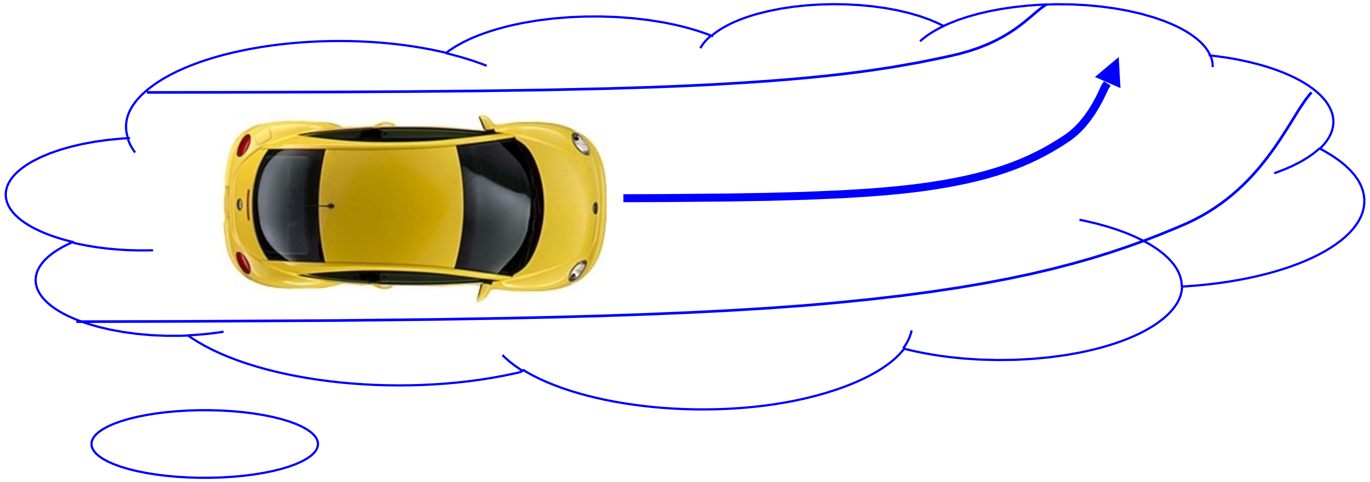
Real-Time Optimization for Nonlinear Model Predictive Control

Moritz Diehl

(First a Distillation NMPC Story for Warm-Up)

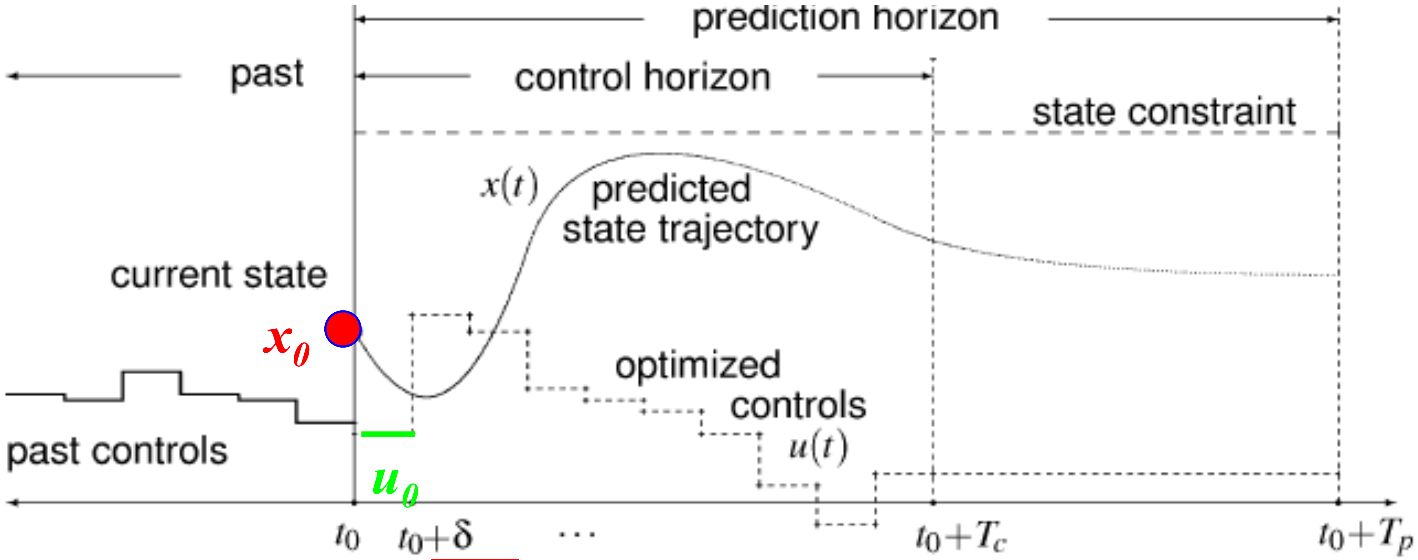
Model Predictive Control (MPC)

Always look a bit into the future.



Brain predicts and optimizes:
e.g. slow down **before** curve

Computations in Model Predictive Control (MPC)



1. Estimate current system state x_0 (and parameters) from measurements.

2. Solve *in real-time* an optimal control problem:

$$\min_{x,z,u} \int_{t_0}^{t_0+T_p} L(x,z,u)dt \; + E\left(x(t_0+T_p)\right) \; s.t. \; \begin{cases} x(t_0)-x_0 = 0, \\ \dot{x}-f(x,z,u) = 0, \; t \in [t_0, t_0+T_p] \\ g(x,z,u) = 0, \; t \in [t_0, t_0+T_p] \\ h(x,z,u) \geq 0, \; t \in [t_0, t_0+T_p] \\ r\left(x(t_0+T_p)\right) \geq 0. \end{cases}$$

3. Implement first control u_0 for time δ at real plant. Set $t_0 = t_0 + \delta$ and go to 1.

Main challenge for MPC: fast and reliable real-time optimization

Example: Distillation Column (ISR, Stuttgart)

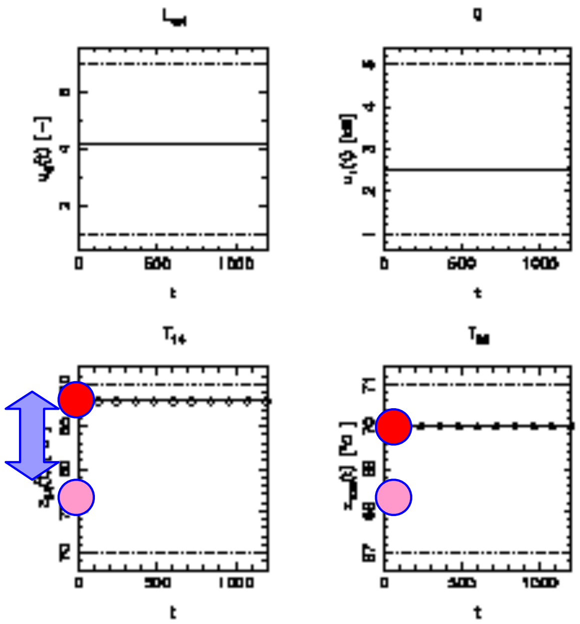
- Aim: to ensure product purity, keep two temperatures (T_{14}, T_{28}) constant despite disturbances
- least squares objective:

$$\min \int_{t_0}^{t_0+T_p} \left\| \begin{array}{c} T_{14}(t) - T_{14}^{\mathrm{ref}} \\ T_{28}(t) - T_{28}^{\mathrm{ref}} \end{array} \right\|_2^2 dt$$

- control horizon 10 min
- prediction horizon 10 h
- stiff DAE model with 82 differential and 122 algebraic state variables
- Desired sampling time: 30 seconds.

Distillation Online Scenario

- System is in steady state, optimizer predicts constant trajectory:

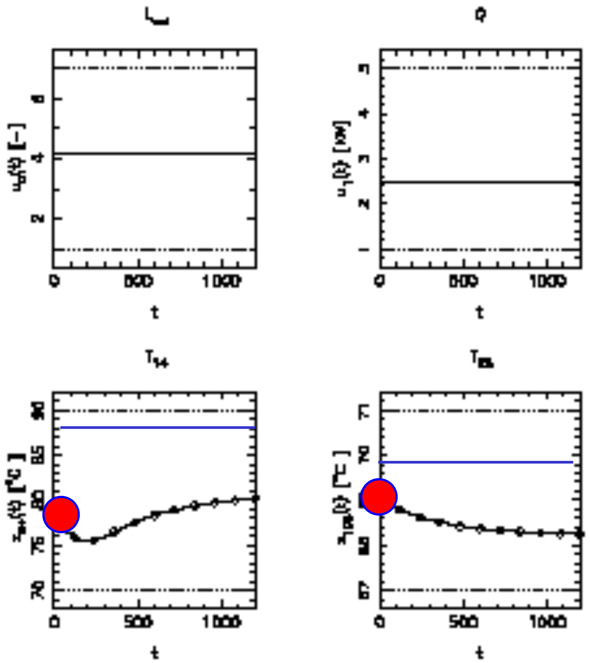


- **Suddenly**, system state x_0 is disturbed.
- What to do with optimizer?

Conventional Approach

- use offline method, e.g. MUSCOD-II with BFGS (Leineweber, 1999).
- initialize with **new** initial value x_0 and integrate system with **old** controls.
- iterate until convergence.

Initialization



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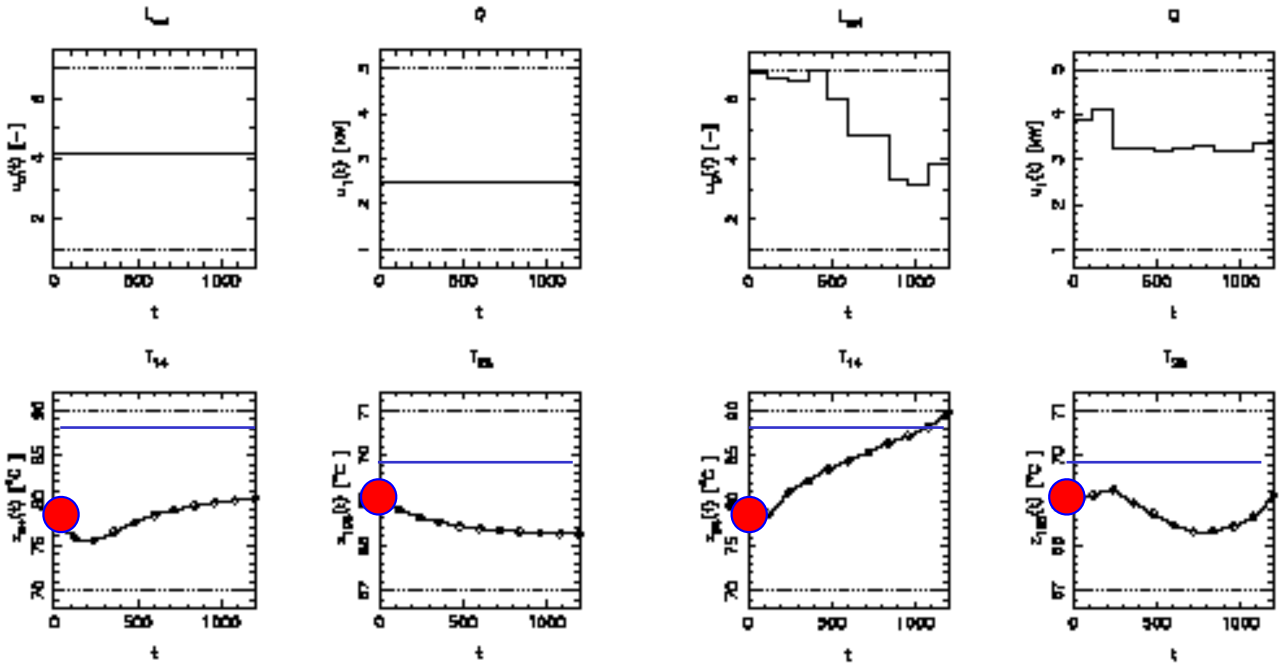
Initialization 16th Iteration



Conventional Approach

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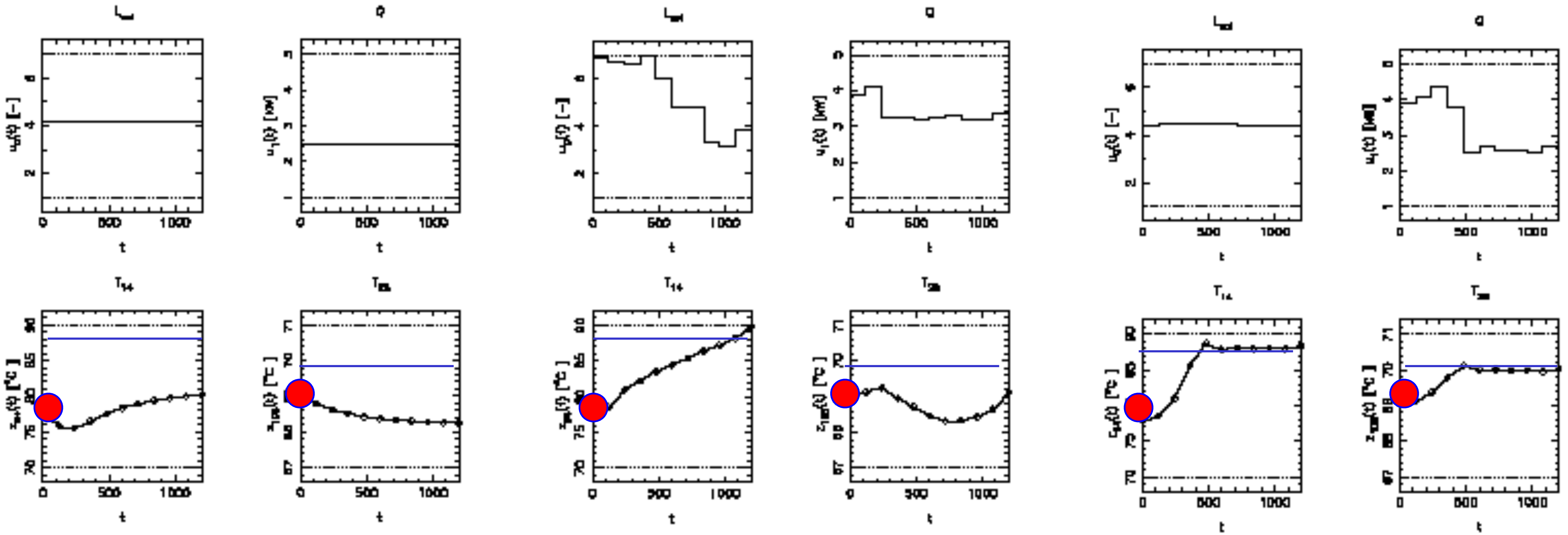
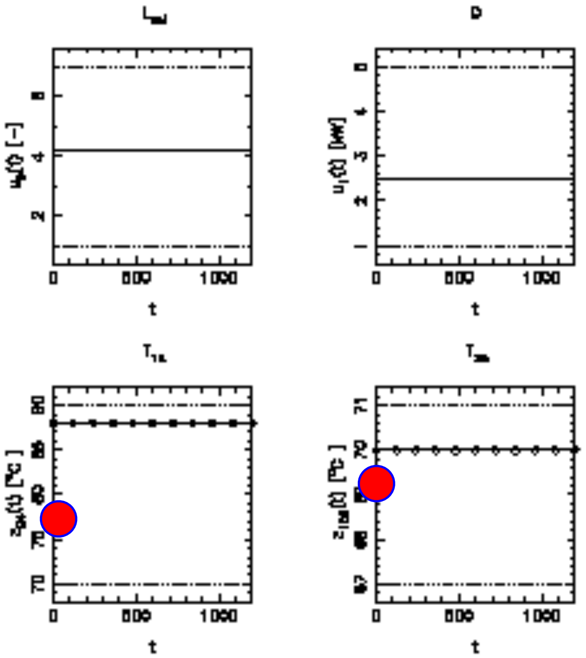
Initialization 16th Iteration Solution (32nd Iteration



