Solution for Exercise 10: ACADO code generation for Nonlinear MPC

TEMPO Summer School on Numerical Optimal Control and Embedded Optimization
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Contents

Swing-up of an inverted pendulum

```
clc;
clear all;
close all;

Ts = 0.05;
EXPORT = 1;

DifferentialState x1 theta v1 dtheta;
Control F;

M = 1;
m = 0.1;
g = 9.81;
l = 0.8;

% Differential Equation

f_expl = [   dot(x1) == v1; ...
             dot(theta) == dtheta; ...
             dot(v1) == (- l*m*sin(theta)*dtheta^2 + F + g*m*cos(theta)*sin(theta))/(M
+ m - m*cos(theta)^2); ...
             dot(dtheta) == (- l*m*cos(theta)*sin(theta)*dtheta^2 + F*cos(theta) + g*m*
sin(theta) + M*g*sin(theta))/(l*(M + m - m*cos(theta)^2)) ];

% SIMexport
acadoSet('problemname', 'sim');

numSteps = 2;
sim = acado.SIMexport( Ts );
sim.setModel(f_expl);
sim.set( 'INTEGRATOR_TYPE',             'INT_IRK_GL2' );
sim.set( 'NUM_INTEGRATOR_STEPS',        numSteps        );

if EXPORT
    sim.exportCode( 'export_SIM' );

    cd export_SIM
    make_acado_integrator('../integrate_pendulum')
    cd ..
end

% MPCexport
acadoSet('problemname', 'mpc');

N = 40;
```

```
ocp = acado.OCP( 0.0, N*Ts, N );

h = [x1 theta v1 dtheta F];
hN = [x1 theta v1 dtheta];

W = acado.BMatrix(eye(length(h)));
WN = acado.BMatrix(eye(length(hN)));

ocp.minimizeLSQ( W, h );
ocp.minimizeLSQEndTerm( WN, hN );

xmin = -2; xmax = 2;
Fmin = -20; Fmax = 20;

ocp.subjectTo( xmin <= x1 <= xmax );
ocp.subjectTo( Fmin <= F <= Fmax );
% ocp.subjectTo( 'AT_END', [x1 theta v1 dtheta] == 0 );

ocp.setModel(f_expl);

mpc = acado.OCPexport( ocp );
mpc.set( 'HESSIAN_APPROXIMATION',       'GAUSS_NEWTON'      );
mpc.set( 'DISCRETIZATION_TYPE',         'MULTIPLE_SHOOTING' );
mpc.set( 'SPARSE_QP_SOLUTION',          'FULL_CONDENSING_N2');
mpc.set( 'LEVENBERG_MARQUARDT',          1e-5               );
mpc.set( 'INTEGRATOR_TYPE',             'INT_RK4'           );
mpc.set( 'NUM_INTEGRATOR_STEPS',        2*N                 );
mpc.set( 'QP_SOLVER',                   'QP_QPOASES'            );

if EXPORT
    mpc.exportCode( 'export_MPC' );

    global ACADO_;
    copyfile([ACADO_.pwd '/../../external_packages/qpoases'], 'export_MPC/qpoases')

    cd export_MPC
    make_acado_solver('../acado_MPCstep')
    cd ..
