Exercises for Lecture Course on Optimal Control and Estimation (OCE)
Albert-Ludwigs-Universität Freiburg – Summer Term 2015

# Exercise 8 - Time Optimal Direct Multiple Shooting (deadline: 29.6.2015, 2:15pm)

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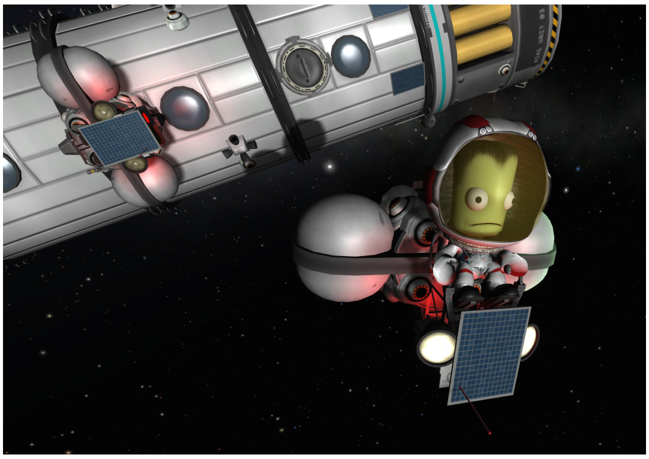
The aim of this exercise is to setup and solve a time optimal control problem using direct multiple shooting.

## Minimum time Kerbal Retrieval



Jebediah Kerman has gone for an EVA and lost track of time. He can't remember when atmospheric re-entry is scheduled, but he believes it is very soon. He needs to get back to his spaceship as quickly as possible.

Jebediah has mass $30\mathrm{kg}$ including space suit but not including fuel. He is currently carrying $10\mathrm{kg}$ of fuel. He is $50\mathrm{m}$ away from his ship, with zero relative velocity. He wants to return to the ship as quickly as possible (to have equal position and zero relative velocity), while still conserving $4\mathrm{kg}$ of fuel for emergencies.

Model the Kerbal as having three states: position $p$, velocity $v$, and fuel mass $m_F$. The space suit has a rocket booster (control $u$) which can fire forwards or reverse, and the differential equation governing the state is:

$$\frac{d}{dt}\begin{pmatrix} p \\ v \\ m_F \end{pmatrix} = \begin{pmatrix} v \\ u/\left(30 + m_F\right) \\ -u^2 \end{pmatrix} \tag{1}$$

## Tasks

1. Write down the continuous time optimal control problem with a minimum time objective $T$.

   (3 points)

2. Discretize this problem using direct multiple shooting, and write down the NLP.

   Use the shooting function $x_{k+1} = f_{\mathrm{rk4}}(x_k, u_k, \Delta t)$ with $\Delta t = \frac{T}{N}$ being an optimization variable, so your vector of optimization variables is $y = [x_0, u_0, \ldots, u_{N-1}, x_N, \Delta t]^\top$.

   (2 points)

3. Using an RK4 integrator, implement this NLP and solve it with `fmincon`. Use number of control intervals $N = 40$. Use nonlinear constraints ONLY for the dynamics constraints. For all other constraints, use lower and upper bounds on design variables (this includes equality constraints where upper bounds equal lower bounds). Think of a proper initialization.

4. Plot $p$, $v$, $m_F$, and $u$ versus time.

   (5 points)

5. **Bonus question:** Make a sketch of the Hessian of the Lagrange function.

   You will see that the Hessian is sparse but not block diagonal. Can you find a problem reformulation with a block diagonal Hessian? Make a sketch of the new Hessian.

   (Hint: Introduce multiple copies of you timestep $\Delta t$ and make it a pseudo state.)

   (2 bonus points)