New Approach: Initial Value Embedding

- Initialize with **old** trajectory, accept violation of $s_0^x - x_0 = 0$

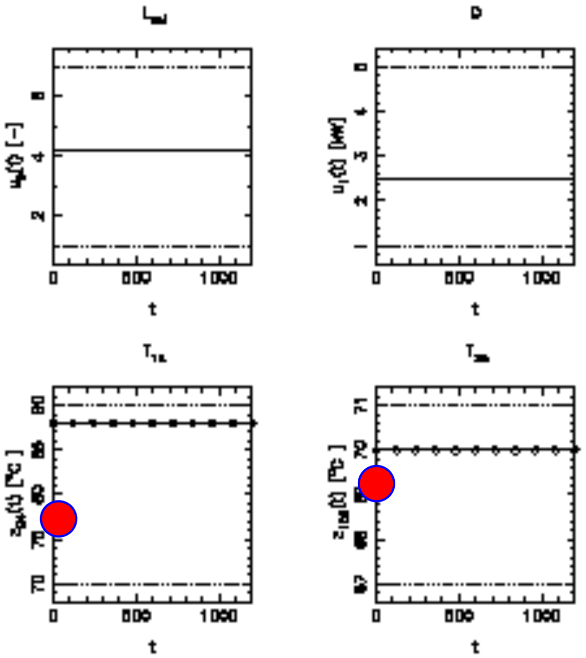
Initialization



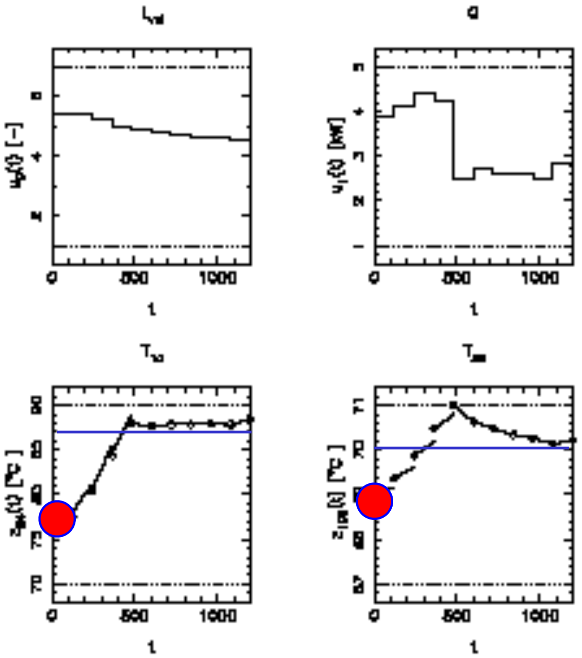
New Approach: Initial Value Embedding

- Initialize with **old** trajectory, accept violation of $s_0^x - \boxed{x_0} = 0$

Initialization



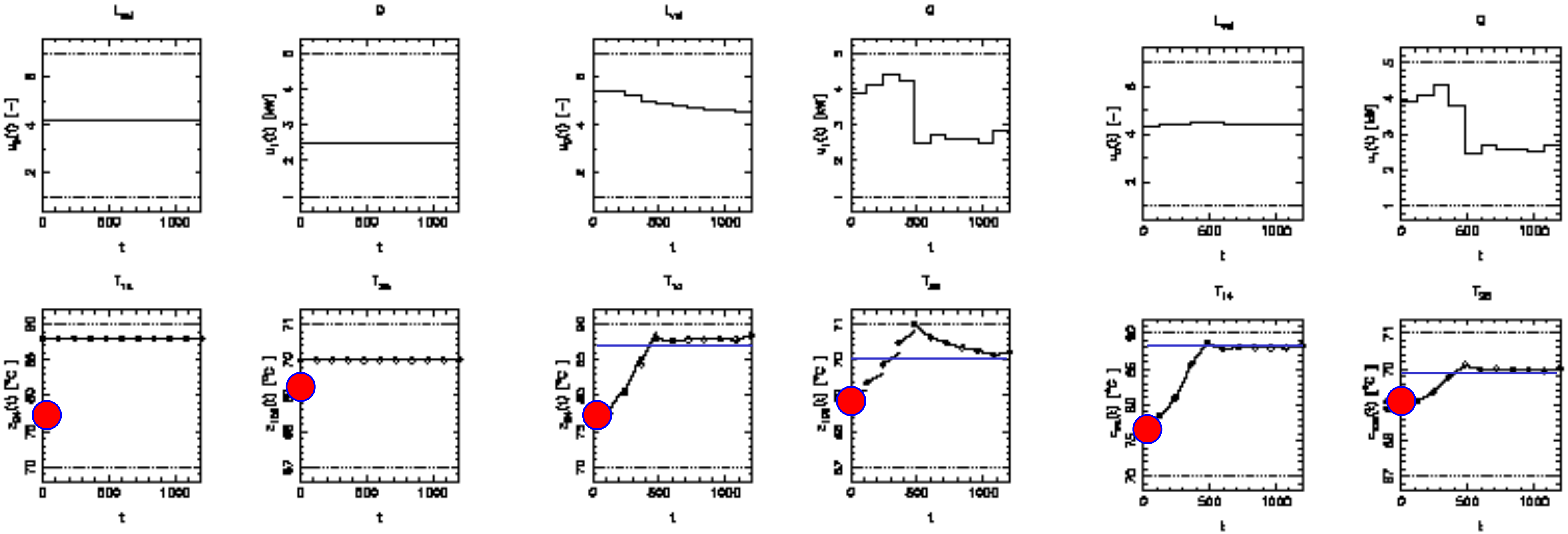
First Iteration



New Approach: Initial Value Embedding

- Initialize with **old** trajectory, accept violation of $s_0^x - \boxed{x_0} = 0$

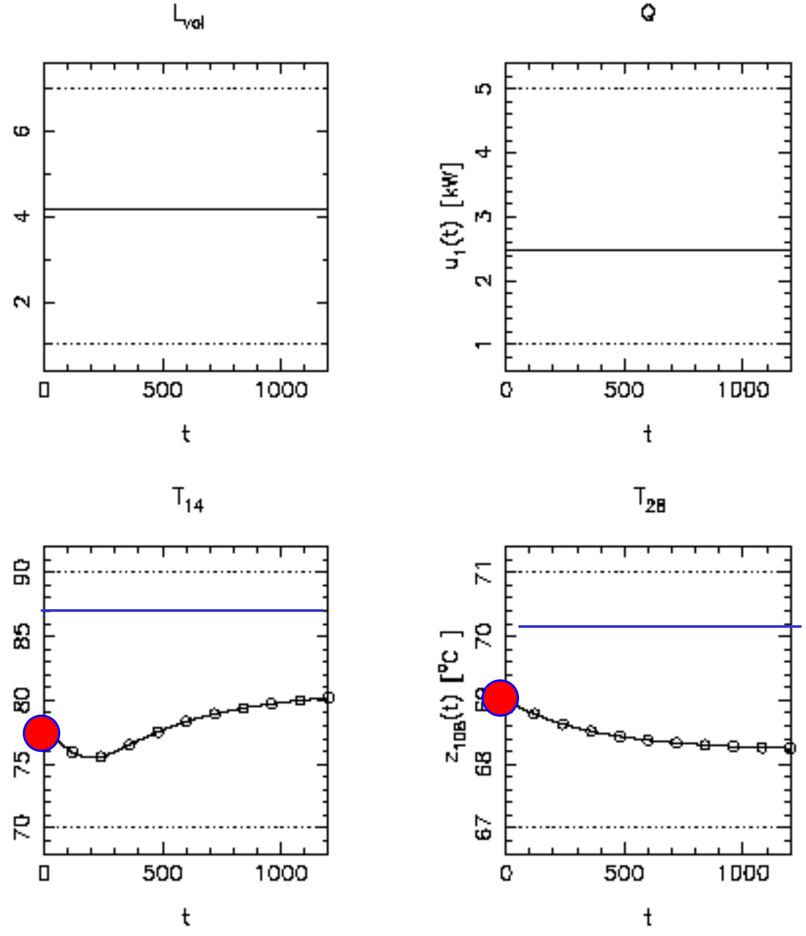
Initialization | First Iteration | Solution (3rd Iteration)



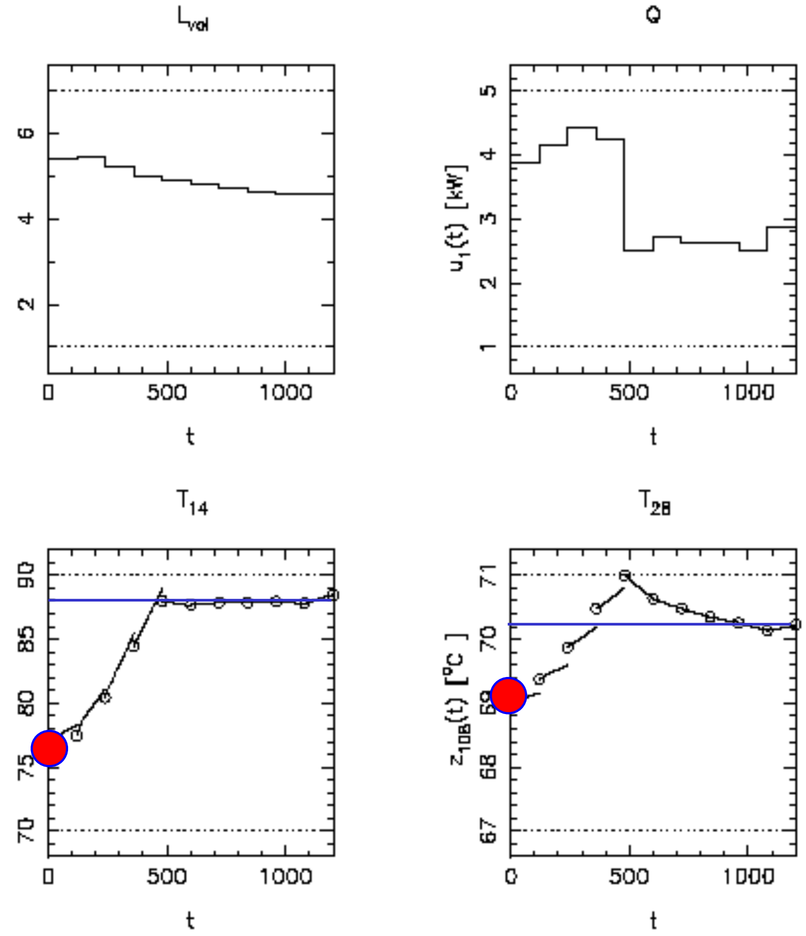
First iteration nearly solution!

Very different results after first iteration!

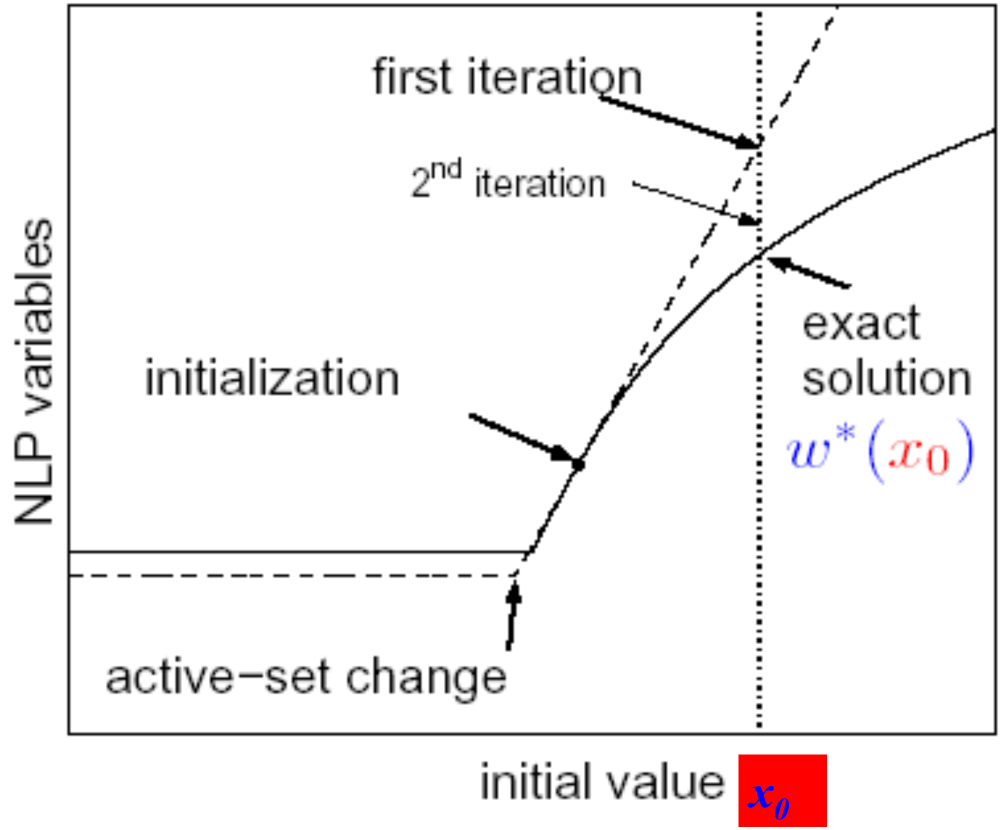
Conventional:



Initial Value Embedding:

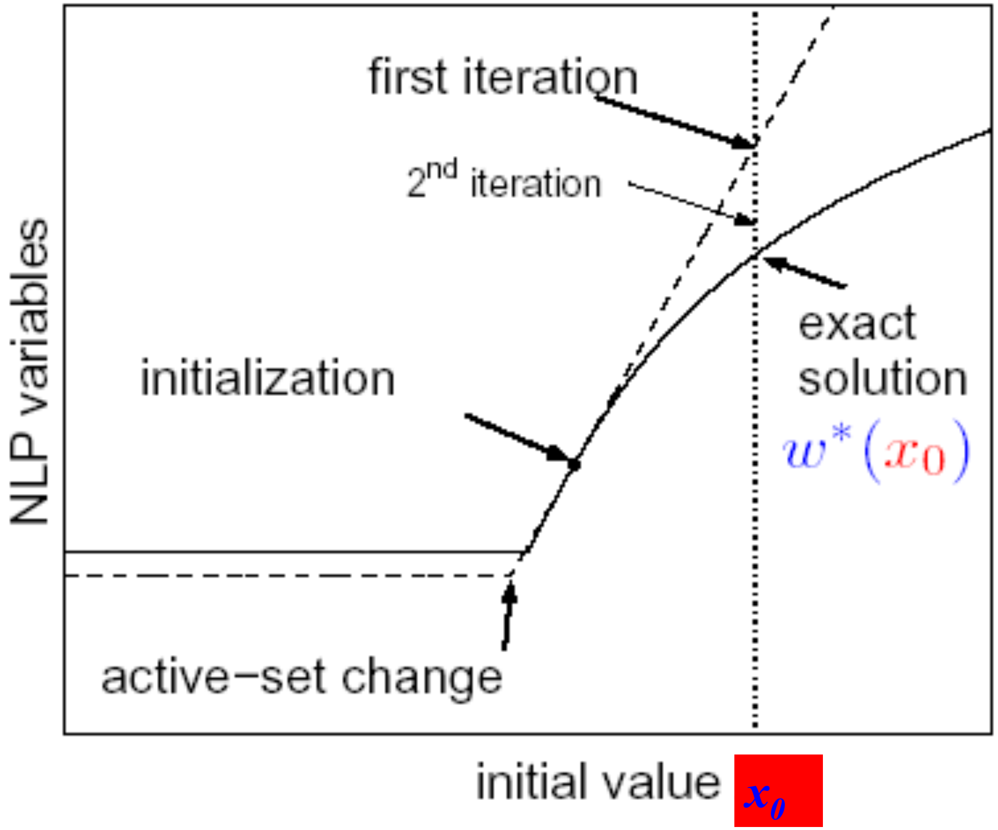


Initial Value Embedding



- first iteration is tangential predictor for exact solution (for exact hessian SQP)
- also valid for active set changes
- derivative can be computed *before* x_0 is known: first iteration nearly without delay

Initial Value Embedding

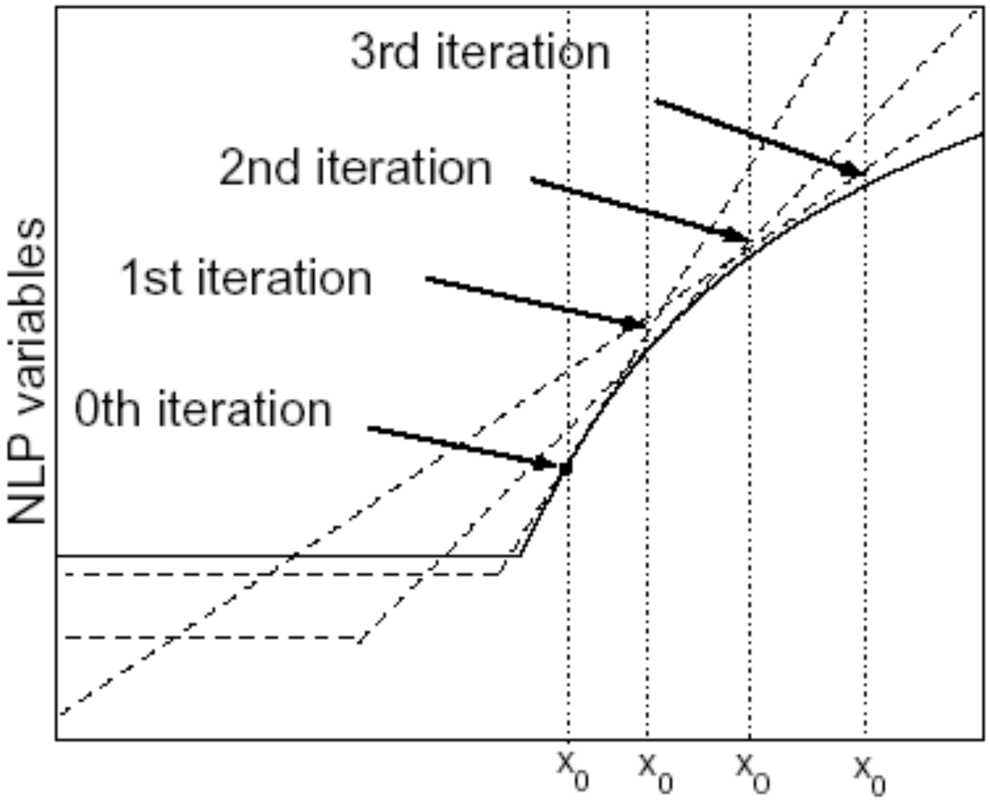


- first iteration is tangential predictor for exact solution (for exact hessian SQP)
- also valid for active set changes
- derivative can be computed *before* x_0 is known: first iteration nearly without delay

Why wait until convergence and do nothing in the meantime?

Real-Time Iterations [D. 2001]

Iterate, *while* problem is changing!



- tangential prediction after each change in x_0
- solution accuracy is increased with each iteration when x_0 changes little
- iterates stay close to solution manifold

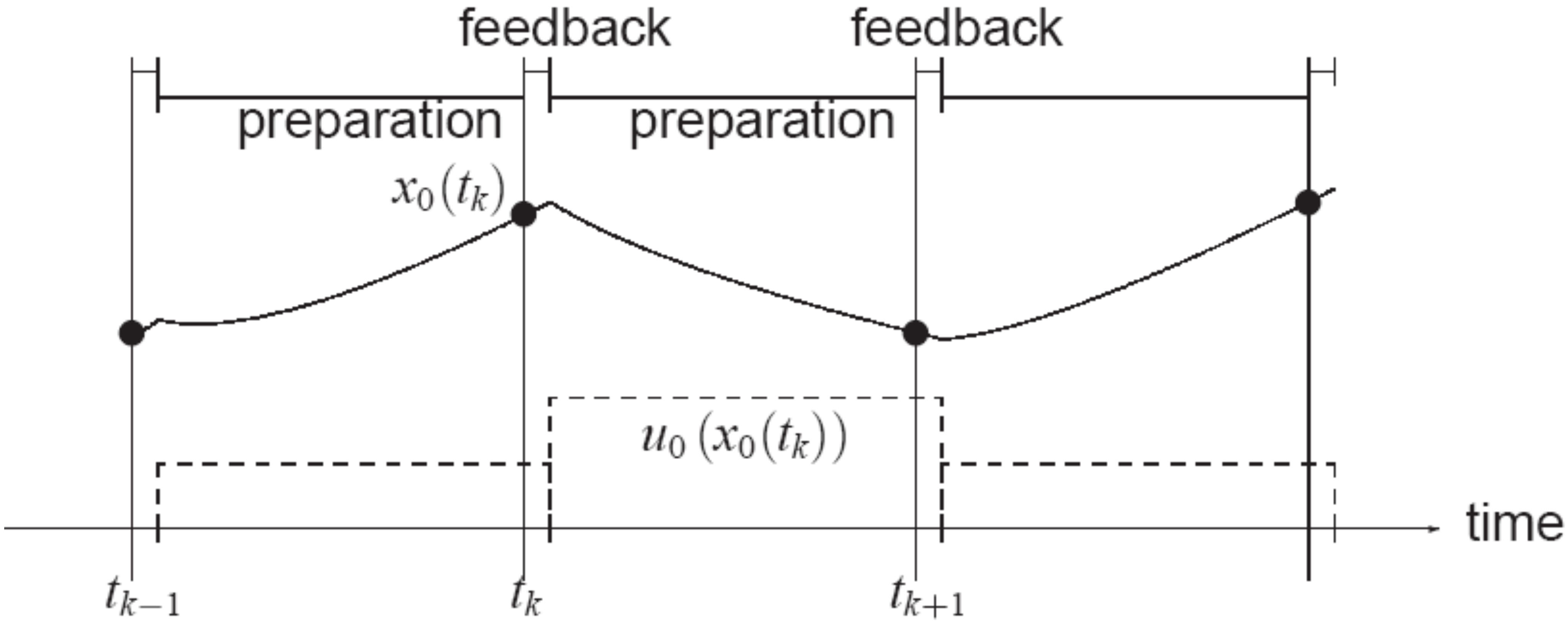
Real-Time Iteration Algorithm:

1. **Preparation Step (costly):**
   Linearize system at current iterate, perform partial reduction and condensing of quadratic program.

2. **Feedback Step (short):**
   When new x_0 is known, solve condensed QP and implement control u_0 immediately.
   Complete SQP iteration. Go to 1.

- short cycle-duration (as **one** SQP iteration)
- negligible feedback delay ($\approx$ 1 % of cycle)
- nevertheless fully nonlinear optimization

Real-Time Iterations minimize feedback delay

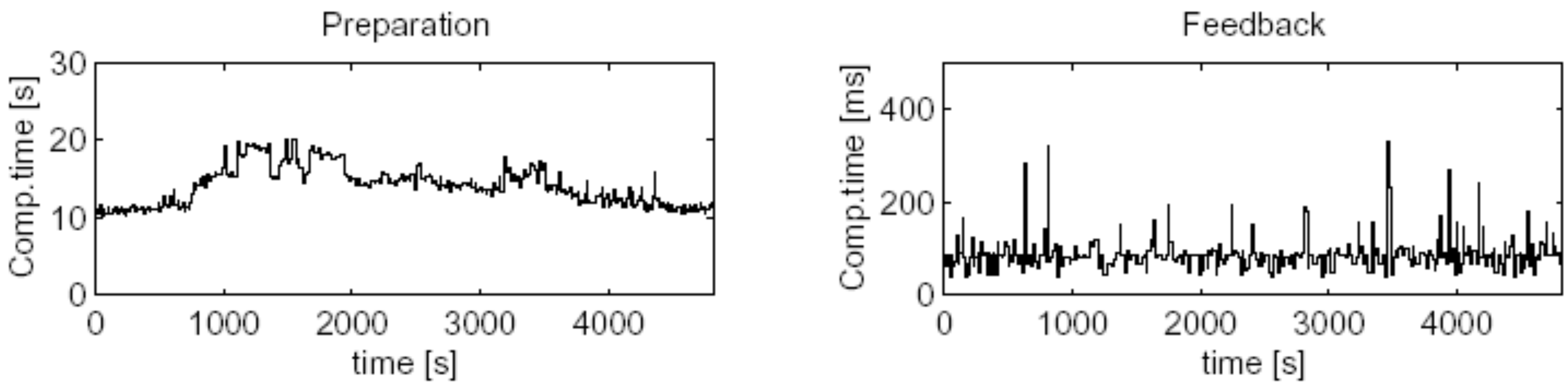


Realization at Distillation Column

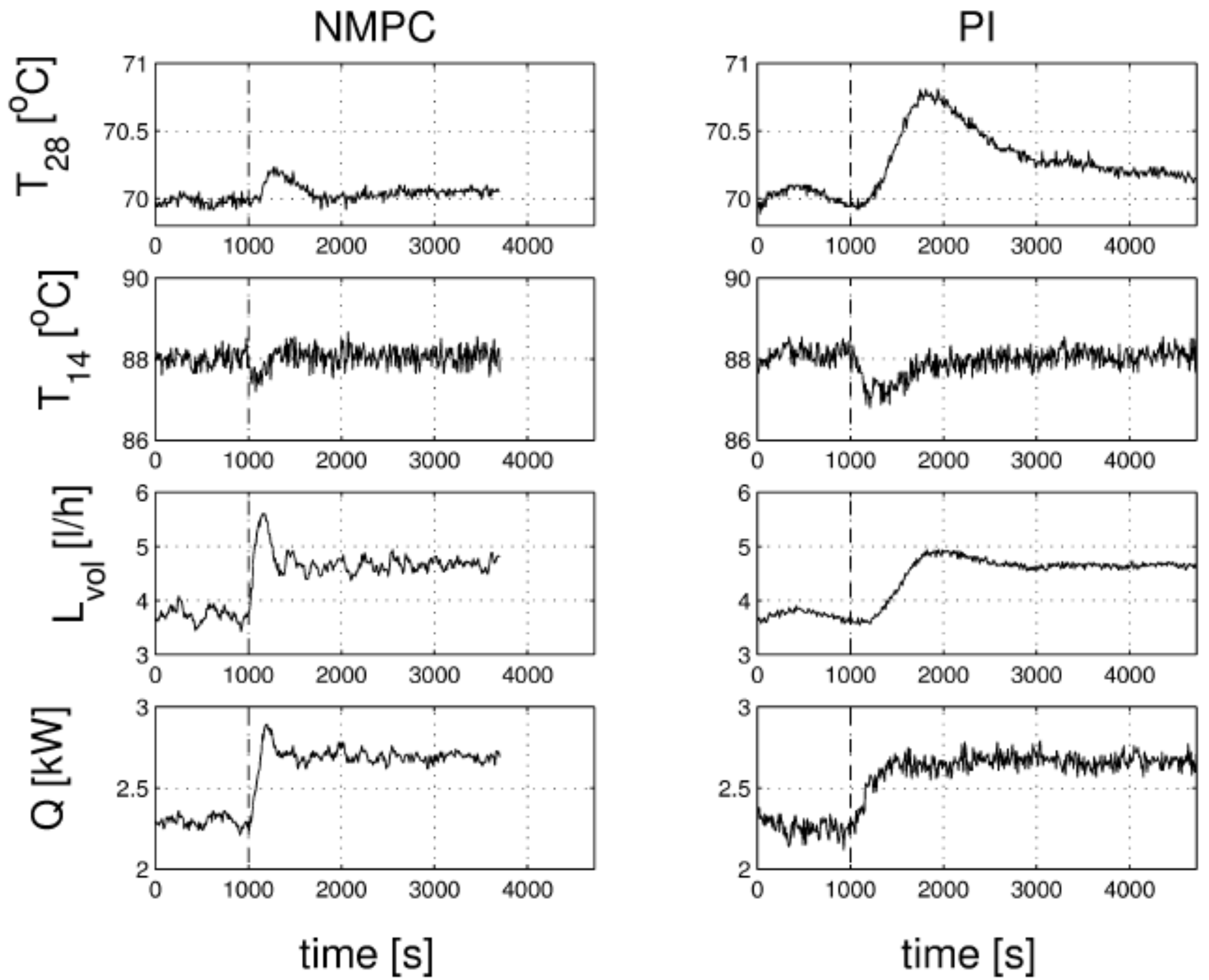
[D., Findeisen, Schwarzkopf, Uslu, Allgöwer, Bock, Schlöder,2002]

- Parameter estimation using dynamic experiments
- Online state estimation with Extended Kalman Filter variant, using only 3 temperature measurements to infer all 82 system states
- Implementation of estimator and optimizer on Linux Workstation.
- Communication with Process Control System via FTP all 10 seconds.
- Self-synchronizing processes.