end

% PARAMETERS SIMULATION
X0 = [0 pi 0 0];
Xref = [0 0 0 0];
input.x = [repmat(X0,N/2,1); repmat(Xref,N/2+1,1)];
input.od = [];

Uref = zeros(N,1);
input.u = Uref;

input.y = [repmat(Xref,N,1) Uref];
input.yN = Xref.';

input.W = diag([5e-1 1 2e-3 2e-3 1e-4]);
input.WN = diag([5e-1 1 2e-3 2e-3]);

input.shifting.strategy = 1;

% SIMULATION LOOP
display('------------------------------------------------------------------')
display('               Simulation Loop'                                    )
display('------------------------------------------------------------------')
```

```
iter = 0; time = 0;
Tf = 4;
KKT_MPC = []; INFO_MPC = [];
controls_MPC = [];
state_sim = X0;

while time(end) < Tf
    tic
    % Solve NMPC OCP
    input.x0 = state_sim(end,:).';
    output = acado_MPCstep(input);

    % Save the MPC step
    INFO_MPC = [INFO_MPC; output.info];
    KKT_MPC = [KKT_MPC; output.info.kktValue];
    controls_MPC = [controls_MPC; output.u(1,:)];

    input.x = output.x;
    input.u = output.u;

    % Simulate system
    sim_input.x = state_sim(end,:).';
    sim_input.u = output.u(1,:).';
    sim_input.od = 0.2;
    states = integrate_pendulum(sim_input);
    state_sim = [state_sim; states.value'];

    iter = iter+1;
    nextTime = iter*Ts;
    disp(['current time: ' num2str(nextTime) '    ' char(9) ' (RTI step -- QP error: ' n
um2str(output.info.status) ',' ' ' char(2) ' KKT val: ' num2str(output.info.kktValue,'%
1.2e') ',' ' ' char(2) ' CPU time: ' num2str(round(output.info.cpuTime*1e6)) ' µs)'])
    time = [time nextTime];

    visualize(time, state_sim, Xref, xmin, xmax, l);
end

figure;
subplot(2,2,1);
plot(time, state_sim(:,1)); hold on;
plot([0 time(end)], [0 0], 'r:');
plot([0 time(end)], [xmin xmin], 'g--');
plot([0 time(end)], [xmax xmax], 'g--');
xlabel('time(s)');
ylabel('x');
ylim([1.5*xmin 1.5*xmax])

subplot(2,2,2);
plot(time, state_sim(:,2)); hold on;
plot([0 time(end)], [0 0], 'r:');
xlabel('time(s)');
ylabel('theta');

subplot(2,2,[3 4]);
stairs(time(1:end-1), controls_MPC); hold on;
plot([0 time(end)], [0 0], 'r:');
plot([0 time(end)], [Fmin Fmin], 'g--');
plot([0 time(end)], [Fmax Fmax], 'g--');
xlabel('time(s)');
ylabel('F');
```

```
ylim([1.5*Fmin 1.5*Fmax])


figure;
semilogy(time(1:end-1), KKT_MPC, ':bx');
xlabel('time(s)')
ylabel('KKT')
```

```
Writing c++ files...
Compiling c++ files...
Building with 'g++'.
MEX completed successfully.

ACADO Toolkit for Matlab - Developed by David Ariens and Rien Quirynen, 2009-2013
Support available at http://www.acadotoolkit.org/matlab

[ACADO] Information: Code generation successful


compiling... Building with 'gcc'.
MEX completed successfully.
done! --> ../integrate_pendulum.mexa64
Writing c++ files...
Compiling c++ files...
Building with 'g++'.
MEX completed successfully.

ACADO Toolkit for Matlab - Developed by David Ariens and Rien Quirynen, 2009-2013
Support available at http://www.acadotoolkit.org/matlab


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compiling... Building with 'g++'.
