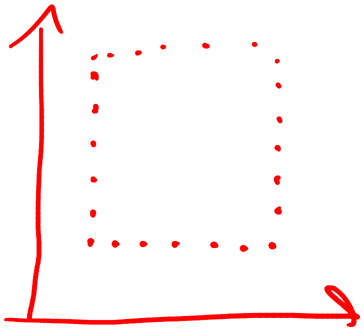
DP - CONVENTIONAL : DISCRETIZE STATE SPACE



$n_x \gg 6$: DP NOT APPLICABLE DUE TO "CURSE OF DIMENSIONALITY" (R. BELLMAN)

LINEAR-QUADRATIC : REPRESENT J AS QUADRATICS

$$\boxed{x^T P x = \min_u \begin{bmatrix} x \\ u \end{bmatrix}^T \begin{bmatrix} Q & S^T \\ S & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}}$$

SCHUR-COMPLEMENT LEMMA : IF $R \succ 0$

$$P = Q - S^T R^{-1} S$$

DIFFERENCE RICCATI EQUATION:

$$\boxed{P_{NEW} = \underbrace{Q + A^T P A}_{=} - (S^T + A^T P B)\underline{(R + B^T P B)^{-1}(S + B^T P A)}}$$

INFINITE HORIZON,

$\Rightarrow$ LQR

$$u^*(x) = -\underbrace{(R + B^T P B)^{-1}(S + B^T P A)}_{=:K}\, x$$

MORE GENERAL ~~EX~~. (EG. (7.8)): AFFINE SYSTEMS

$$* + \begin{bmatrix} x \\ u \end{bmatrix}^T \begin{bmatrix} Q & S^T \\ S & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + \underline{2 \begin{bmatrix} q \\ s \end{bmatrix}^T \begin{bmatrix} x \\ u \end{bmatrix}} = \begin{bmatrix} 1 \\ x \\ u \end{bmatrix}^T \begin{bmatrix} * & q^T & s^T \\ q & Q & S^T \\ s & S & R \end{bmatrix} \begin{bmatrix} 1 \\ x \\ u \end{bmatrix}$$

$$x_+ = Ax + Bu + \underline{c} = \begin{bmatrix} c \mid A \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} + Bu$$

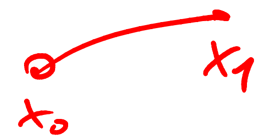
$$\boxed{\tilde{x}_n = \begin{bmatrix} 1 \\ x_n \end{bmatrix}}$$

$$\tilde{x}_{n+1} = \left[\begin{array}{c|c} 1 & 0 \\ \hline \underline{\underline{c_n}} & A_n \end{array}\right] \cdot \tilde{x}_n + \begin{bmatrix} 0 \\ B \end{bmatrix} u_n$$

$$\tilde{x}_0 = \begin{bmatrix} 1 \\ \bar{x}_0 \end{bmatrix}$$

SIMPLE EXAMPLE: $x, u \in \mathbb{R}$

$N = 1$



$$\boxed{\begin{aligned} &\min_{x_0, u_0, x_1} \quad \underline{\underline{u_0^2}} + \underline{10\, x_1^2} \\ &\qquad\qquad x_1 = 2 \cdot x_0 + 1 \cdot u_0 \\ &\qquad\qquad x_0 = \bar{x}_0 \end{aligned}}$$

$P_N = P_1 = 10$ $\quad \begin{bmatrix} Q & S^T \\ S & R \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$A = 2, \quad B = 1$

$$P_0 = A^T P_1 A - (A^T P_1 B)(R + A^T P_1 A)^{-1}(B^T P_1 A) =$$

$$= 40 - (20)\,\underline{(1 + 40)^{-1}(1 \cdot 10 \cdot 2)}$$

$$P_0 = 40 - \frac{400}{41} \approx 30$$

$$u_0^*(\bar{x}_0) = -\left(41 \right)^{-1} \left(20 \right) \bar{x}_0 = -\frac{20}{41} \cdot \bar{x}_0$$

$$\bar{x}_0 = 1 = x_0^* \qquad u_0^* = -\frac{20}{41}$$

$$x_1^* = 2 \cdot x_0^* + 1 u_0^* = 2 - \frac{20}{41} \approx 1.5$$

KNAPSACK AS DISCRETE TIME OPTIMAL CONTROL PROBLEM: N OBJECTS, EACH WITH VALUE $-L(o_i)$

$o_1, \dots, o_N$

$$\min \sum_{k=1}^{N} L(o_i) + E(x_N)$$

$$\begin{bmatrix} x_{k+1} \\ s_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \cup \{u_k\} \\ \text{SIZE} \end{bmatrix}$$

$$x_0 = \begin{bmatrix} \{\ \} \\ 0 \end{bmatrix}$$

$X \equiv \begin{bmatrix} \text{SET OF OBJECT CHOSEN ALREADY} \\ \text{SIZE SO FAR} \end{bmatrix}$

$x_0 = \begin{bmatrix} \{\ \} \\ 0 \end{bmatrix}$

$\mathcal{O}$ = SET OF ALL OBJECTS

u = OBJECT WE CHOOSE TO ADD

o_i : VALUE $v(o_i) =:$

SIZE $s(o_i) =:$

$$E(x_N) = \begin{cases} 0 \\ \infty & \text{IF } \sum \text{SIZE} \geq \text{LIMIT} \end{cases}$$

$$L(x_k, u_k) = \begin{cases} -v(u_k) \\ \infty & \text{IF } u_k \in x_k \end{cases}$$

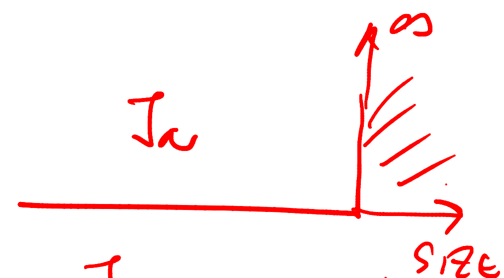
$$x_{k+1} = x_k + s(o_k) \cdot u_k$$

x : SIZE SO FAR

$u \in \{0, 1\}$ TAKE OBJECT k OR NOT

$$L_k(x_k, u_k) = -v(o_k) \cdot u_k$$

$$E(x_N) = \begin{cases} 0 \\ \infty & \text{IF } x_N \geq \text{TOTAL SIZE} \end{cases}$$



$N = 100$

2^{100}

$n_x = 1000$

$N \cdot n_x \cdot 2$

200 000