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Exercise 5: YALMIP for convex optimization

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http://syscop.de/teaching/numerical-optimal-control/

Short introduction to YALMIP YALMIP (**Y**et **A**nother **LMI** (linear matrix inequality) **P**arser) is a modeling language for advanced modeling and solution of convex and nonconvex optimization problems. It is implemented as a free (as in no charge) toolbox for MATLAB. Further information at:

`http://users.isy.liu.se/johanl/yalmip/`

Note: YALMIP is only the modeling language, to solve the problems, it needs some installed underlying solvers such as `quadprog`, qpOASES, CPLEX, Sedumi, SDPT3, etc.

Equilibrium position for a hanging chain We want to model a chain attached to two supports and hanging in between. Let us discretize it with N mass points connected by $N-1$ springs. Each mass i has position $(y_i, z_i),\ i = 1, \ldots, N$. The equilibrium point of the system minimizes the potential energy. The potential energy of each spring is

$$V_{\mathrm{el}}^i = \frac{1}{2} D \left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 \right).$$

The gravitational potential energy of each mass is

$$V_{\mathrm{g}}^i = m\, g_0\, z_i.$$

The total potential energy is thus given by:

$$V_{\mathrm{chain}}(y, z) = \frac{1}{2} \sum_{i=1}^{N-1} D \left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 \right) + g_0 \sum_{i=1}^{N} m\, z_i, \tag{1}$$

where $y = [y_1, \cdots, y_N]^T$ and $z = [z_1, \cdots, z_N]^T$. We wish to solve:

$$\begin{array}{ll} \underset{y,z}{\text{minimize}} & V_{\mathrm{chain}}(y,z) \\ \text{subject to} & (y_1, z_1) \ = (-2, 1) \\ & (y_N, z_N) = (2, 1) \end{array}$$

The problem we want to solve is relatively simple; this gives us the possibility to get to know the tools to solve this and other more practical problems. This particular optimization problem can be made a bit more interesting by adding inequality constraints, modeling a plane that the chain might touch.

5.1 First, let us formulate the problem using $N = 40,\ m = 4/N$ kg, $D = \frac{70}{40} N$ N/m, $g_0 = 9.81$ m/s^2 with the first and last mass point fixed to $(-2, 1)$ and $(2, 1)$, respectively. You can start from the template code `hanging_chain.m` on our event page.

5.2 Solve this simple problem using `quadprog` from YALMIP and interpret the results.

5.3 Introduce ground constraints: $z_i \geq 0.5$ and $z_i - 0.1\, y_i \geq 0.5$. Solve the resulting Quadratic Program (QP) and plot the result. Compare the result with the previous one.

5.4 What would happen if you add instead of the piecewise linear ground constraints, the nonlinear ground constraints $z_i \geq -0.2 + 0.1 y_i^2$ to your problem? Do not start coding yet! The resulting problem is no longer a QP, but do you think the problem is still convex?

5.5 What would happen if you add instead the nonlinear ground constraints $z_i \geq -y_i^2$? Just thinking about it, do you expect this optimization problem to be convex?

5.6 **Extra**: Check the above results from Tasks 5.4 and 5.5 numerically using YALMIP. Note that, since the problem is no longer a QP, you will have to use a solver different from `quadprog` such as e.g. `fmincon` or `SDPT3`. If any of these 2 optimization problems is nonconvex, does it have multiple local minima? If yes, can you confirm that numerically by initializing the solver differently? Note that you can provide an initialization in YALMIP by setting the option `usex0`:

```
options = sdpsettings('solver', 'fmincon','verbose',2,'usex0',1);
```