/mnt/HDD-home/rien/Google Drive/GIT/tempo-noc/exercises/e10_ACADO/solution/export_MPC/a
cado_solver_mex.c: In function ‘int getArray(unsigned int, const mxArray*, int, const c
har*, real_t*, unsigned int, unsigned int)’:
/mnt/HDD-home/rien/Google Drive/GIT/tempo-noc/exercises/e10_ACADO/solution/export_MPC/a
cado_solver_mex.c:181:47: warning: deprecated conversion from string constant to ‘char*
’ [-Wwrite-strings]
    mexErrMsgTxtAdv("Field %s not found.", name);
                                           ^
/mnt/HDD-home/rien/Google Drive/GIT/tempo-noc/exercises/e10_ACADO/solution/export_MPC/a
cado_solver_mex.c:185:64: warning: deprecated conversion from string constant to ‘char*
’ [-Wwrite-strings]
   mexErrMsgTxtAdv("Field %s must be an array of doubles.", name);
                                                            ^
/mnt/HDD-home/rien/Google Drive/GIT/tempo-noc/exercises/e10_ACADO/solution/export_MPC/a
cado_solver_mex.c:188:75: warning: deprecated conversion from string constant to ‘char*
```
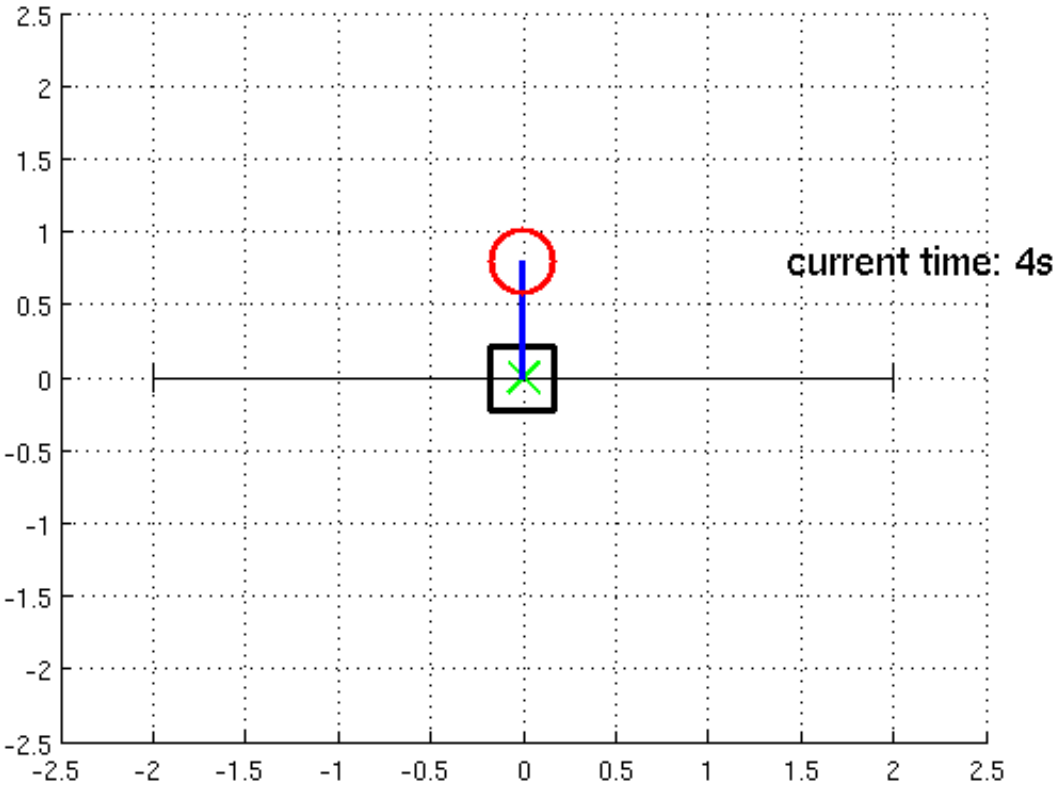
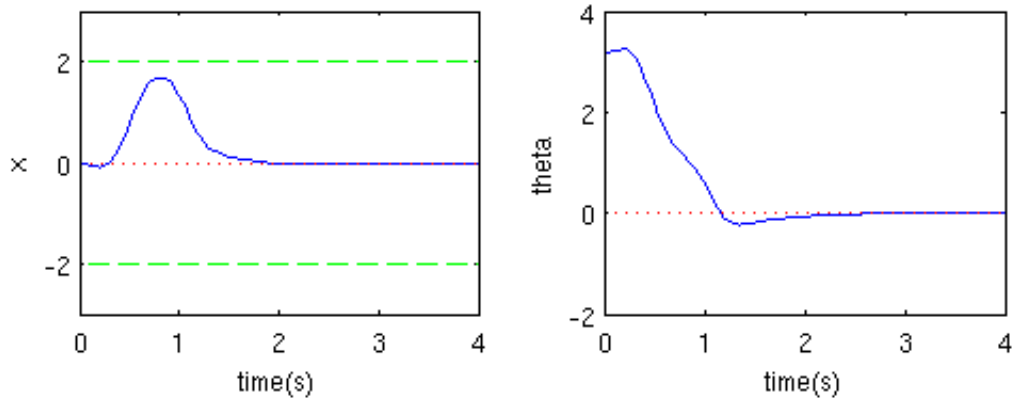
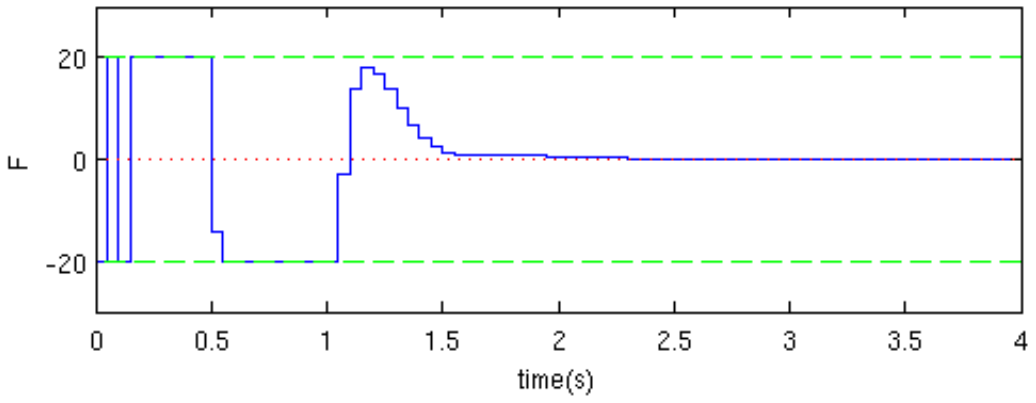

```
’ [-Wwrite-strings]
   mexErrMsgTxtAdv("Field %s must be of size: %d x %d.", name, nRows, nCols);
                                                                          ^


MEX completed successfully.
done! --> ../acado_MPCstep.mexa64
-----------------------------------------------------------------
               Simulation Loop
-----------------------------------------------------------------
current time: 0.05       (RTI step -- QP error: 0,  KKT val: 1.29e+04,  CPU time: 1144
µs)
current time: 0.1        (RTI step -- QP error: 0,  KKT val: 2.38e+03,  CPU time: 636 µ
s)
current time: 0.15       (RTI step -- QP error: 0,  KKT val: 5.23e+03,  CPU time: 566 µ
