Solution for Exercise 4: Direct single and multiple shooting

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TEMPO Summer School on Numerical Optimal Control and Embedded Optimization
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Contents

Part 0: Define parameters inverted pendulum swing-up

```
clc;
close all;
clear all;

global N;
global Q;
global R;
global x0;
global xref;

Nx = 4;     % number of differential states
Nu = 1;     % number of control inputs
Nz = Nx+Nu;
N = 30;     % number of shooting intervals
T = 1.5;      % Horizon length
Ts = T/N;
x0 = [0; pi; 0; 0];
xref = [0; 0; 0; 0];
Q = diag([0 0 0 0]);
R = 1;
umax = 20;

global input;
input.Ts = Ts;
input.nSteps = 1;
input.sens = 1;
```

Part 1: Single shooting method

```
% Bound values:
LB_SS = -umax*ones(N*Nu,1);
UB_SS = umax*ones(N*Nu,1);

% Initialize single shooting variables:
Z_SS = zeros(Nu,N);

% Setting options and solving the problem with fmincon + SINGLE SHOOTING:
options = optimoptions(@fmincon,'Display','iter','Algorithm','interior-point','GradObj'
,'on','GradConstr','on','Hessian','user-supplied','HessFcn',@hessian_single);
sol_single = fmincon(@cost_single,Z_SS,[],[],[],[],LB_SS,UB_SS,@constr_single,options);
```

```
Z_SS = sol_single;
U = Z_SS;

X = zeros(Nx,N+1);
X(:,1) = x0;
for k = 1:N
    u_k = U((k-1)*Nu+1:k*Nu);
    input.x = X(:,k);
    input.u = u_k;
    output = RK4_integrator( @ode, input );
    x_k = output.value;
    X(:,k+1) = x_k;
end
```

```
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       1    0.000000e+00    3.142e+00    0.000e+00
    1       3    2.981066e+02    2.488e+00    4.010e+00    2.442e+01
    2       5    6.047679e+02    2.152e+00    7.137e+00    1.079e+01
    3       7    7.736411e+02    1.913e+00    8.297e+00    6.198e+00
    4       9    8.773237e+02    1.688e+00    1.242e+01    6.222e+00
    5      12    9.486421e+02    1.472e+00    1.759e+01    7.702e+00
    6      16    9.789313e+02    1.344e+00    2.191e+01    6.141e+00
    7      20    9.977155e+02    1.251e+00    2.522e+01    5.389e+00
    8      24    1.009914e+03    1.179e+00    2.896e+01    4.825e+00
    9      28    1.018358e+03    1.120e+00    3.218e+01    4.386e+00
   10      32    1.025158e+03    1.071e+00    3.495e+01    4.035e+00
   11      36    1.031849e+03    1.028e+00    3.800e+01    3.744e+00
   12      39    1.059324e+03    1.003e+00    4.444e+01    6.991e+00
   13      42    1.089674e+03    1.029e+00    4.978e+01    6.148e+00
   14      45    1.125743e+03    1.033e+00    5.506e+01    5.422e+00
   15      48    1.166900e+03    1.024e+00    5.920e+01    4.757e+00
   16      51    1.211140e+03    1.007e+00    6.160e+01    4.144e+00
   17      54    1.256323e+03    9.857e-01    6.260e+01    3.589e+00
   18      57    1.300908e+03    9.632e-01    6.308e+01    3.111e+00
   19      60    1.344481e+03    9.403e-01    6.261e+01    2.740e+00
   20      63    1.388362e+03    9.174e-01    6.131e+01    2.530e+00
   21      66    1.436638e+03    8.934e-01    5.961e+01    2.570e+00
   22      69    1.497718e+03    8.663e-01    5.800e+01    2.989e+00
   23      72    1.585712e+03    8.349e-01    5.676e+01    3.894e+00
   24      76    1.650263e+03    8.067e-01    5.654e+01    2.666e+00
   25      80    1.715137e+03    7.787e-01    5.553e+01    2.528e+00
