Solution for Exercise 3: Newton's method for optimization

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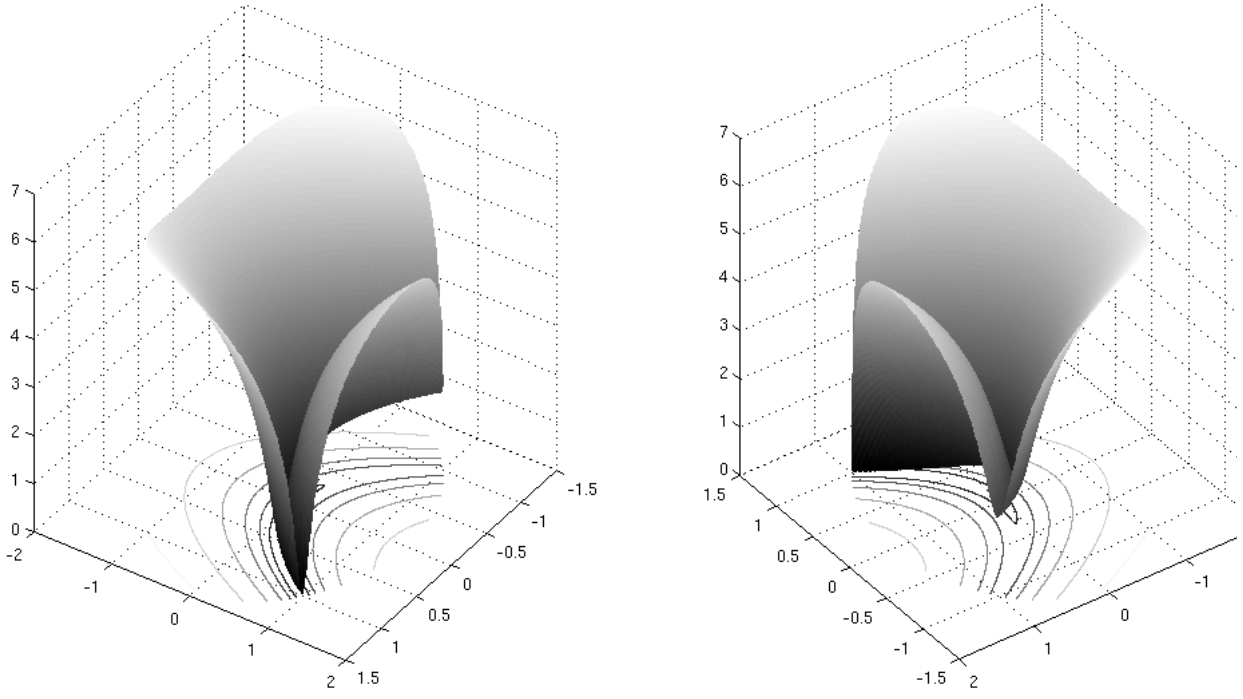
Contents

Part 0: Plotting Rosenbrock's function

```
clc;
clear all;
close all;

% Create figure
figure1 = figure('Position',[1 400 1200 600]);
colormap('gray');
axis square;
R=0:.002:sqrt(2);
TH=2*pi*(0:.002:1);
X=R'*cos(TH);
Y=R'*sin(TH);
Z=log(1+vrosenbrock(X,Y));

% Create subplot
subplot1 = subplot(1,2,1,'Parent',figure1);
view([124 34]);
grid('on');
hold('all');

% Create surface
surf(X,Y,Z,'Parent',subplot1,'LineStyle','none');

% Create contour
contour(X,Y,Z,'Parent',subplot1);

% Create subplot
subplot2 = subplot(1,2,2,'Parent',figure1);
view([234 34]);
grid('on');
hold('all');

% Create surface
surf(X,Y,Z,'Parent',subplot2,'LineStyle','none');

% Create contour
contour(X,Y,Z,'Parent',subplot2);
```



Part 1: Implementing Newton's method

```
z = [0; 0];
res = 1; iter = 0;
while norm(res) > 1e-5
    [~,res,H] = D2f_eval(z);
    dz = -H\res
    z = z + dz;
    iter = iter+1;
end
disp(['solution found in ' num2str(iter) ' iterations: x = ' num2str(z(1)) ', y = ' num2str(z(2))])
```

```
dz =

     1
     0


dz =

     0
     1


dz =

     0
     0

solution found in 3 iterations: x = 1, y = 1
```