s)
current time: 0.2        (RTI step -- QP error: 0,  KKT val: 6.09e+03,  CPU time: 577 µ
s)
current time: 0.25       (RTI step -- QP error: 0,  KKT val: 3.63e+04,  CPU time: 549 µ
s)
current time: 0.3        (RTI step -- QP error: 0,  KKT val: 1.18e+03,  CPU time: 531 µ
s)
current time: 0.35       (RTI step -- QP error: 0,  KKT val: 9.45e-02,  CPU time: 518 µ
s)
current time: 0.4        (RTI step -- QP error: 0,  KKT val: 8.35e-02,  CPU time: 464 µ
s)
current time: 0.45       (RTI step -- QP error: 0,  KKT val: 1.83e-01,  CPU time: 440 µ
s)
current time: 0.5        (RTI step -- QP error: 0,  KKT val: 7.29e-01,  CPU time: 416 µ
s)
current time: 0.55       (RTI step -- QP error: 0,  KKT val: 2.00e+00,  CPU time: 383 µ
s)
current time: 0.6        (RTI step -- QP error: 0,  KKT val: 9.39e-02,  CPU time: 395 µ
s)
current time: 0.65       (RTI step -- QP error: 0,  KKT val: 2.10e-03,  CPU time: 369 µ
s)
current time: 0.7        (RTI step -- QP error: 0,  KKT val: 9.82e-03,  CPU time: 350 µ
s)
current time: 0.75       (RTI step -- QP error: 0,  KKT val: 3.29e-02,  CPU time: 341 µ
s)
current time: 0.8        (RTI step -- QP error: 0,  KKT val: 6.75e-02,  CPU time: 316 µ
s)
current time: 0.85       (RTI step -- QP error: 0,  KKT val: 1.19e-01,  CPU time: 296 µ
s)
current time: 0.9        (RTI step -- QP error: 0,  KKT val: 1.95e-01,  CPU time: 444 µ
s)
current time: 0.95       (RTI step -- QP error: 0,  KKT val: 3.06e-01,  CPU time: 263 µ
s)
current time: 1          (RTI step -- QP error: 0,  KKT val: 4.71e-01,  CPU time: 250 µ
s)
current time: 1.05       (RTI step -- QP error: 0,  KKT val: 7.15e-01,  CPU time: 220 µ
s)
current time: 1.1        (RTI step -- QP error: 0,  KKT val: 1.07e+00,  CPU time: 208 µ
s)
current time: 1.15       (RTI step -- QP error: 0,  KKT val: 4.45e-02,  CPU time: 206 µ