   26      83    1.840882e+03    7.434e-01    5.168e+01    4.488e+00
   27      86    1.944236e+03    7.079e-01    4.495e+01    3.475e+00
   28      89    2.027522e+03    6.757e-01    3.723e+01    2.697e+00
   29      92    2.093971e+03    6.478e-01    2.972e+01    2.105e+00
   30      94    2.203501e+03    6.130e-01    1.669e+01    3.346e+00

                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
   31      96    2.276408e+03    5.845e-01    1.197e+01    2.297e+00
   32      99    2.329480e+03    5.618e-01    1.208e+01    1.887e+00
   33     102    2.401515e+03    5.366e-01    1.160e+01    2.604e+00
   34     105    2.470983e+03    5.123e-01    1.128e+01    2.526e+00
   35     108    2.546241e+03    4.882e-01    1.153e+01    2.737e+00
   36     111    2.606464e+03    4.656e-01    1.164e+01    2.223e+00
   37     114    2.657877e+03    4.445e-01    1.168e+01    1.925e+00
   38     117    2.706662e+03    4.242e-01    1.179e+01    1.855e+00
   39     120    2.759453e+03    4.037e-01    1.240e+01    2.047e+00
```

```
    40     123    2.805183e+03    3.843e-01    1.249e+01    1.838e+00
    41     126    2.844811e+03    3.661e-01    1.229e+01    1.660e+00
    42     129    2.882606e+03    3.485e-01    1.205e+01    1.658e+00
    43     132    2.923255e+03    3.307e-01    1.184e+01    1.870e+00
    44     135    2.955778e+03    3.142e-01    1.145e+01    1.583e+00
    45     138    2.983962e+03    2.986e-01    1.102e+01    1.430e+00
    46     141    3.011358e+03    2.833e-01    1.061e+01    1.439e+00
    47     144    3.041321e+03    2.674e-01    1.023e+01    1.627e+00
    48     147    3.075876e+03    2.509e-01    1.001e+01    1.940e+00
    49     150    3.111085e+03    2.341e-01    9.940e+00    2.072e+00
    50     153    3.137144e+03    2.171e-01    9.497e+00    1.654e+00
    51     156    3.157629e+03    2.012e-01    8.844e+00    1.389e+00
    52     158    3.194355e+03    1.843e-01    7.680e+00    2.522e+00
    53     160    3.227813e+03    1.642e-01    6.456e+00    2.612e+00
    54     162    3.253869e+03    1.399e-01    5.105e+00    2.477e+00
    55     163    3.286750e+03    1.169e-01    3.645e+00    3.716e+00
    56     164    3.293548e+03    8.242e-02    2.287e+00    2.082e+00
    57     165    3.306329e+03    3.635e-02    3.362e+00    3.731e+00
    58     166    3.301133e+03    6.170e-03    2.292e+00    1.597e+00
    59     167    3.300412e+03    8.463e-04    5.925e-01    7.346e-01
    60     168    3.299606e+03    8.192e-05    1.504e-01    2.945e-01

                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    61     169    3.299396e+03    1.405e-05    8.263e-02    1.188e-01
    62     170    3.299389e+03    1.934e-06    1.980e-02    3.711e-02
    63     171    3.299389e+03    2.770e-07    1.125e-02    1.351e-02
    64     172    3.299389e+03    3.709e-08    2.453e-03    5.060e-03
    65     173    3.299387e+03    5.322e-09    1.528e-03    1.929e-03
    66     174    3.299387e+03    7.407e-10    3.602e-04    6.974e-04
    67     175    3.299387e+03    1.029e-10    1.867e-04    2.767e-04
    68     176    3.299387e+03    1.503e-11    6.140e-05    9.568e-05
    69     177    3.299387e+03    2.071e-12    2.571e-05    3.570e-05
    70     178    3.299387e+03    3.133e-13    8.425e-06    1.333e-05

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.
```