Computation Times During Application

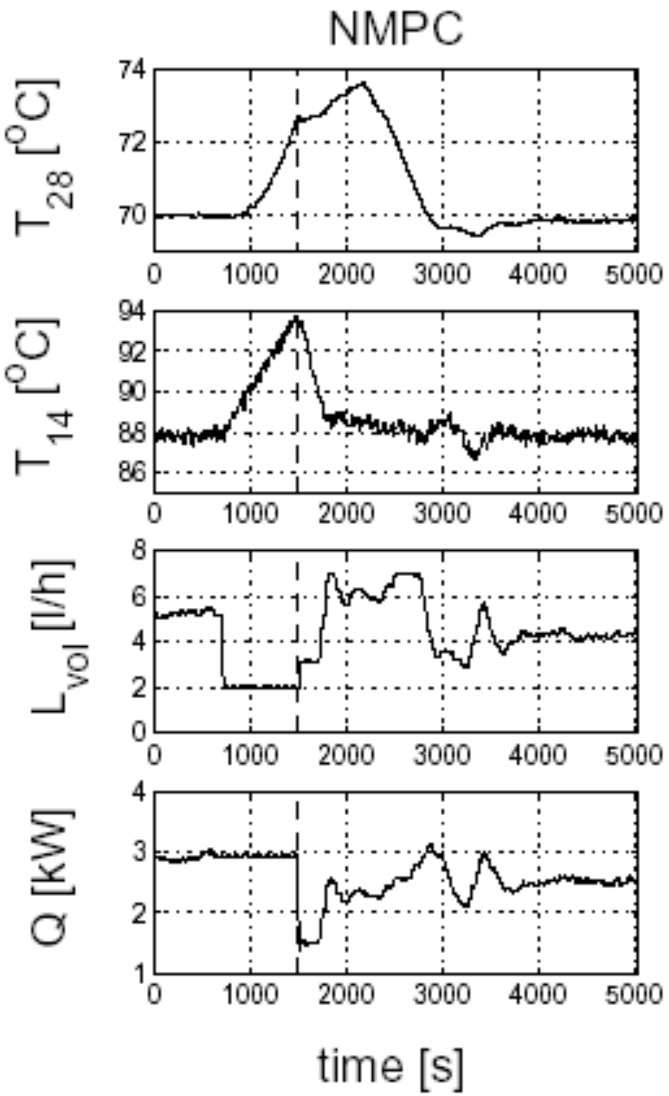


Feedflow Change by 20%: Transient Phase (Comparison with PI-Controller)



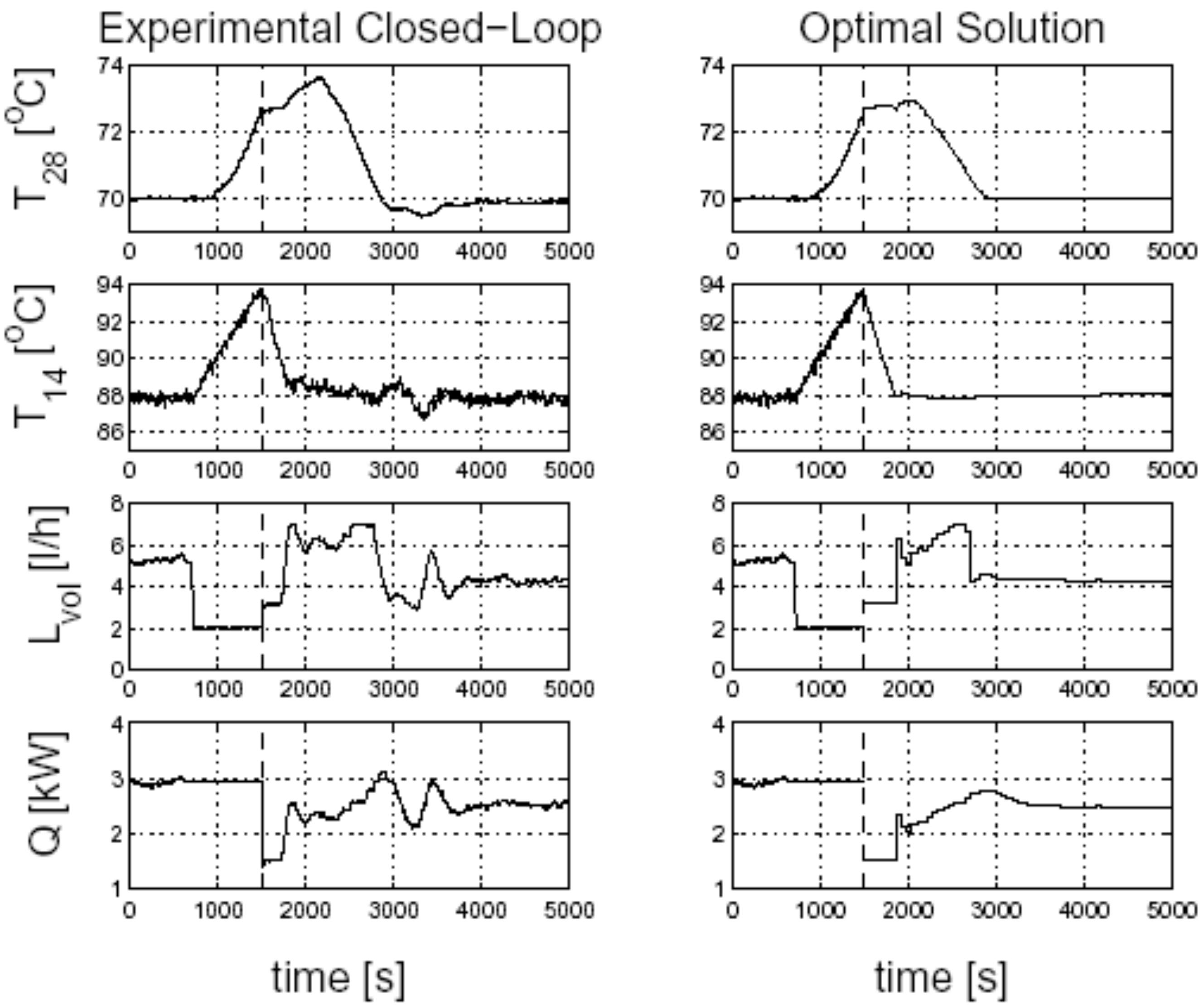
Transient in 15 minutes instead of 2 hours!

Large Disturbance (Heating), then NMPC



- Overheating by manual control
- NMPC only starts at $t = 1500$ s
- PI-controller not implementable, as disturbance too large (valve saturation)
- NMPC: at start control bound active $\Rightarrow T_{28}$ rises further
- Disturbance attenuated after half an hour

Real vs. Theoretical Optimal Solution



(Now back to the history of NMPC)

Outline of the Talk

PART I: Offline Optimal Control

- NMPC and MHE Problem Statement
- Simultaneous vs. Sequential Formulation
- Newton Type Optimization: IP vs. SQP Methods

PART II: Online Algorithms

- Parametric Sensitivities
- Review of Three Classical Algorithms

NMPC Optimal Control Problem in Continuous Time

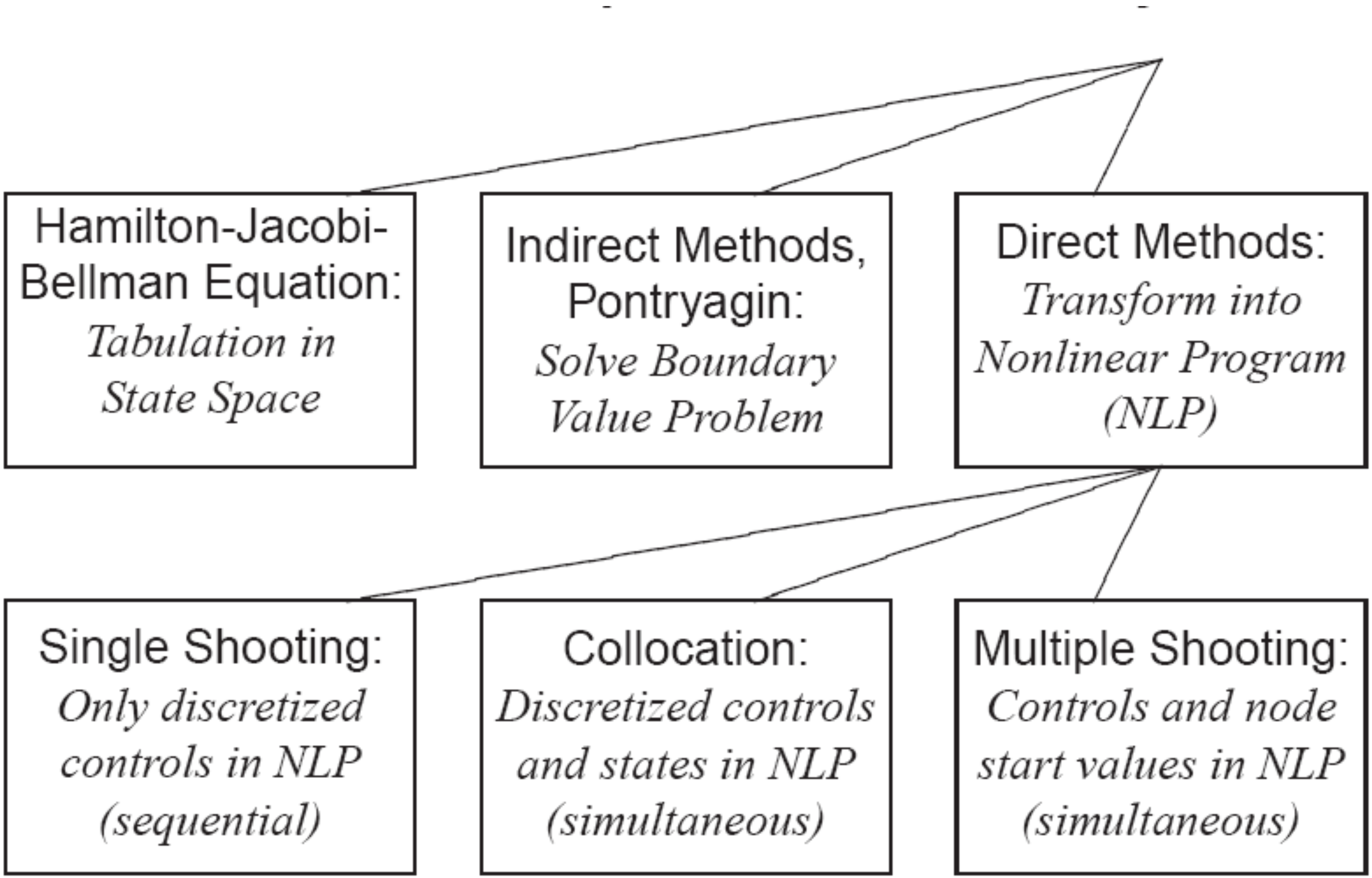
$$\underset{x(\cdot),\, u(\cdot)}{\text{minimize}} \quad \int_0^T L(x(t), u(t))\, dt \;+\; E\,(x(T))$$

subject to

$$\begin{aligned}
x(0) - x_0 &= 0, && && \text{(fixed initial value)}\\
\dot{x}(t) - f(x(t), u(t)) &= 0, && t \in [0, T], && \text{(ODE model)}\\
h(x(t), u(t)) &\geq 0, && t \in [0, T], && \text{(path constraints)}\\
r\,(x(T)) &= 0 && && \text{(terminal constraints).}
\end{aligned}$$

How to solve these nonlinear problems reliably and fast?