s)
current time: 1.2        (RTI step -- QP error: 0,  KKT val: 8.84e-02,  CPU time: 204 µ
s)
current time: 1.25       (RTI step -- QP error: 0,  KKT val: 1.11e-01,  CPU time: 198 µ
s)
current time: 1.3        (RTI step -- QP error: 0,  KKT val: 7.87e-02,  CPU time: 287 µ
s)
```

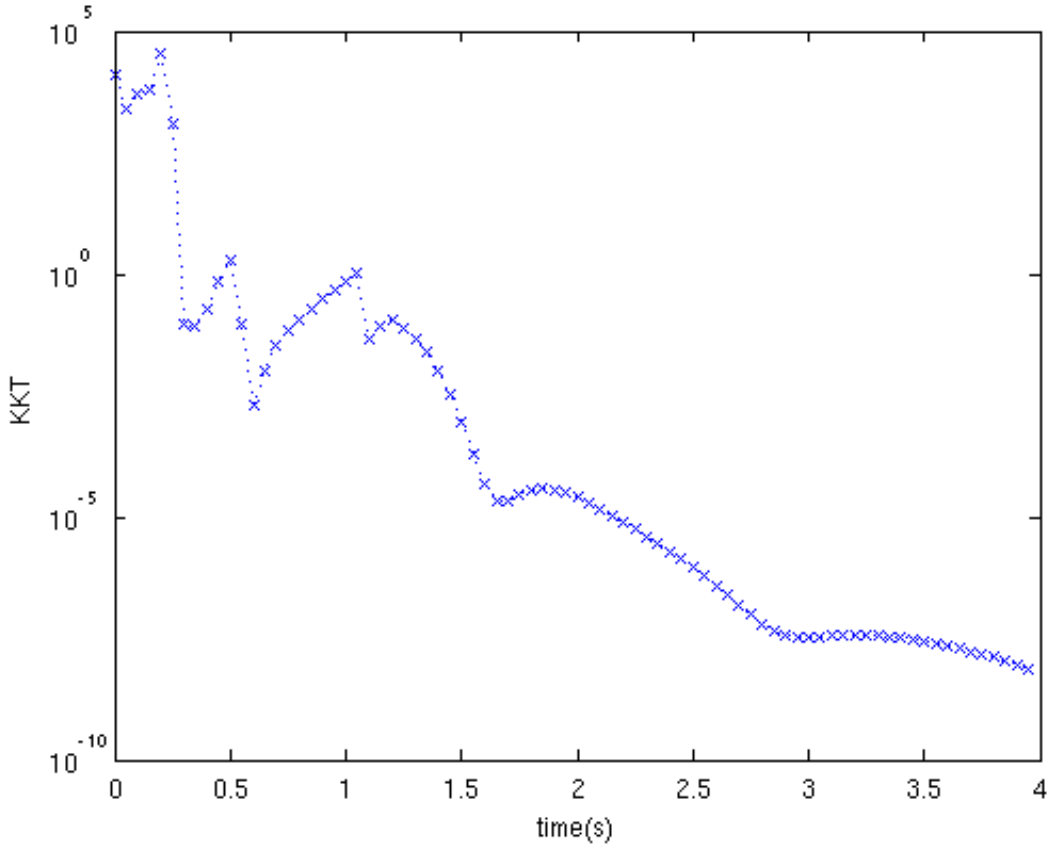
```
current time: 1.35       (RTI step -- QP error: 0,  KKT val: 4.79e-02,  CPU time: 208 µ
s)
current time: 1.4        (RTI step -- QP error: 0,  KKT val: 2.42e-02,  CPU time: 210 µ
s)
current time: 1.45       (RTI step -- QP error: 0,  KKT val: 9.87e-03,  CPU time: 200 µ
s)
current time: 1.5        (RTI step -- QP error: 0,  KKT val: 3.24e-03,  CPU time: 209 µ
s)
current time: 1.55       (RTI step -- QP error: 0,  KKT val: 8.63e-04,  CPU time: 200 µ
s)
current time: 1.6        (RTI step -- QP error: 0,  KKT val: 1.96e-04,  CPU time: 197 µ
s)
current time: 1.65       (RTI step -- QP error: 0,  KKT val: 4.83e-05,  CPU time: 197 µ
s)
current time: 1.7        (RTI step -- QP error: 0,  KKT val: 2.20e-05,  CPU time: 215 µ
s)
current time: 1.75       (RTI step -- QP error: 0,  KKT val: 2.17e-05,  CPU time: 219 µ
s)
current time: 1.8        (RTI step -- QP error: 0,  KKT val: 2.84e-05,  CPU time: 198 µ
s)
current time: 1.85       (RTI step -- QP error: 0,  KKT val: 3.51e-05,  CPU time: 205 µ
s)
current time: 1.9        (RTI step -- QP error: 0,  KKT val: 3.78e-05,  CPU time: 209 µ
s)
current time: 1.95       (RTI step -- QP error: 0,  KKT val: 3.60e-05,  CPU time: 205 µ
s)
current time: 2          (RTI step -- QP error: 0,  KKT val: 3.12e-05,  CPU time: 207 µ
s)
current time: 2.05       (RTI step -- QP error: 0,  KKT val: 2.53e-05,  CPU time: 206 µ
s)
current time: 2.1        (RTI step -- QP error: 0,  KKT val: 1.95e-05,  CPU time: 203 µ
s)