Part 2: Multiple shooting method

```
% Bound values:
LB_MS = -inf*ones(Nz,N);
UB_MS = inf*ones(Nz,N);
LB_MS(end,:) = -umax*ones(1,N);
UB_MS(end,:) = umax*ones(1,N);
% terminal state node
LB_MS = [LB_MS(:); -inf*ones(Nx,1)];
UB_MS = [UB_MS(:); inf*ones(Nx,1)];

% Initialize multiple shooting variables:
Z_MS = repmat([x0; 0],1,N);
Z_MS = [Z_MS(:); x0];

% Setting options and solving the problem with fmincon + MULTIPLE SHOOTING:
```

```
options = optimoptions(@fmincon,'Display','iter','Algorithm','interior-point','GradObj'
,'on','GradConstr','on','Hessian','user-supplied','HessFcn',@hessian_multiple);
sol_multiple = fmincon(@cost_multiple,Z_MS,[],[],[],[],LB_MS,UB_MS,@constr_multiple,opt
ions);

Z_MS = sol_multiple;
X2 = zeros(Nx,N+1);
U2 = zeros(Nu,N);
for k = 1:N
   X2(:,k) = Z_MS((k-1)*Nz+1:(k-1)*Nz+Nx);
   U2(:,k) = Z_MS((k-1)*Nz+Nx+1:k*Nz);
end
X2(:,N+1) = Z_MS(N*Nz+1:end);
```

```
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       1    0.000000e+00    3.142e+00    0.000e+00
    1       3    2.981066e+02    2.467e+00    9.950e+00    2.604e+01
    2       4    1.033911e+03    1.783e+00    1.995e+01    2.314e+01
    3       5    1.014719e+03    1.518e+00    2.000e+01    1.027e+01
    4       7    1.284018e+03    9.796e-01    1.997e+01    2.982e+01
    5       9    1.275335e+03    8.678e-01    1.962e+01    1.662e+01
    6      10    1.358311e+03    7.093e-01    1.997e+01    2.414e+01
    7      11    1.358141e+03    7.028e-01    2.000e+01    8.784e-01
    8      13    1.599682e+03    5.373e-01    1.999e+01    2.252e+01
    9      14    2.403451e+03    2.964e-01    1.998e+01    2.764e+01
   10      15    2.456398e+03    2.776e-01    2.000e+01    2.062e+00
   11      16    2.631491e+03    2.207e-01    1.999e+01    6.564e+00
   12      17    2.645206e+03    2.159e-01    2.000e+01    6.663e-01
   13      18    2.811561e+03    1.649e-01    2.000e+01    7.218e+00
   14      19    3.010416e+03    1.087e-01    2.000e+01    8.701e+00
   15      20    3.068306e+03    8.904e-02    2.000e+01    3.775e+00
   16      21    3.264969e+03    3.613e-02    2.000e+01    1.131e+01
   17      22    3.276768e+03    2.882e-02    2.000e+01    2.574e+00
   18      23    3.300222e+03    1.786e-02    2.000e+01    4.844e+00
   19      24    3.306206e+03    1.003e-02    2.000e+01    4.303e+00
   20      25    3.307921e+03    2.742e-03    1.999e+01    4.311e+00
   21      26    3.301795e+03    6.370e-04    2.000e+01    1.660e+00
   22      27    3.300502e+03    1.013e-04    1.469e+01    9.383e-01
   23      28    3.300280e+03    1.154e-05    5.230e+00    3.161e-01
   24      29    3.300257e+03    1.744e-06    1.950e+00    1.180e-01
   25      30    3.299592e+03    6.563e-07    1.126e+00    1.597e-01
   26      31    3.299566e+03    1.025e-07    5.815e-01    4.464e-02
   27      32    3.299565e+03    1.392e-08    2.215e-01    1.539e-02
   28      33    3.299424e+03    1.839e-08    3.274e-01    3.696e-02
   29      34    3.299423e+03    2.045e-09    8.107e-02    5.602e-03
   30      35    3.299423e+03    3.381e-10    3.399e-02    2.123e-03

                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
   31      36    3.299394e+03    7.393e-10    7.111e-02    7.799e-03
   32      37    3.299394e+03    1.122e-10    1.909e-02    1.239e-03
   33      38    3.299394e+03    1.998e-11    8.211e-03    5.054e-04
   34      39    3.299388e+03    3.316e-11    1.476e-02    1.582e-03
   35      40    3.299388e+03    4.026e-12    3.588e-03    2.343e-04
   36      41    3.299388e+03    7.295e-13    1.567e-03    9.606e-05
   37      42    3.299387e+03    1.368e-12    2.958e-03    3.170e-04
   38      43    3.299387e+03    1.679e-13    7.342e-04    4.779e-05
   39      44    3.299387e+03    3.041e-14    3.205e-04    1.965e-05
```

```
   40      45    3.299387e+03    5.507e-14    5.933e-04    6.345e-05
   41      46    3.299387e+03    6.765e-15    1.455e-04    9.485e-06
   42      47    3.299387e+03    1.180e-15    6.362e-05    3.899e-06
   43      48    3.299387e+03    2.665e-15    1.186e-04    1.269e-05
   44      51    3.299387e+03    1.776e-15    9.149e-05    4.756e-07
   45      53    3.299387e+03    1.776e-15    4.903e-05    6.172e-07
   46      57    3.299387e+03    8.882e-16    4.315e-05    4.656e-08

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.
```