Optimal Control Family Tree



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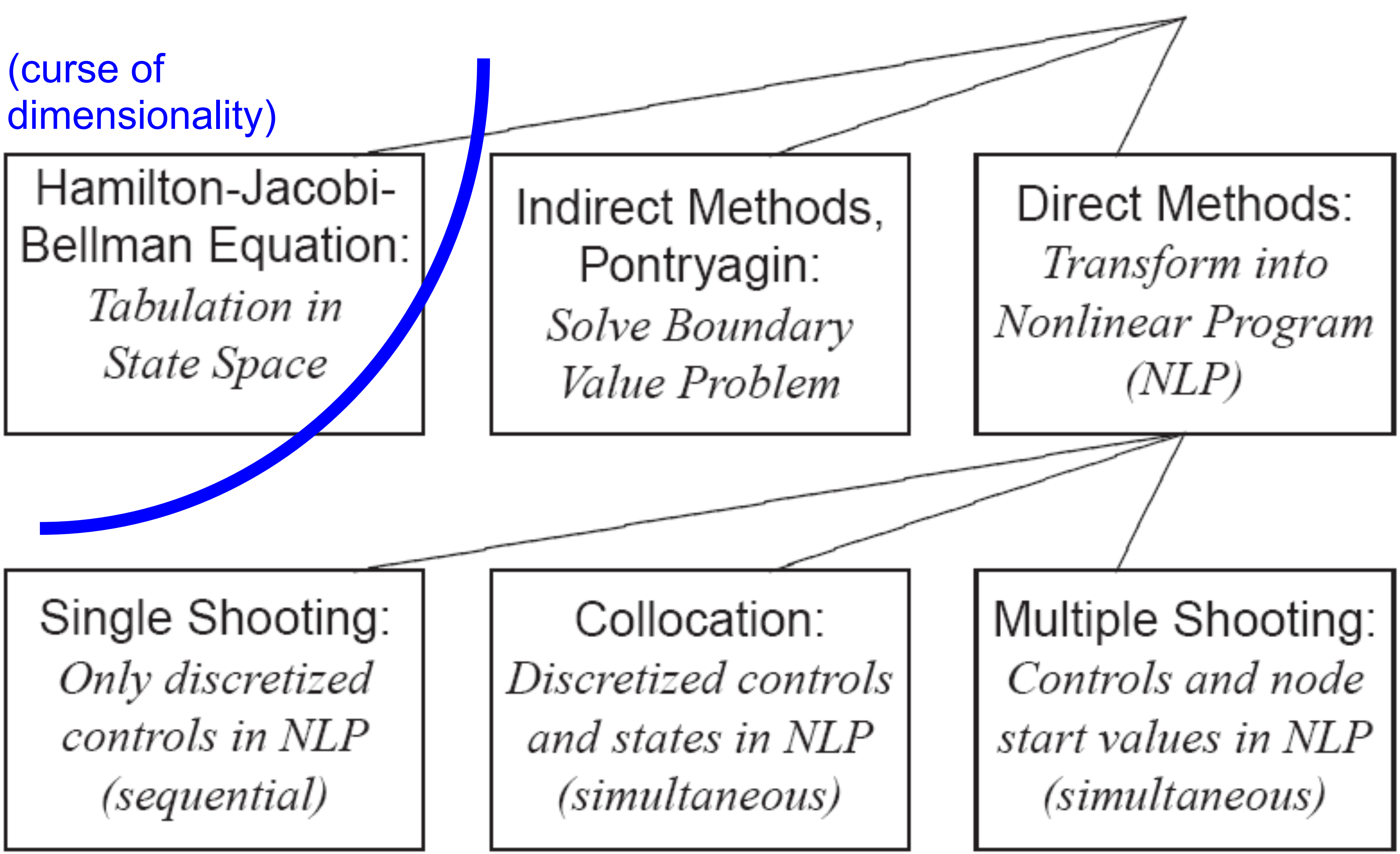

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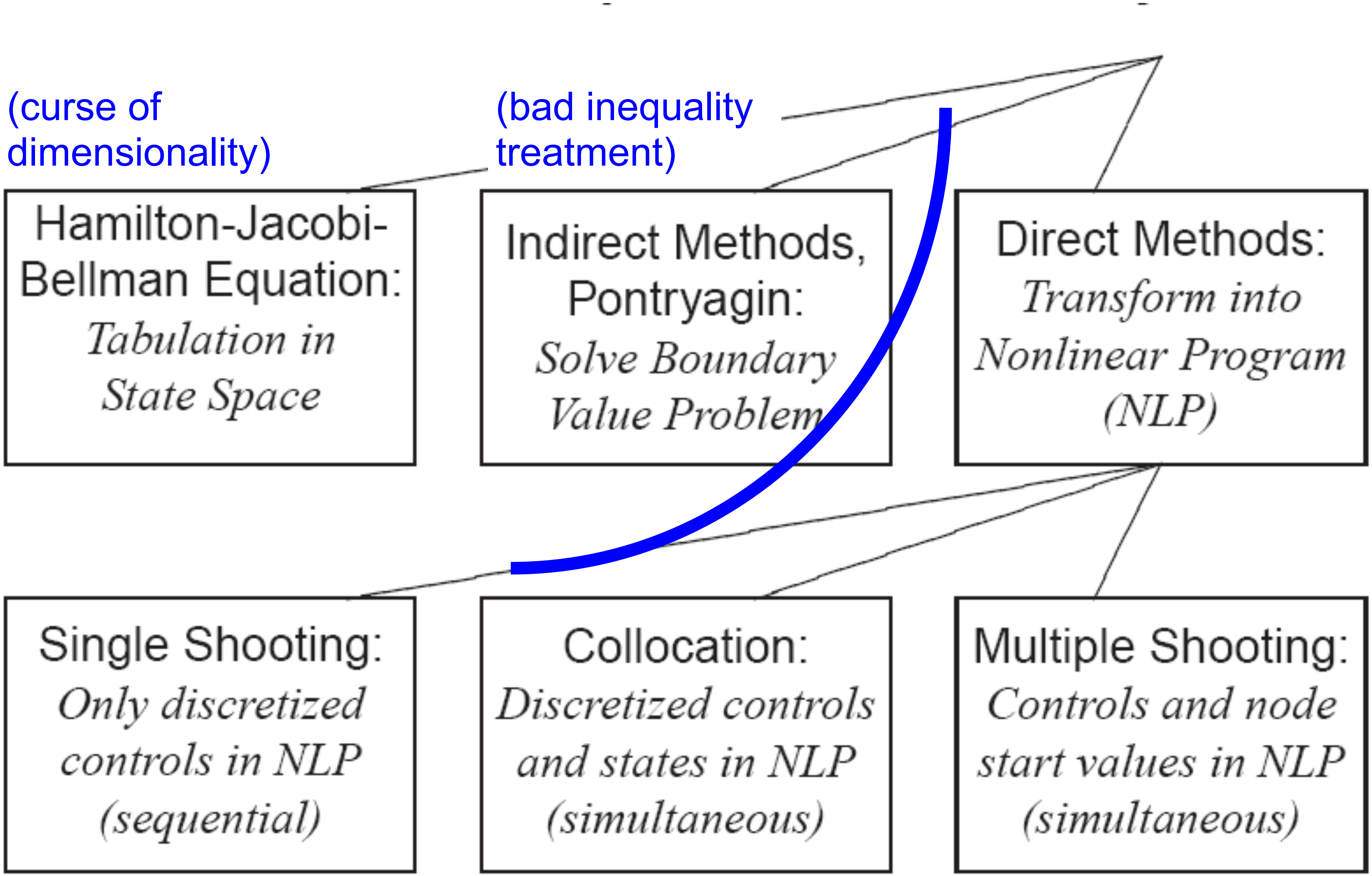

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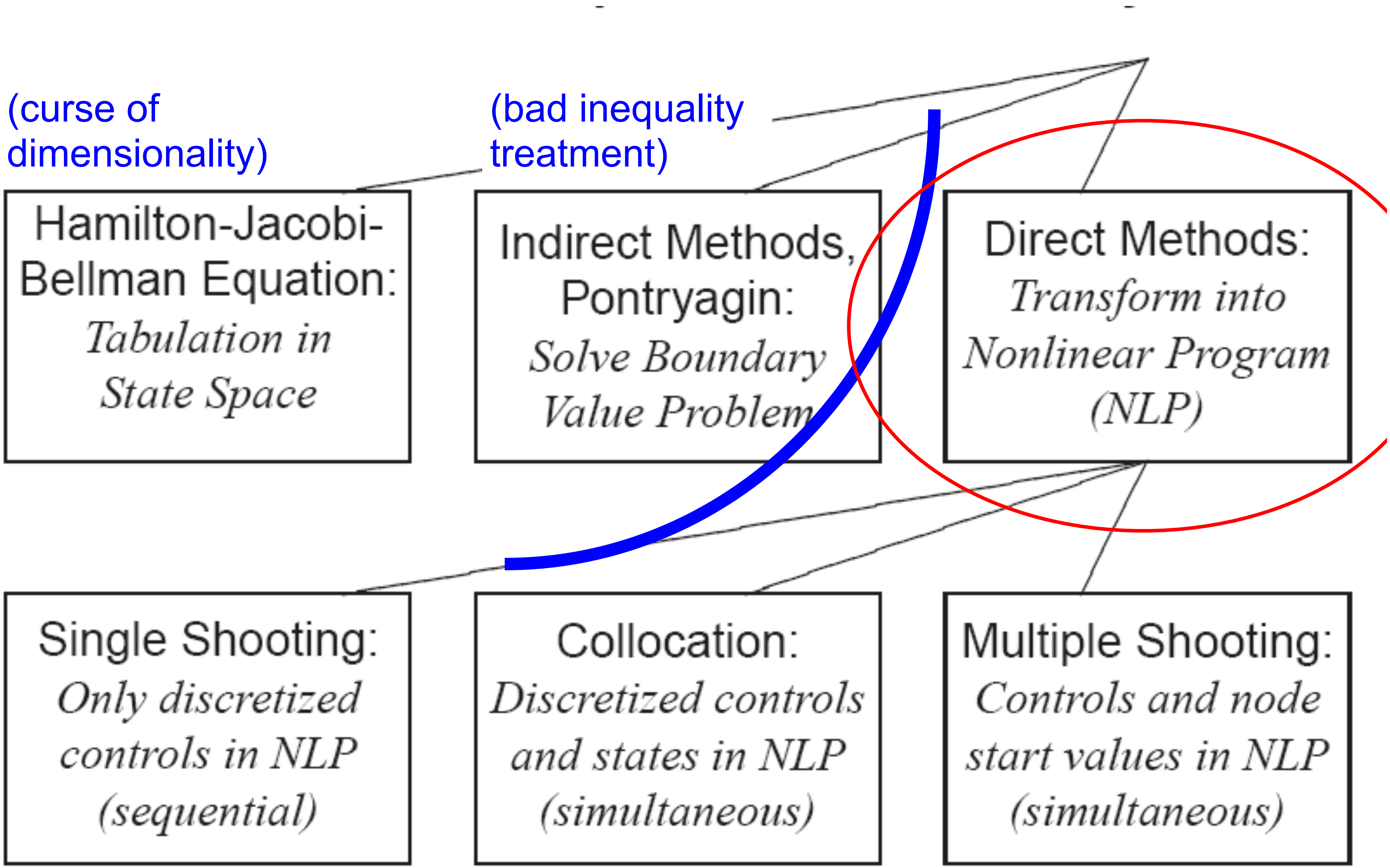


NMPC Problem in Discrete Time

$$
\begin{array}{lrcll}
\underset{x,\, z,\, u}{\text{minimize}} & \displaystyle\sum_{i=0}^{N-1} L_i(x_i, z_i, u_i) & + & E\left(x_N\right) & \\
\text{subject to} & x_0 - \bar{x}_0 & = & 0, & \\
& x_{i+1} - f_i(x_i, z_i, u_i) & = & 0, & i = 0, \ldots, N-1, \\
& g_i(x_i, z_i, u_i) & = & 0, & i = 0, \ldots, N-1, \\
& h_i(x_i, z_i, u_i) & \leq & 0, & i = 0, \ldots, N-1, \\
& r\left(x_N\right) & \leq & 0. &
\end{array}
$$

Structured parametric Nonlinear Program, “mp-NLP”

- Initial Value $\bar{x}_0$ is not known beforehand (“online data”)
- Discrete time dynamics often come from ODE simulation (“shooting”)
- “Algebraic States” z implicitly defined via third condition, can come from DAEs or from collocation discretization

[Moving Horizon Estimation (MHE) Problem]

$$\underset{x,\,z,\,w}{\text{minimize}} \quad \|x_0 - \bar{x}_0\|_P^2 + \sum_{i=0}^{N-1} \|y_i - m_i(x_i, z_i, u_i, w_i)\|_Q^2 + \|w_i\|_R^2$$

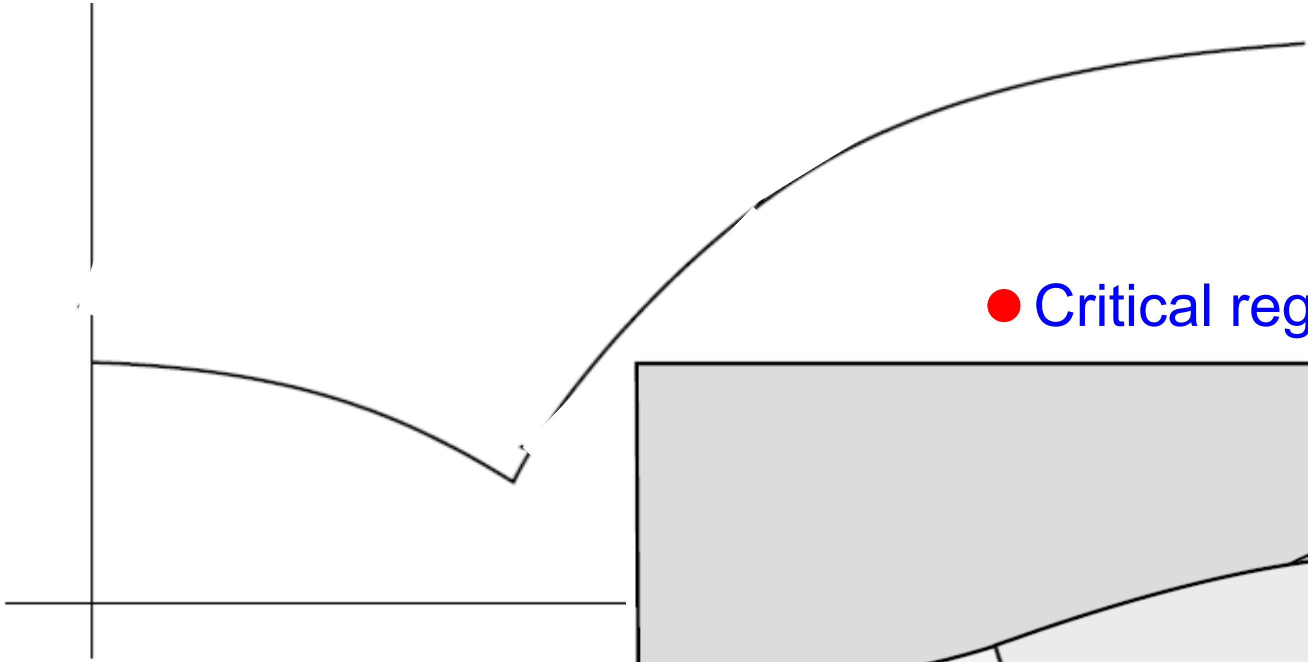
$$\text{subject to}$$

$$\begin{aligned}
x_{i+1} - f_i(x_i, z_i, u_i, w_i) &= 0, \quad i = 0, \ldots, N-1, \\
g_i(x_i, z_i, u_i, w_i) &= 0, \quad i = 0, \ldots, N-1, \\
h_i(x_i, z_i, u_i, w_i) &\leq 0, \quad i = 0, \ldots, N-1,
\end{aligned}$$

