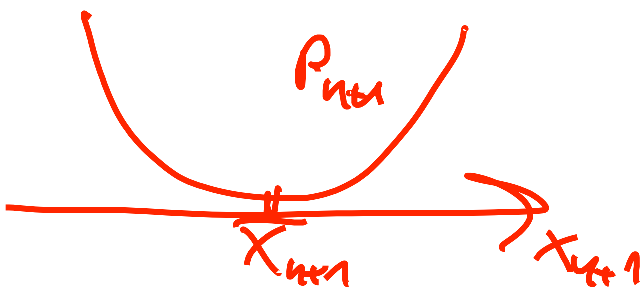
DP - LQ

$$\alpha_k(x_k) = \frac{1}{2}\|x_k - \bar{x}_k\|^2_{P_k}$$

$$\alpha_{k+1}(x_{k+1}) = \min_{x_k} \frac{1}{2}\Big(\|x_k - \bar{x}_k\|^2_{P_k} + \|y_k - C_k x_k\|^2_{Q} + \|x_{k+1} - A_k x_k - b_k\|^2_{R}\Big)$$

$$= \frac{1}{2}\|x_{k+1} - \bar{x}_{k+1}\|^2_{P_{k+1}} + \text{const}$$

$$P_{k+1} = R - (A_k^T R)^T \big(\underbrace{P_k + C_k^T Q C_k + A_k^T R A_k}_{=:\tilde{R}}\big)^{-1} (A_k^T R)$$



12.5 REC. BAYESIAN ESTIMATION

$$P(x_{n+1}|y_0,\dots,y_n) = \int_{x_{n+1}=f_n(x_n,w_n)} P(w_n)\cdot \frac{P(x_n|y_0,\dots,y_{n-1})}{(\cdot\ \text{const})}\cdot P(y_n|x_n,w_n)\, dx_n\, dw_n$$

$\boxed{12.6}$

ASS: $P(x_n|y_0,\dots,y_{n-1}) = \text{const}\cdot \exp\left(-\frac{1}{2}\|x_n-\bar{x}_n\|^2_{P_n}\right)$

USE: $P(w_n) = \exp\left(-\frac{1}{2}\|w_n\|^2_R\right)\cdot \text{const}.$

$P(y_n|x_n,w_n) = \exp\left(-\frac{1}{2}\|y_n-C_nx_n\|^2_Q\right)\cdot \text{const}.$

$x_{n+1}=A_nx_n+b_n+w_n \iff w_n = x_{n+1}-A_nx_n-b_n$

$$P(x_{n+1}|y_0,\dots,y_n) = \int \exp\left(-\frac{1}{2}\|x_{n+1}-A_nx_n-b_n\|^2_R\right)\cdot \exp\left(-\frac{1}{2}\|x_n-\bar{x}_n\|^2_{P_n}\right)\cdot \exp\left(-\frac{1}{2}\|y_n-C_nx_n\|^2_Q\right) dx_n \cdot \text{const}$$

...

$$= \int \exp\left(-\frac{1}{2}\underbrace{\left[\|x_{n+1} - A_n x_n - b_n\|^2_R + \|x_n - \bar{x}_n\|^2_{P_n} + \|y_n - C_n x_n\|^2_Q\right]}_{= \text{const} + \|x_{n+1} - \bar{x}_{n+1}\|^2_{P_{n+1}} + \|m + M x_{n+1} + x_n\|^2_{\tilde{R}}}\right) dx_n \cdot \text{const}$$

$$= \exp\left(-\frac{1}{2}\|x_{n+1} - \bar{x}_{n+1}\|^2_{P_{n+1}}\right) \cdot \text{const} \cdot \underbrace{\int \exp\left(-\frac{1}{2}\|m + M x_{n+1} + x_n\|^2_{\tilde{R}}\right) dx_n}_{= \text{const}}$$

$$= \exp\left(-\frac{1}{2}\|x_{n+1} - \bar{x}_{n+1}\|^2_{P_{n+1}}\right) \cdot \text{const}$$

12.7 KF & EKF

$\underset{n}{|} \longrightarrow \underset{n+1}{|}$

LQ - CASE:

INPUT: $\bar{x}_n$, P_n, y_n, Q, C_n, R, A_n, b_n

$[x_{n+1} = A_n x_n + b_n + w_n]$, ...

$\frac{1}{2} \| x_{n+1} - \bar{x}_{n+1} \|^2_{P_{n+1}}$

OUTPUT: $\bar{x}_{n+1}$, P_{n+1}

COMPUTATIONAL STEPS:

$\tilde{R} = \ldots\ldots$, $\tilde{S} = -P_n \bar{x}_n - C_n^T Q y_n + A_n^T P_n b_n$

$P_{n+1} = \left(R - (A_n^T R)^T \tilde{R} (A^T R) \right)$, $\underline{\underline{\bar{x}_{n+1} = \ldots (\bar{x}_n, y_n)}}$

EXTENDED KF:

$$x_{n+1} = f_n(x_n) + w_n$$

$$y_n' = g_n(x_n) + v_n$$

KF

(1) $x_{n+1} = A_n x_n + b_n + w_n$

(2) $y_n = C_n x_n + v_n$

IDEA: LINEARIZE f_n, g_n AT $\bar{x}_n$ (TAYLOR)

$$x_{n+1} = \frac{\partial f_n}{\partial x_n}(\bar{x}_n)\cdot(x_n - \bar{x}_n) + f_n(\bar{x}_n) + w_n$$

$$= \underbrace{\frac{\partial f_n}{\partial x}(\bar{x}_n)}_{A_n}\cdot x_n + \underbrace{f_n(\bar{x}_n) - \frac{\partial f_n}{\partial x_n}\cdot \bar{x}_n}_{b_n} + w_n \longrightarrow (1)$$

$$y_n' = \underbrace{\frac{\partial g_n}{\partial x_n}(\bar{x}_n)}_{C_n}\cdot x_n + g_n(\bar{x}_n) - \frac{\partial g_n}{\partial x_n}(\bar{x}_n)\cdot \bar{x}_n + v_n$$

$$y_n = y_n' - g_n(\bar{x}_n) + \frac{\partial g_n}{\partial x_n}(\bar{x}_n)\cdot \bar{x}_n \longrightarrow (2)$$