Exercises for Lecture Course on Optimal Control and Estimation (OCE)
Albert-Ludwigs-Universität Freiburg – Summer Term 2015

Exercise 1 - Discrete Dynamic Systems (deadline: 4.5.2015)

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1 Dynamic Systems

Give two different examples of dynamics systems. To which of the following system classes do your example dynamic systems belong?

- Continuous or Discrete Time System
- Continuous or Discrete State Space System
- Finite or infinite dimensional state space system
- Time-Variant or Time-Invariant Systems
- Linear or Non-linear Systems
- Controlled or Uncontrolled Dynamic System
- Stable or Unstable Dynamic Systems
- Deterministic or Stochastic Systems

(2 points)

2 Simulation

A discrete time LTI system is given by:

$$x_{k+1} = Ax_k + Bu_k. \tag{1}$$

For a given initial state x_0 and given sequence of controls $u_0, \ldots, u_{N-1}$ we can compute all states $x_0, \ldots, x_N$. Assume matrices A and B to be given.

a) Give the forward simulation solution x_N after N simulation steps. Try to write it down in a compact form.

b) Calculate the map to the final state x_5 for given scalar system and control matrices $A = 0.1$, $B = 1.0$, and $N = 5$.

(2 points)

3 Population Model

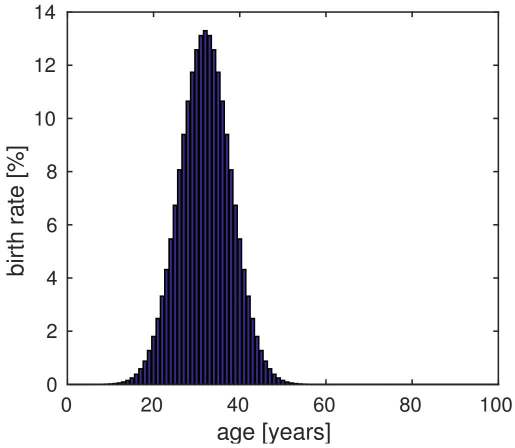
Consider a linear model of a population. State vector $x \in \mathbb{R}^{100}$ represents the population of each age group. Let $x_i(k)$ mean the number of people of age i during year k. For instance, $x_6(2014)$ would be the number of people who are 6 years old in year 2014. Each year babies (0-year-olds) are formed depending on a linear birth rate:

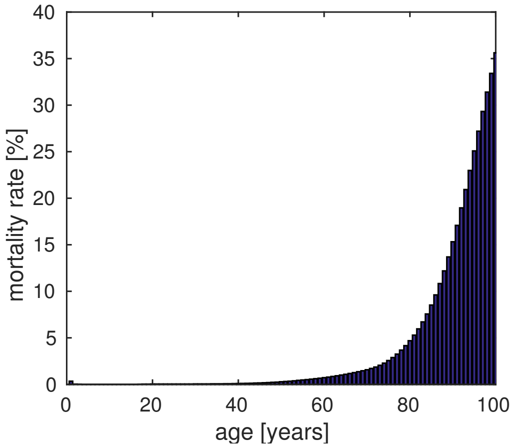
$$x_0(k+1) = \sum_{j=0}^{99} \beta_j x_j(k) \tag{2}$$

Each year most of the population ages by one year, except for a fraction who die according to mortality rate μ:

$$x_{i+1}(k+1) = x_i(k) - \mu_i x_i(k) \qquad i = 0, \ldots, 98 \tag{3}$$

β and μ will be provided for you by the file *birth_mortality_rates.m* on the course website. [1]
They look like:





Tasks

1. Write the discrete time model in the form of

$$x(k+1) = A\,x(k) \tag{4}$$

2. Lord of the Flies: Setting an initial population of 100 four-year-olds, and no other people, simulate the system for 150 years. Make a 3-d plot of the population, with axes {year, age, population}.

3. Eigen decomposition: Plot the eigenvalues of A in the complex plane. Plot the real part of the two eigenvectors of A which have largest eigenvalue magnitude.

   Is this system stable? What is the significance of these eigenvectors with large eigenvalues?

4. Run two simulations: in each simulation, use for $x(0)$ the real part of an eigenvector from the previous question. What is the significance of this result?

(6 points)

[1] Mortility data from Sterbetafel 2010/2012 Statistisches Bundesamt