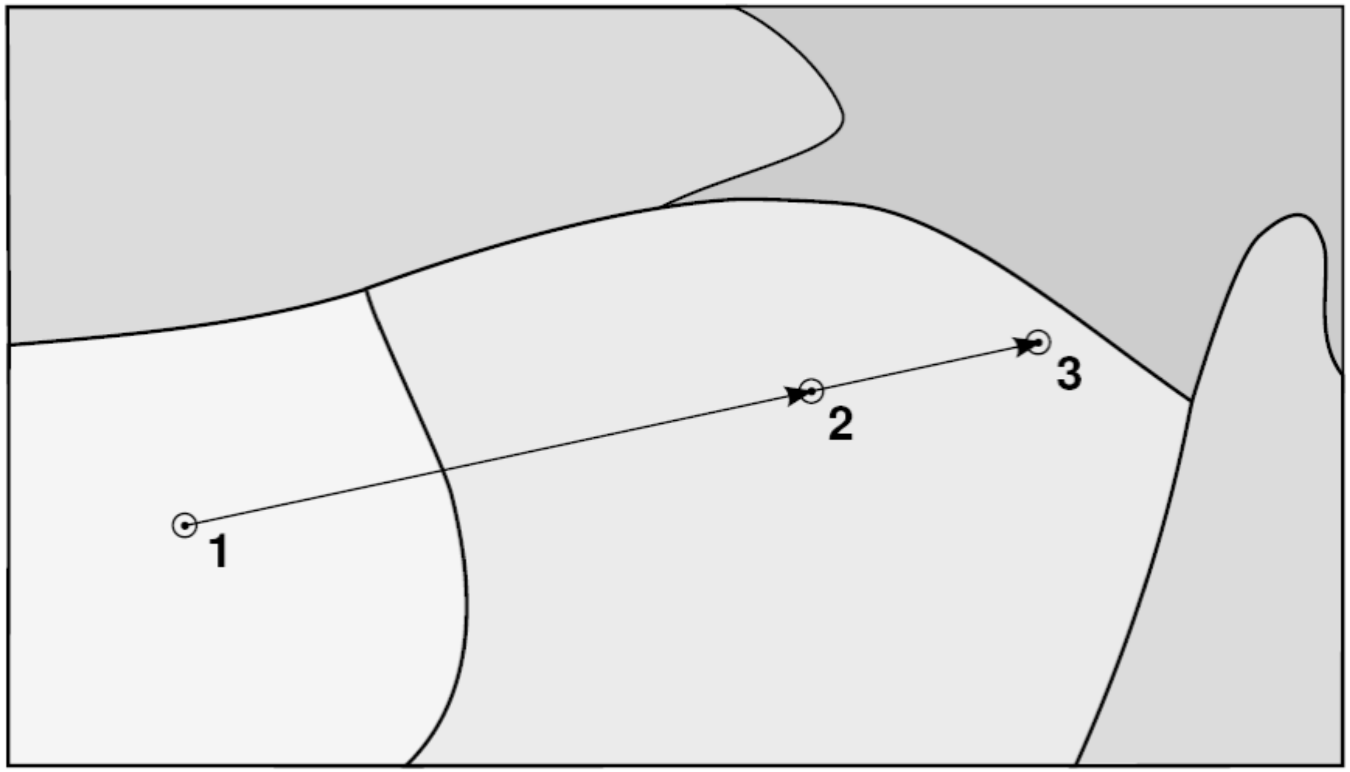
- Online problem data: y_i
- “Controls” w account for unknown disturbances. Often many w.
- Initial value is free

NMPC = mp-NLP

- Solution manifold is piecewise differentiable

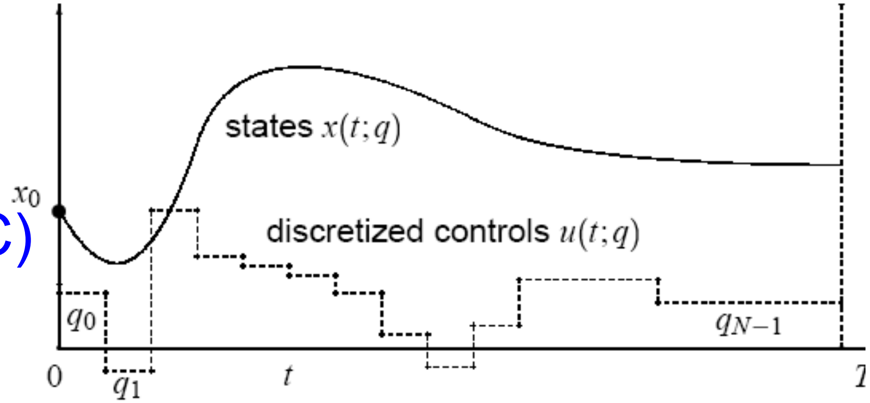


- Critical regions are non-polyhedral



Sequential Approach (Single Shooting): Eliminate States

$$
\begin{array}{lrcl}
\underset{u}{\text{minimize}} & \displaystyle\sum_{i=0}^{N-1} L_i(\tilde{x}_i(u), \tilde{z}_i(u), u_i) & + & E\left(\tilde{x}_N(u)\right) \\
\text{subject to} & h_i(\tilde{x}_i(u), \tilde{z}_i(u), u_i) & \leq & 0, \quad i = 0, \ldots, N-1, \\
 & r\left(\tilde{x}_N(u)\right) & \leq & 0.
\end{array}
$$



Pros:
- Only control degrees of freedom (for NMPC)
- Can couple with "Vanilla NLP" solver

Cons:
- Sparsity of problem lost
- Unstable systems cannot be treated

Historically first "direct" approach ("single shooting", Sargent&Sullivan 1978)

Simultaneous Approach: Keep States in NLP

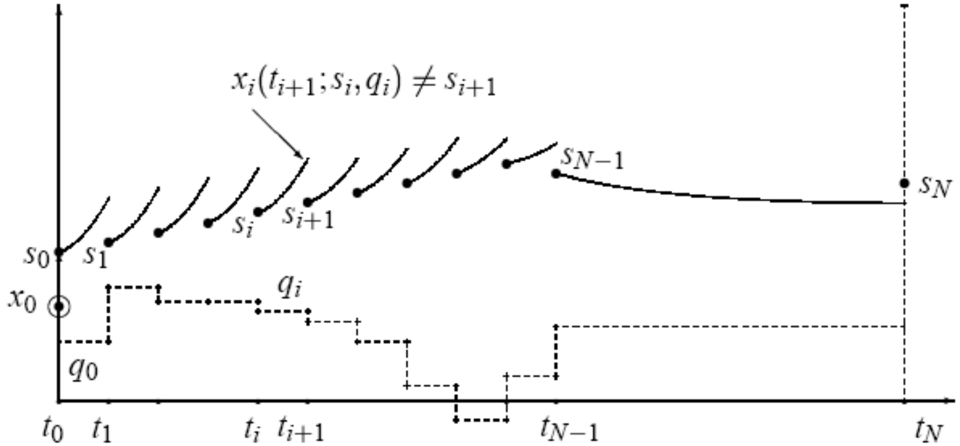
INTERNATIONAL FEDERATION
OF AUTOMATIC CONTROL
9TH WORLD CONGRESS

BUDAPEST, HUNGARY
JULY 2-6 1984

A MULTIPLE SHOOTING ALGORITHM FOR DIRECT SOLUTION OF OPTIMAL CONTROL PROBLEMS*

Hans Georg Bock and Karl J. Plitt

Institut für Angewandte Mathematik, SFB 72, Universität Bonn, 5300 Bonn, Federal Republic of Germany



Variants:
Multiple Shooting and Collocation

Pros:
- Sparsity of problem kept
- Unstable systems can be treated, nonlinearity reduced

Cons:
- Large scale problems
- Need to develop (or use) structure exploiting NLP solver

Outline of the Talk

PART I: Offline Optimal Control

- NMPC and MHE Problem Statement
- Simultaneous vs. Sequential Formulation
- **Newton Type Optimization: IP vs. SQP Methods**

PART II: Online Algorithms

- Parametric Sensitivities
- Review of Three Classical Algorithms

PART III: Software and Mechatronic Applications

How to solve Nonlinear Programs (NLPs) ?

$$\boxed{\underset{X}{\text{minimize}} \; F(X) \quad \text{s.t.} \quad \left\{ \begin{array}{rcl} G(X) & = & 0 \\ H(X) & \leq & 0 \end{array} \right.}$$

Lagrangian: $\mathcal{L}(X, \lambda, \mu) = F(X) + G(X)^T \lambda + H(X)^T \mu$

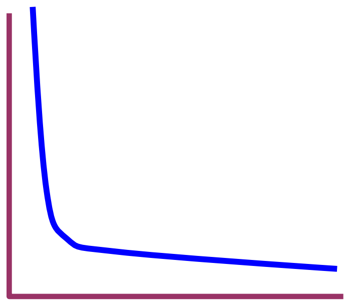
Karush Kuhn Tucker (KKT) conditions: for optimal X^* exist λ^*, μ^* such that:

$$\boxed{\begin{array}{rcl} \nabla_X \mathcal{L}(X^*, \lambda^*, \mu^*) & = & 0 \\ G(X^*) & = & 0 \\ 0 \geq H(X^*) \quad \perp \quad \mu^* & \geq & 0. \end{array}}$$

Newton type methods try to find points satisfying these conditions. But last condition non-smooth: cannot apply Newton's method directly. What to do?

Approach 1: Interior Point (IP) Methods

- Replace last condition by smoothed version:



$$\boxed{\begin{aligned} \nabla_X \mathcal{L}(X^*, \lambda^*, \mu^*) &= 0 \\ G(X^*) &= 0 \\ -\,H_i(X^*)\,\mu_i^* &= \tau, \quad i = 1, \ldots, n_H. \end{aligned}}$$

Summarize as $R(W) = 0$

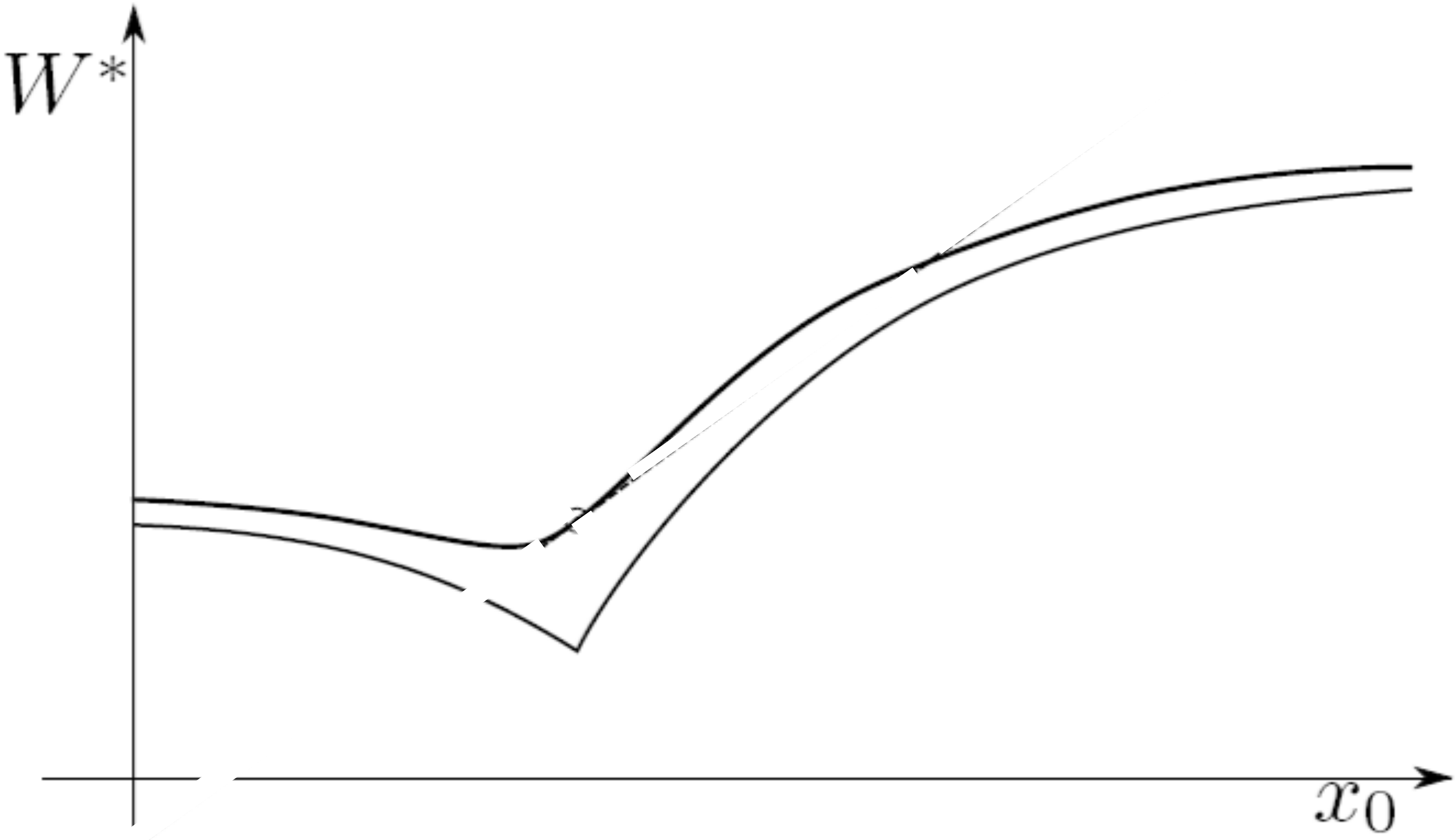
- Solve with Newton's method, i.e.,
  - Linearize at current guess $W^k = (X^k, \lambda^k, \mu^k)$

$$\boxed{R(W^k) + \nabla R(W^k)^T (W^{k+1} - W^k) = 0}$$

  - solve linearized system, get new trial point

- For τ small, IP problem gets close to original (path-following, self-concordance, polynomial time for convex problems, …)

(Note: IP with fixed τ makes mp-NLP smooth)



Approach 2: Sequential Quadratic Programming (SQP)

Mathematical Programming 14 (1978) 224–248.

ALGORITHMS FOR NONLINEAR CONSTRAINTS THAT USE LAGRANGIAN FUNCTIONS*

M.J.D. POWELL

University of Cambridge, Cambridge, United Kingdom

Received 10 October 1976

- Linearize all problem functions, solve Quadratic Program (QP):

$$\underset{X}{\text{minimize}} \; F_{\mathrm{QP}}^{k}(X) \quad \text{s.t.} \quad \left\{ \begin{array}{rcl} G(X^k) + \nabla G(X^k)^T (X - X^k) & = & 0 \\ H(X^k) + \nabla H(X^k)^T (X - X^k) & \leq & 0 \end{array} \right.$$

with convex quadratic objective using an approximation of Hessian.

