = \dots \quad \delta_{n_k-2} \cdot \underbrace{u((n_k-1)-(n_k-2))}_{u(1)} + \delta_{n_k-1} \cdot u((n_k-1)-(n_k-1))$$

$$= \delta_{n_k-2}$$

$$Y(n_k) = \delta_{n_k-1}$$

$$Y(n_k+1) = \delta_{n_k}$$

$$Y(n_k+2) = 0$$

$$\vdots$$

SYSTEM ID FOR FIR MODELS.

GIVEN $y(1), \dots, y(N)$

$u(1), \dots, u(N)$

ORDER $n_u =: n$ AND FIR MODEL STRUCTURE $\boxed{y(k) = \sum_{i=0}^{n} b_i \, u(k-i)}$

AIM: FIND $\theta = [b_0, \dots, b_n]^T$ THAT FITS DATA "BEST"

IDEA: REGARD INPUTS AS <u>REGRESSORS</u>, USE LINEAR LEAST SQUARES

$$\boxed{\underset{\theta \in \mathbb{R}^{n+1}}{\text{minimize}} \sum_{k=n+1}^{N} \Big(\underbrace{y(k)}_{\text{MEASUREMENTS}} - \underbrace{\sum_{i=0}^{n} b_i \, u(k-i)}_{\text{MODEL PREDICTIONS}} \Big)^2} \qquad \| y_N - \Phi_N \cdot \theta \|_2^2$$

$$y_N = \begin{bmatrix} y(n+1) \\ y(n+2) \\ \vdots \\ y(N) \end{bmatrix} \qquad \Phi_N = \begin{array}{c} \begin{array}{cccccc} b_0 & b_1 & & & & b_n \end{array} \\ \begin{bmatrix} u(n+1) & u(n) & \dots & & - & u(1) \\ u(n+2) & u(n+1) & u(n) & \dots & & u(2) \\ \vdots & & & & & \vdots \\ u(N) & \dots & & & & u(N-n) \end{bmatrix} \end{array} \qquad \theta = \begin{bmatrix} b_0 \\ \vdots \\ b_n \end{bmatrix} = \Phi_N^+ y_N$$