Part 3: Compare OCP solutions

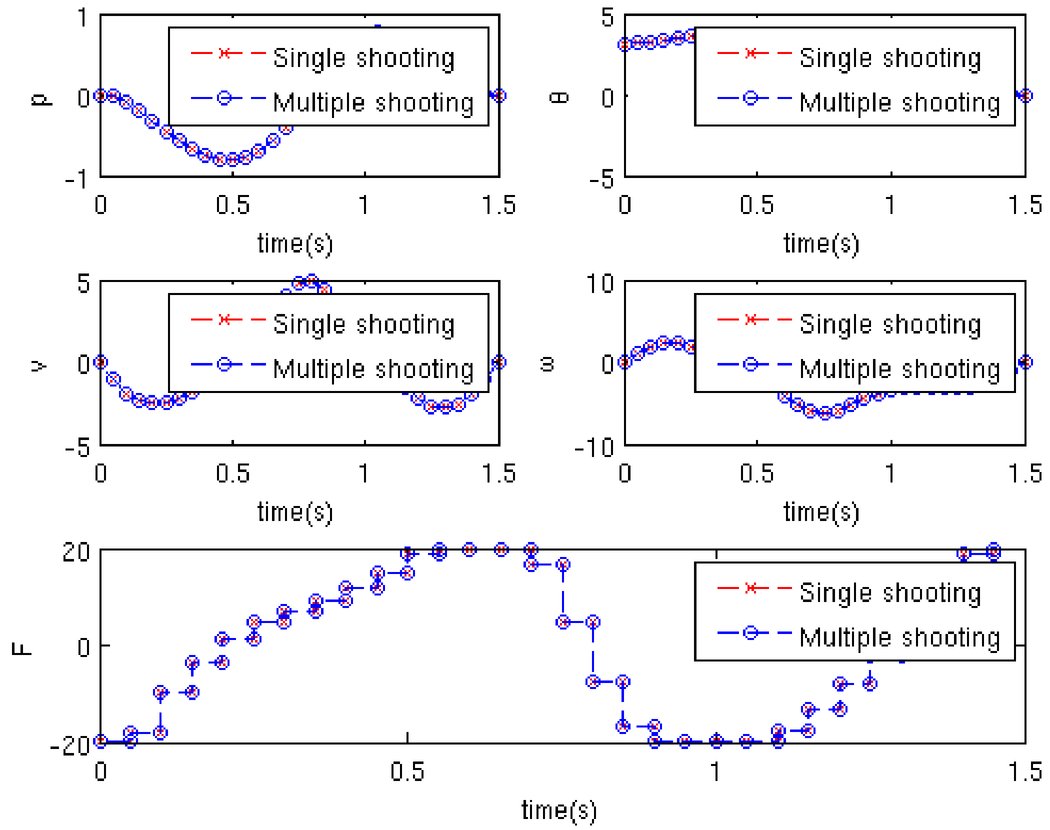
```
figure;
subplot(321);
plot([0:N]*Ts,X(1,:),'--rx'); hold on;
plot([0:N]*Ts,X2(1,:),'--bo');
xlabel('time(s)'); ylabel('p');
legend('Single shooting', 'Multiple shooting')

subplot(322);
plot([0:N]*Ts,X(2,:),'--rx'); hold on;
plot([0:N]*Ts,X2(2,:),'--bo');
xlabel('time(s)'); ylabel('\theta');
legend('Single shooting', 'Multiple shooting')

subplot(323);
plot([0:N]*Ts,X(3,:),'--rx'); hold on;
plot([0:N]*Ts,X2(3,:),'--bo');
xlabel('time(s)'); ylabel('v');
legend('Single shooting', 'Multiple shooting')

subplot(324);
plot([0:N]*Ts,X(4,:),'--rx'); hold on;
plot([0:N]*Ts,X2(4,:),'--bo');
xlabel('time(s)'); ylabel('\omega');
legend('Single shooting', 'Multiple shooting')

subplot(3,2,[5 6]);
stairs([0:N-1]*Ts,U,'--rx'); hold on;
stairs([0:N-1]*Ts,U2,'--bo');
xlabel('time(s)'); ylabel('F');
legend('Single shooting', 'Multiple shooting')
```



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