Obtain new guesses for both X^* and λ^*, μ^* .

(Important Variant of SQP: Generalized Gauss-Newton)

If objective is sum of squares: $F(X) = \frac{1}{2}\|R(X)\|_2^2.$

then QP objective can well be approximated by:

$$F_{\mathrm{QP}}^k(X) = \frac{1}{2}\|R(X^k) + \nabla R(X^k)^T(X - X^k)\|_2^2$$

(often extremely good linear convergence)

Difference between IP and SQP ?

- Both generate sequence of iterates X^k, λ^k, μ^k
- Both need to linearize problem functions in each iteration.

- IP iterations cheaper:
  - IP solves only linear system
  - SQP solves a QP in each iteration (maybe even with an IP method!)

- IP needs more iterations:
  - IP multipliers change slowly, iterates always in interior
  - SQP multipliers jump, active set can quickly be identified

SQP good if problem function evaluations are expensive (shooting methods)

(Adjoint Based SQP: can use old Jacobians)

- SQP even works if all QP matrices are old. Only constraints and Lagrange gradient (cheap by adjoint differentiation) need to be exact.

- Trick: use "modified gradient" $a_k = \nabla_X \mathcal{L}(X^k, \lambda^k, \mu^k) - B_k \lambda^k - C_k \mu^k$

  in QP objective $F^k_{\mathrm{adjQP}}(X) = a_k^T X + \frac{1}{2}(X - X^k)^T A_k (X - X^k).$

Solve QP with inexact constraints

$$\boxed{\underset{X}{\text{minimize}} \quad F^k_{\mathrm{adjQP}}(X) \quad \text{s.t.} \quad \left\{ \begin{array}{lcl} G(X^k) + B_k^T (X - X^k) & = & 0 \\ H(X^k) + C_k^T (X - X^k) & \leq & 0. \end{array} \right.}$$

Can prove stability of active set and linear convergence [Bock, D., Kostina, Schloeder 2007]. Adjoint SQP iterations often orders of magnitude cheaper than full SQP iterations.

Linear Algebra Issues in Optimal Control

- In each SQP iteration, solve structured QP:

$$
\begin{array}{rcll}
\underset{x,\,z,\,u}{\text{minimize}} \quad \displaystyle\sum_{i=0}^{N-1} L_{\mathrm{QP},i}(x_i, z_i, u_i) & + & E_{\mathrm{QP}}(x_N) & \\
\text{subject to} \qquad x_0 - \bar{x}_0 & = & 0, & \\
x_{i+1} - f_i' - F_i^x x_i - F_i^z z_i - F_i^u u_i & = & 0, & i = 0, \ldots, N-1, \\
g_i' + G_i^x x_i + G_i^z z_i + G_i^u u_i & = & 0, & i = 0, \ldots, N-1, \\
h_i' + H_i^x x_i + H_i^z z_i + H_i^u u_i & \leq & 0, & i = 0, \ldots, N-1, \\
r' + R x_N & \leq & 0. &
\end{array}
$$

- Algebraic Reduction/compression: first eliminate z

QP after Algebraic Reduction

$$
\begin{aligned}
\underset{x,\,u}{\text{minimize}} \quad & \sum_{i=0}^{N-1} L_{\text{redQP},i}(x_i, u_i) \quad + \quad E_{\text{QP}}(x_N) \\
\text{subject to} \quad x_0 - \bar{x}_0 &= 0, \\
x_{i+1} - c_i - A_i x_i - B_i u_i &= 0, \quad i = 0, \ldots, N-1, \\
\bar{h}_i + \bar{H}_i^x x_i + \bar{H}_i^u u_i &\leq 0, \quad i = 0, \ldots, N-1, \\
r' + R x_N &\leq 0.
\end{aligned}
$$

How to solve this structured QP?

Approach 1: Banded Factorization

- Factorize large banded KKT Matrix e.g. by Riccati based recursion

$$M = \begin{bmatrix} & \mathbb{I} & & & & \\ \mathbb{I} & Q_0 & S_0 & -A_0^T & & \\ & S_0^T & R_0 & -B_0^T & & \\ & -A_0 & -B_0 & \ddots & \mathbb{I} & \\ & & & \mathbb{I} & Q_N & \end{bmatrix}$$

- Advantageous for long horizons and many controls

Approach 2: Condensing - Eliminate all States

- Eliminate states by linear system simulation, keep only controls in QP, solve QP with dense solver

$$\begin{array}{lll} & \underset{u}{\text{minimize}} & f_{\text{condQP},i}(\bar{x}_0, u) \\ \text{subject to} & \bar{r} + \bar{R}^{x0}\bar{x}_0 + \bar{R}^u u & \leq & 0. \end{array}$$

- Note: mp-QP in same form as needed by qpOASES and explicit MPC
- Can use this QP as fast feedback law for several $\bar{x}_0$
- But QP matrices change after each SQP re-linearization

Outline of the Talk

PART I: Offline Optimal Control

- NMPC and MHE Problem Statement
- Simultaneous vs. Sequential Formulation
- Newton Type Optimization: IP vs. SQP Methods

PART II: Online Algorithms

- Parametric Sensitivities
- Review of Three Classical Algorithms

The start: “Newton-Type Controller” by Li and Biegler

Chem Eng Res Des, Vol. 67, November 1989

MULTISTEP, NEWTON-TYPE CONTROL STRATEGIES FOR CONSTRAINED, NONLINEAR PROCESSES

W. C. LI and L. T. BIEGLER
Carnegie-Mellon University, Department of Chemical Engineering, Pittsburgh, USA

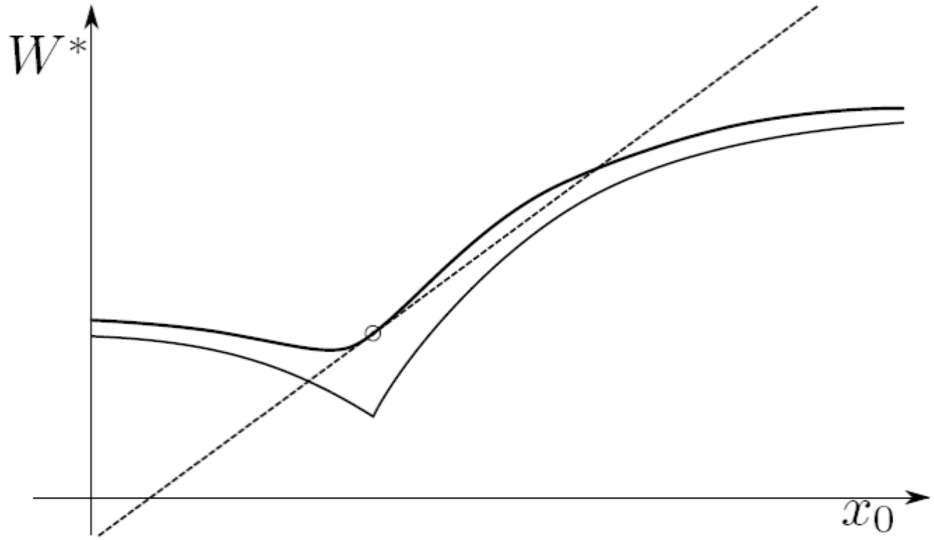
- SQP type method
- single shooting
- perform only one SQP iteration per problem (“real-time iteration”)
- Method also implemented in “NEPSAC Algorithm” by [De Keyser 1998] and many others [IPCOS, GE, …]. Many applications.
- was missing one important feature: *parametric sensitivities*

Parametric Sensitivities

- In IP case, smoothed KKT conditions are equivalent to parametric root finding problem: $$R(\bar{x}_0, W) = 0$$

  with solution $W^*(\bar{x}_0)$ depending on initial condition

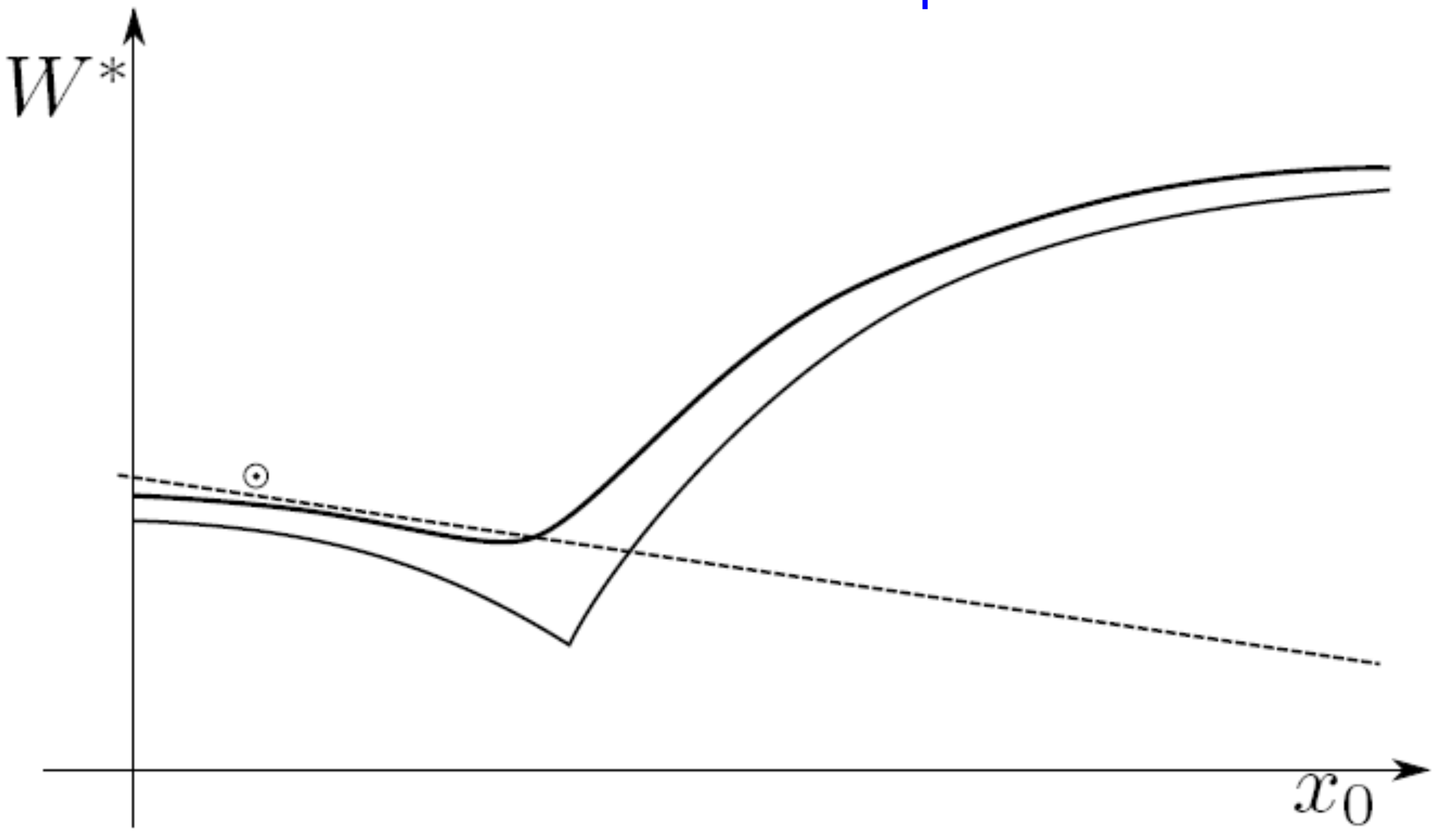
Based on old solution, can get "tangential predictor" for new one:



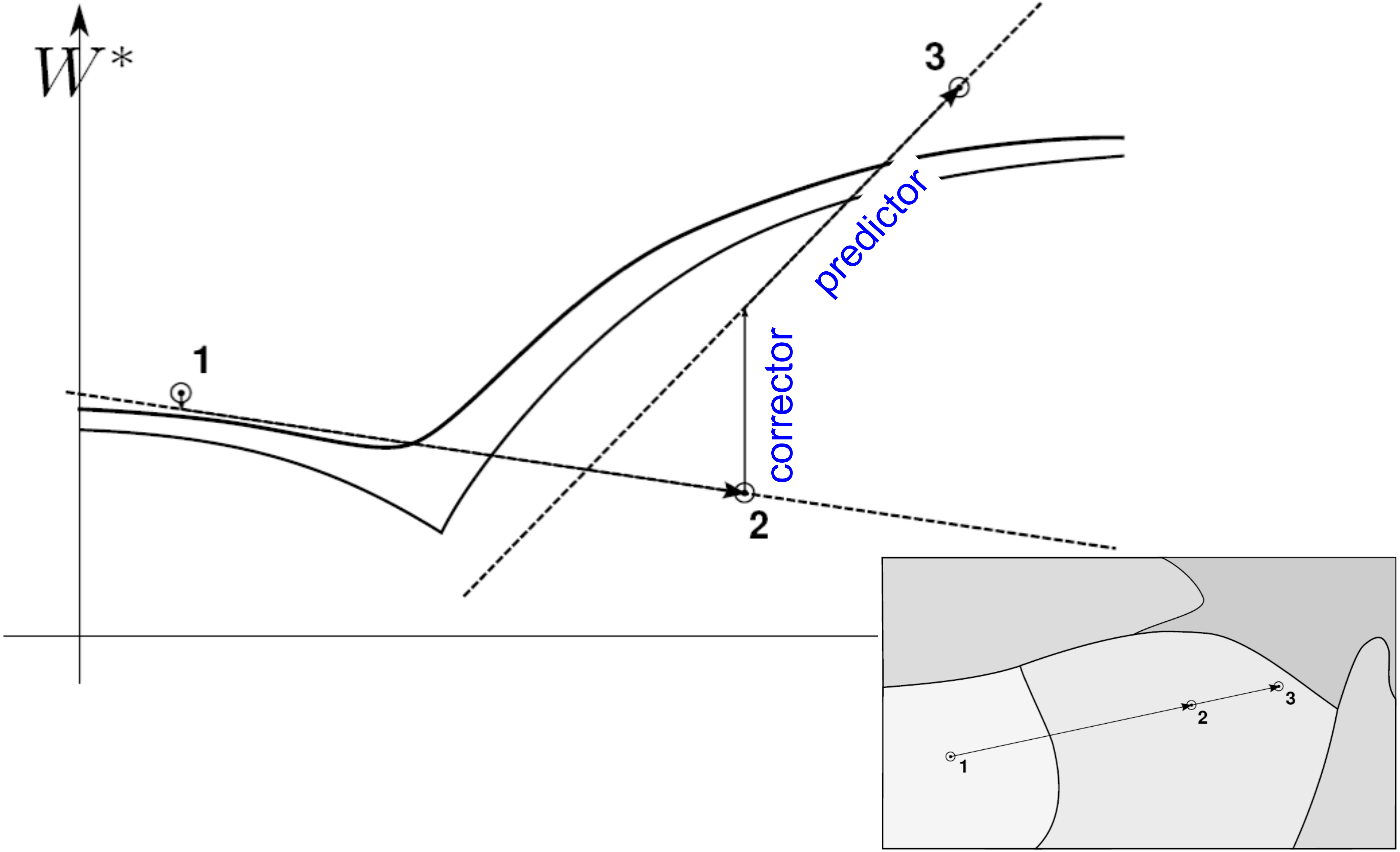
Initial Value Embedding Idea

- Can obtain sensitivity nearly for free in Newton type methods:

$$W' = W - \left(\frac{\partial R}{\partial W}(\bar{x}_0, W)\right)^{-1} \left[\underbrace{\frac{\partial R}{\partial \bar{x}_0}(\bar{x}_0, W)\left(\bar{x}_0' - \bar{x}_0\right)}_{\text{predictor}} + \underbrace{R(\bar{x}_0, W)}_{\text{corrector}}\right]$$