current time: 2.15       (RTI step -- QP error: 0,  KKT val: 1.46e-05,  CPU time: 213 µ
s)
current time: 2.2        (RTI step -- QP error: 0,  KKT val: 1.08e-05,  CPU time: 204 µ
s)
current time: 2.25       (RTI step -- QP error: 0,  KKT val: 7.80e-06,  CPU time: 205 µ
s)
current time: 2.3        (RTI step -- QP error: 0,  KKT val: 5.60e-06,  CPU time: 205 µ
s)
current time: 2.35       (RTI step -- QP error: 0,  KKT val: 3.99e-06,  CPU time: 205 µ
s)
current time: 2.4        (RTI step -- QP error: 0,  KKT val: 2.81e-06,  CPU time: 198 µ
s)
current time: 2.45       (RTI step -- QP error: 0,  KKT val: 1.95e-06,  CPU time: 263 µ
s)
current time: 2.5        (RTI step -- QP error: 0,  KKT val: 1.34e-06,  CPU time: 196 µ
s)
current time: 2.55       (RTI step -- QP error: 0,  KKT val: 9.04e-07,  CPU time: 260 µ
s)
current time: 2.6        (RTI step -- QP error: 0,  KKT val: 5.98e-07,  CPU time: 206 µ
s)
current time: 2.65       (RTI step -- QP error: 0,  KKT val: 3.88e-07,  CPU time: 273 µ
s)
current time: 2.7        (RTI step -- QP error: 0,  KKT val: 2.46e-07,  CPU time: 195 µ
s)
current time: 2.75       (RTI step -- QP error: 0,  KKT val: 1.54e-07,  CPU time: 204 µ
s)
current time: 2.8        (RTI step -- QP error: 0,  KKT val: 9.67e-08,  CPU time: 197 µ
s)
current time: 2.85       (RTI step -- QP error: 0,  KKT val: 6.27e-08,  CPU time: 197 µ
```

```
s)
current time: 2.9        (RTI step -- QP error: 0,  KKT val: 4.42e-08,  CPU time: 198 µ
s)
current time: 2.95       (RTI step -- QP error: 0,  KKT val: 3.54e-08,  CPU time: 193 µ
s)
current time: 3          (RTI step -- QP error: 0,  KKT val: 3.24e-08,  CPU time: 202 µ
s)
current time: 3.05       (RTI step -- QP error: 0,  KKT val: 3.24e-08,  CPU time: 206 µ
s)
current time: 3.1        (RTI step -- QP error: 0,  KKT val: 3.38e-08,  CPU time: 200 µ
s)