Part 2: Solving the problem using fminunc

fminunc's solution:

```
options = optimoptions(@fminunc,'Display','iter','Algorithm','quasi-newton');
x_sol = fminunc(@f_eval,[0 0],options)

% fminunc's solution with gradient and hessian:
options = optimoptions(@fminunc,'Display','iter','Algorithm','trust-region','GradObj','on','Hessian','on');
x_sol1 = fminunc(@D2f_eval,[0 0],options)
```

```
                                                 First-order
 Iteration  Func-count       f(x)        Step-size       optimality
     0           3               1                           2
     1          12         0.771192      0.0817341          5.34
     2          15         0.610658              1          6.73
     3          18         0.522451              1          7.11
     4          24         0.261629         0.7075          1.88
     5          30         0.248996            0.5          3.44
     6          33         0.207486              1          2.94
     7          36         0.125351              1           1.5
     8          39        0.0893498              1          3.93
     9          42        0.0308666              1          1.23
    10          48        0.0200762       0.322023          1.95
    11          51        0.0138484              1          1.57
    12          54        0.0044155              1         0.303
    13          60       0.00268685            0.5          1.14
    14          63      0.000276528              1          0.28
    15          66       4.2104e-05              1         0.122
    16          69      1.37272e-06              1       0.00796
    17          72      7.48542e-07              1        0.0303
    18          75      3.34562e-09              1       0.00221
    19          78      6.71604e-11              1      7.22e-05
                                                 First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    20          81      1.94742e-11              1      1.06e-06

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the function tolerance.



x_sol =

    1.0000    1.0000


                             Norm of      First-order
 Iteration        f(x)          step          optimality   CG-iterations
     0                1                             2
     1                1             1               2           1
     2         0.953125          0.25            12.5           0
     3         0.549578        0.0625            1.63           1
     4         0.414158         0.125            2.74           1
     5         0.292376       0.218082            5.67           1
     6         0.155502       0.123894           0.954           1
     7         0.117347          0.25            7.16           1
     8        0.0385147       0.093587           0.308           1
```

```
     9          0.0385147       0.284678          0.308           1
    10          0.0268871         0.0625          0.351           0
    11          0.0118213          0.125           1.38           1
    12         0.00295219       0.113248          0.983           1
    13        0.000358228      0.0664421          0.313           1
    14        1.04116e-05      0.0322022         0.0759           1
    15        1.29552e-08     0.00556519        0.00213           1
    16        2.20449e-14    0.000223683       3.59e-06           1

Local minimum possible.

fminunc stopped because the final change in function value relative to
its initial value is less than the default value of the function tolerance.



x_sol1 =

    1.0000    1.0000
```

Part 2: Solving the constrained problem using fmincon

```
options = optimoptions(@fmincon,'Display','iter', ...
    'Algorithm','interior-point','GradObj','on','GradConstr','on','Hessian','bfgs');
x_sol2 = fmincon(@Df_eval,[0 0],[],[],[],[],[],[],@c_eval,options)

options = optimoptions(@fmincon,'Display','iter', ...
    'Algorithm','interior-point','GradObj','on','GradConstr','on','Hessian','user-supplied','HessFcn',@hessian_fun);
x_sol3 = fmincon(@Df_eval,[0 0],[],[],[],[],[],[],@c_eval,options)
```

```
                                        First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       1    1.000000e+00    0.000e+00    2.000e+00
    1       9    7.753529e-01    0.000e+00    6.250e+00    1.768e-01
    2      12    6.519631e-01    0.000e+00    9.048e+00    1.679e-01
    3      13    5.543204e-01    0.000e+00    8.033e+00    1.203e-01
    4      14    2.985193e-01    0.000e+00    1.790e+00    9.329e-02
    5      15    2.653787e-01    0.000e+00    2.788e+00    5.723e-02
    6      16    1.897171e-01    0.000e+00    2.311e+00    1.147e-01
    7      17    1.513684e-01    0.000e+00    9.707e-01    5.764e-02
    8      18    1.153323e-01    0.000e+00    1.127e+00    8.168e-02
    9      19    1.198046e-01    0.000e+00    1.000e-01    1.521e-02
   10      20    8.909926e-02    0.000e+00    8.379e-01    8.301e-02
   11      21    6.771924e-02    0.000e+00    1.365e+00    7.148e-02
   12      22    6.437627e-02    0.000e+00    1.147e-01    5.702e-03
   13      23    6.329018e-02    0.000e+00    1.883e-02    3.773e-03
   14      24    5.161965e-02    0.000e+00    3.016e-01    4.464e-02
   15      25    4.964186e-02    0.000e+00    7.915e-02    7.896e-03
   16      26    4.955399e-02    0.000e+00    5.465e-03    4.184e-04
   17      27    4.954834e-02    0.000e+00    3.993e-03    2.209e-05
   18      28    4.658288e-02    0.000e+00    1.318e-02    1.255e-02
   19      29    4.647010e-02    0.000e+00    8.006e-04    4.940e-04
   20      30    4.569141e-02    0.000e+00    3.136e-03    3.379e-03
   21      31    4.568281e-02    0.000e+00    6.437e-05    3.974e-05
   22      32    4.568281e-02    0.000e+00    8.000e-06    1.083e-07
   23      33    4.567641e-02    0.000e+00    1.601e-06    2.793e-05
   24      34    4.567482e-02    0.000e+00    2.162e-08    6.916e-06

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.



x_sol2 =

    0.7864    0.6177

                                        First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       1    1.000000e+00    0.000e+00    2.000e+00
    1       4    8.639096e-01    0.000e+00    1.033e+01    2.273e-01
    2       5    5.025640e-01    0.000e+00    8.470e-01    1.041e-01
    3       7    3.812427e-01    0.000e+00    5.829e+00    1.915e-01
    4       8    2.320844e-01    0.000e+00    8.002e-01    1.076e-01
    5      10    1.582944e-01    0.000e+00    5.122e+00    2.085e-01
    6      11    1.133520e-01    0.000e+00    9.882e-02    2.079e-02
    7      12    7.723643e-02    0.000e+00    1.135e+00    1.048e-01
    8      13    6.519418e-02    0.000e+00    7.045e-02    3.596e-02
    9      14    6.317567e-02    0.000e+00    1.915e-02    7.195e-03
   10      15    5.136832e-02    0.000e+00    1.909e-01    4.508e-02
   11      16    4.958223e-02    0.000e+00    4.027e-03    7.310e-03
   12      17    4.590562e-02    0.000e+00    2.208e-02    1.562e-02
   13      18    4.571511e-02    0.000e+00    4.281e-05    8.422e-04
   14      19    4.571479e-02    0.000e+00    4.000e-05    1.392e-06
   15      20    4.568282e-02    0.000e+00    8.015e-06    1.395e-04
   16      21    4.567489e-02    0.000e+00    1.070e-07    3.463e-05

Local minimum found that satisfies the constraints.
```

```
Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.



x_sol3 =

    0.7864    0.6177
```

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