current time: 3.15       (RTI step -- QP error: 0,  KKT val: 3.54e-08,  CPU time: 196 µ
s)
current time: 3.2        (RTI step -- QP error: 0,  KKT val: 3.67e-08,  CPU time: 201 µ
s)
current time: 3.25       (RTI step -- QP error: 0,  KKT val: 3.73e-08,  CPU time: 197 µ
s)
current time: 3.3        (RTI step -- QP error: 0,  KKT val: 3.71e-08,  CPU time: 208 µ
s)
current time: 3.35       (RTI step -- QP error: 0,  KKT val: 3.62e-08,  CPU time: 195 µ
s)
current time: 3.4        (RTI step -- QP error: 0,  KKT val: 3.47e-08,  CPU time: 205 µ
s)
current time: 3.45       (RTI step -- QP error: 0,  KKT val: 3.27e-08,  CPU time: 194 µ
s)
current time: 3.5        (RTI step -- QP error: 0,  KKT val: 3.03e-08,  CPU time: 193 µ
s)
current time: 3.55       (RTI step -- QP error: 0,  KKT val: 2.77e-08,  CPU time: 198 µ
s)
current time: 3.6        (RTI step -- QP error: 0,  KKT val: 2.50e-08,  CPU time: 193 µ
s)
current time: 3.65       (RTI step -- QP error: 0,  KKT val: 2.23e-08,  CPU time: 195 µ
s)
current time: 3.7        (RTI step -- QP error: 0,  KKT val: 1.97e-08,  CPU time: 205 µ
s)
current time: 3.75       (RTI step -- QP error: 0,  KKT val: 1.72e-08,  CPU time: 197 µ
s)
current time: 3.8        (RTI step -- QP error: 0,  KKT val: 1.49e-08,  CPU time: 204 µ
s)
current time: 3.85       (RTI step -- QP error: 0,  KKT val: 1.28e-08,  CPU time: 209 µ
s)
current time: 3.9        (RTI step -- QP error: 0,  KKT val: 1.09e-08,  CPU time: 209 µ
s)
current time: 3.95       (RTI step -- QP error: 0,  KKT val: 9.17e-09,  CPU time: 205 µ
s)
current time: 4          (RTI step -- QP error: 0,  KKT val: 7.67e-09,  CPU time: 198 µ
s)
```


current time: 4s


x
time(s)
theta
time(s)

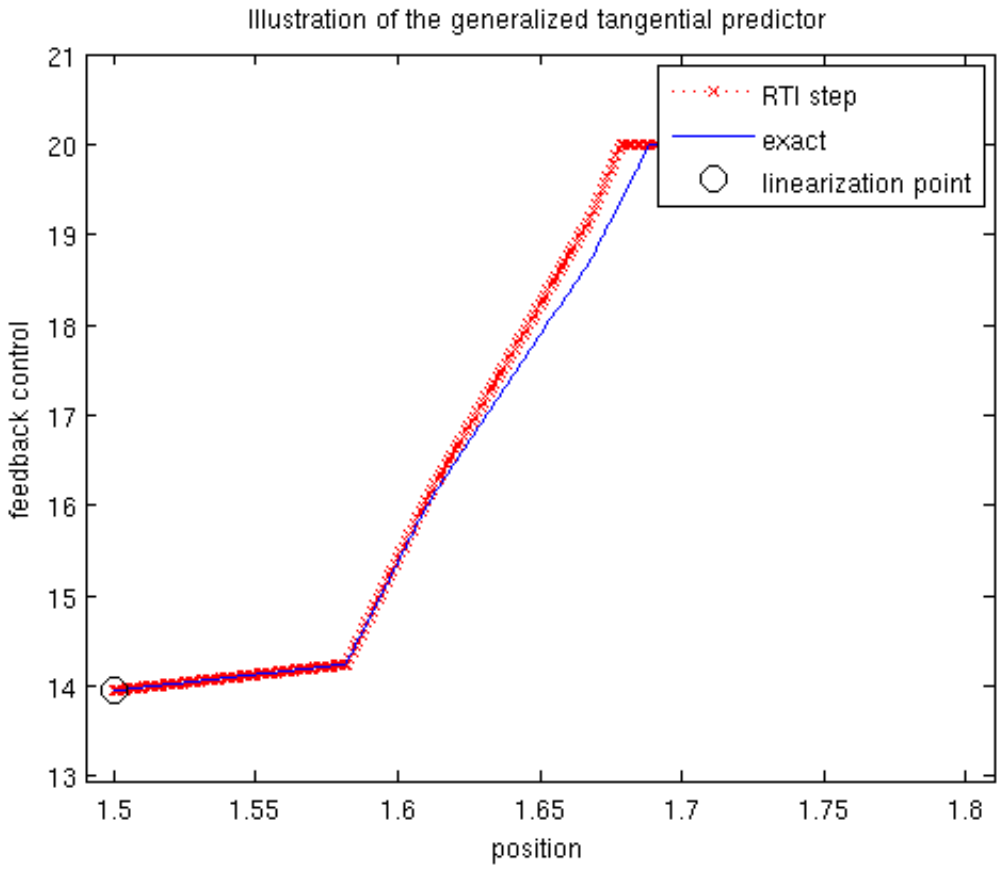

F
time(s)



Generalized tangential predictor of RTI for Nonlinear MPC

```
clc;
clear all;
close all;

% Define parameters:
N = 40;
Xref = [0 0 0 0];
X0 = [1.5 -0.2 0 0].';
input.x = repmat(X0.',N+1,1);
input.x0 = X0;

Uref = zeros(N,1);
input.u = Uref;

input.y = [repmat(Xref,N,1) Uref];
input.yN = Xref.';

input.W = diag([5e-1 1 5e-2 5e-2 2e-2]);
input.WN = diag([5e-1 1 5e-2 5e-2]);

% Converge the SQP method as the linearization point:
for k = 1:30
    output = acado_MPCstep(input);
    input.x = output.x;
    input.u = output.u;
end
X_RTI = output.x; U_RTI = output.u;

cost_RTI = []; u_RTI = [];
cost_SQP = []; u_SQP = [];
p_values = 1.5:0.001:1.8;
```

```
for p = p_values
    X0 = [p -0.2 0 0].';
    input.x0 = X0;

    % Perform one RTI step:
    input.x = X_RTI; input.u = U_RTI; % NOTE: we use the same, original linearization p
oint
    output = acado_MPCstep(input);
    cost_RTI = [cost_RTI; output.info.objValue];
    u_RTI = [u_RTI; output.u(1)];
    if output.info.status ~= 0
       error(['output status = ' num2str(output.info.status) ' for p = ' num2str(p)])
    end

    % Perform a fixed amount of 30 SQP steps:
    for k = 1:30
        output = acado_MPCstep(input);
        input.x = output.x;
        input.u = output.u;
    end
    cost_SQP = [cost_SQP; output.info.objValue];
    u_SQP = [u_SQP; output.u(1)];
    if output.info.status ~= 0
       error(['output status = ' num2str(output.info.status) ' for p = ' num2str(p)])
    end
end

figure;
plot(p_values, u_RTI, ':rx'); hold on;
plot(p_values, u_SQP, '-b');
plot(p_values(1), u_SQP(1), 'ko', 'MarkerSize', 10)
xlabel('position');
ylabel('feedback control')
legend('RTI step', 'exact', 'linearization point');
xlim([min(p_values)-0.01 max(p_values)+0.01])
ylim([min(u_SQP)-1 max(u_SQP)+1]);
title('Illustration of the generalized tangential predictor')
```



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