“IP real-time iteration” for sequence of NLPs



IP Real-Time Iteration ≈ Ohtsuka's Continuation Method

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Available online at www.sciencedirect.com

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Automatica 40 (2004) 563–574

automatica

www.elsevier.com/locate/automatica

A continuation/GMRES method for fast computation of nonlinear receding horizon control☆

Toshiyuki Ohtsuka*

Department of Computer-Controlled Mechanical Systems, Graduate School of Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan

Additional features of Continuation/GMRES:

- matrix free and iterative linear algebra via GMRES
- Interior Point formulation via quadratic slacks
- Single shooting with adjoint gradient computation
- (recently extended to multiple shooting with condensing, faster contraction)

But: Ohtsuka's method "overshoots" at active set changes

(Variant of IP Methods: Quadratic Slacks)

- Ohtsuka (2004) uses for NMPC a variant of IP methods:

$$\underset{X,Y}{\text{minimize}} \;\; F(X) - \tau \sum_{i=1}^{n_H} Y_i \quad \text{s.t.} \begin{cases} G(X) &= 0 \\ H_i(X) + Y_i^2 &= 0, \; i = 1, \ldots, n_H. \end{cases}$$

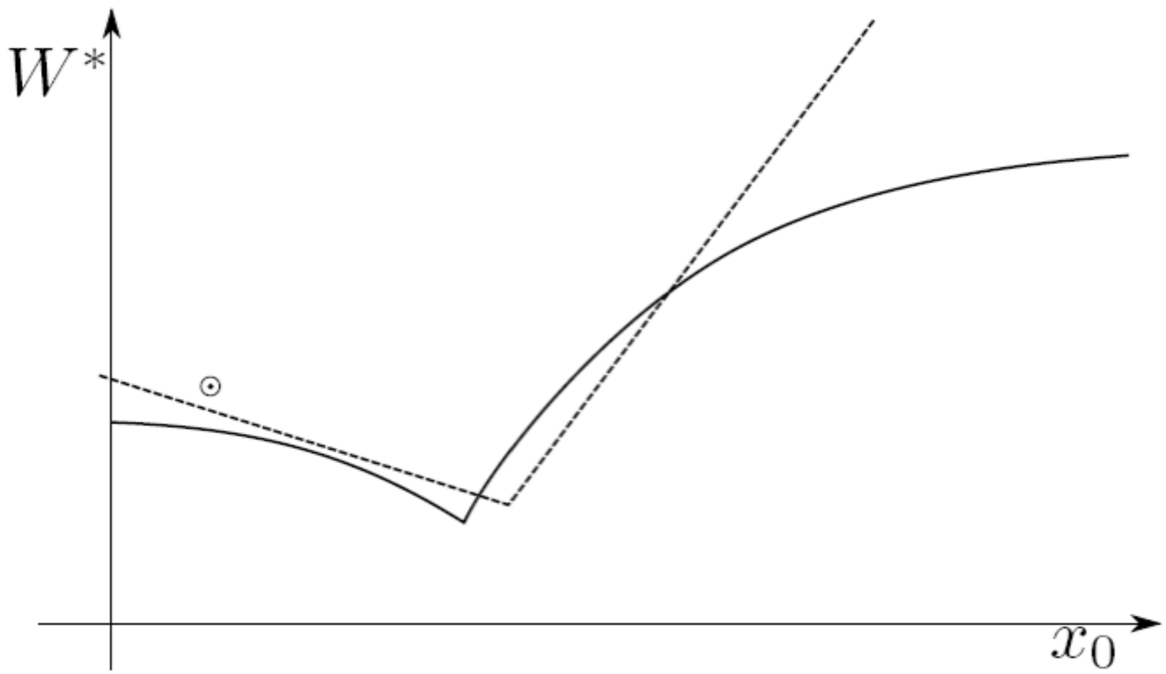
Seems to work well for fixed penalty parameter. But no self-concordance properties as in usual IP methods.

Generalized Tangential Predictor via SQP

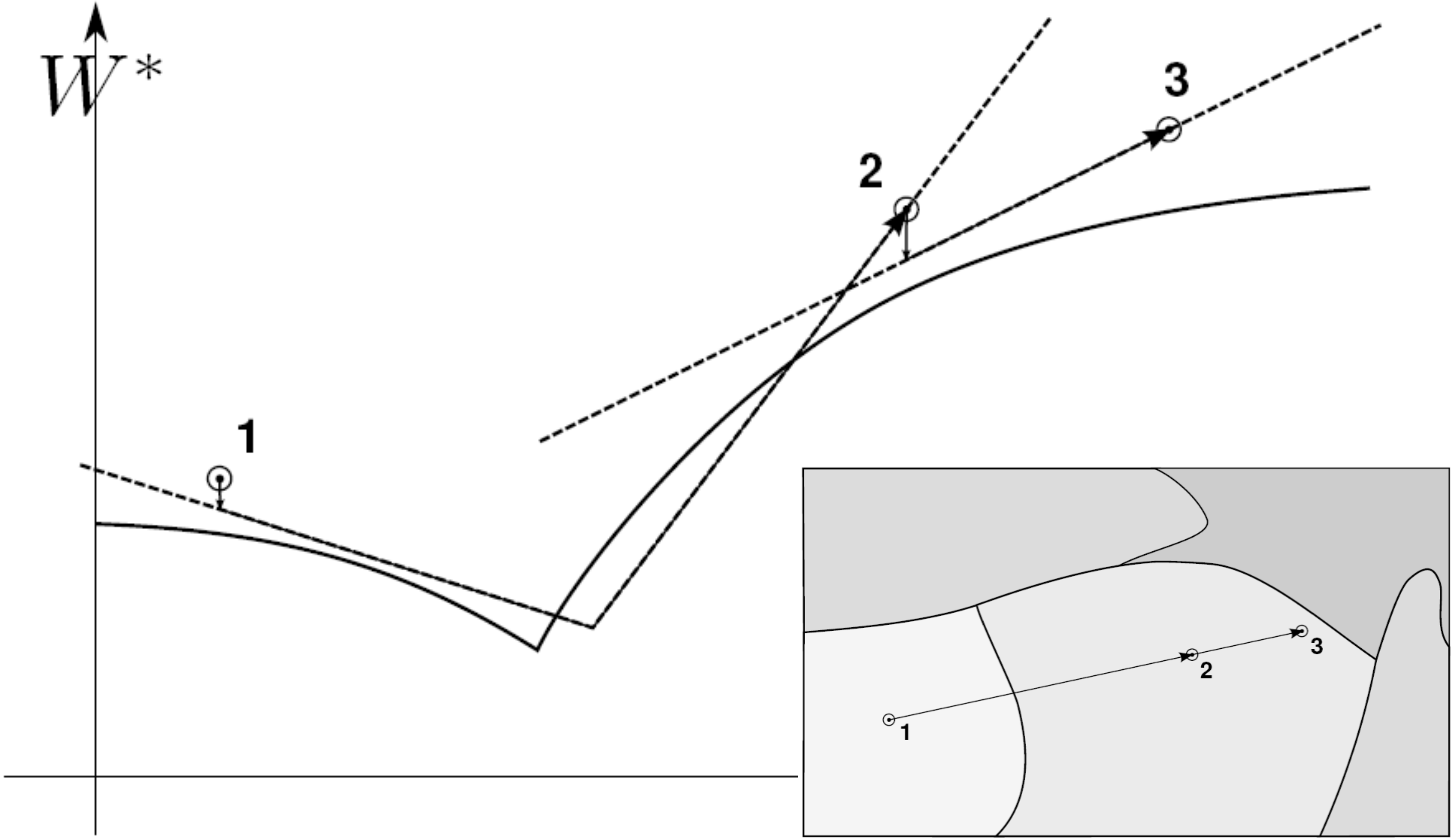
- Solve a full QP with "initial value embedding" [D. et al. 2002].

$$\begin{array}{lll} & \underset{u}{\text{minimize}} & f_{\text{condQP},i}(\bar{x}_0, u) \\ \text{subject to} & & \bar{r} + \bar{R}^{x_0}\bar{x}_0 + \bar{R}^u u \quad \leq \quad 0. \end{array}$$

- At smooth parts, delivers same predictor-corrector step as Newton. But is "Generalized Tangential Predictor" valid also across active set changes:

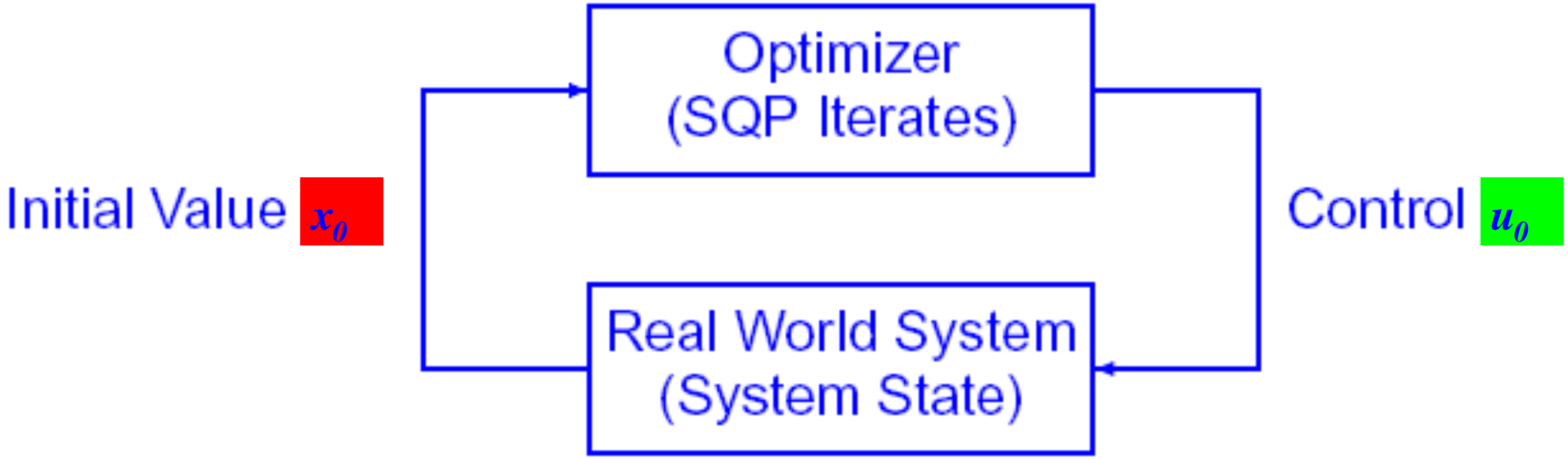


SQP Real-Time Iteration [D. et al 2002]



- long "preparation phase" for linearization, reduction, and condensing
- fast "feedback phase" (QP solution once $\bar{x}_0$ is known). Fast, but…

Stability of System-Optimizer Dynamics?

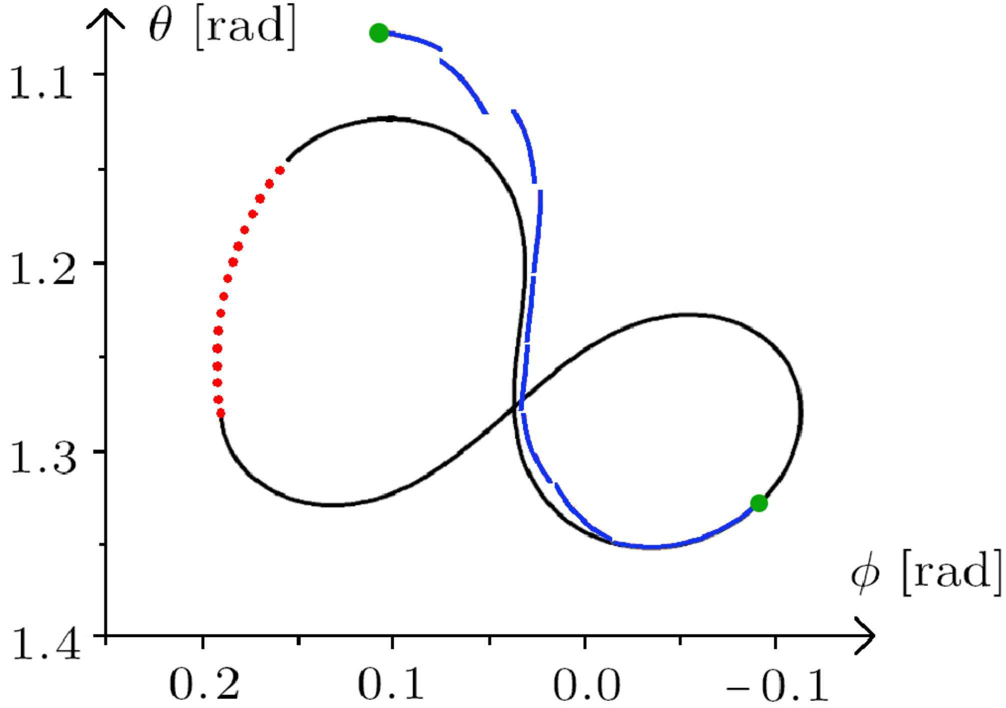


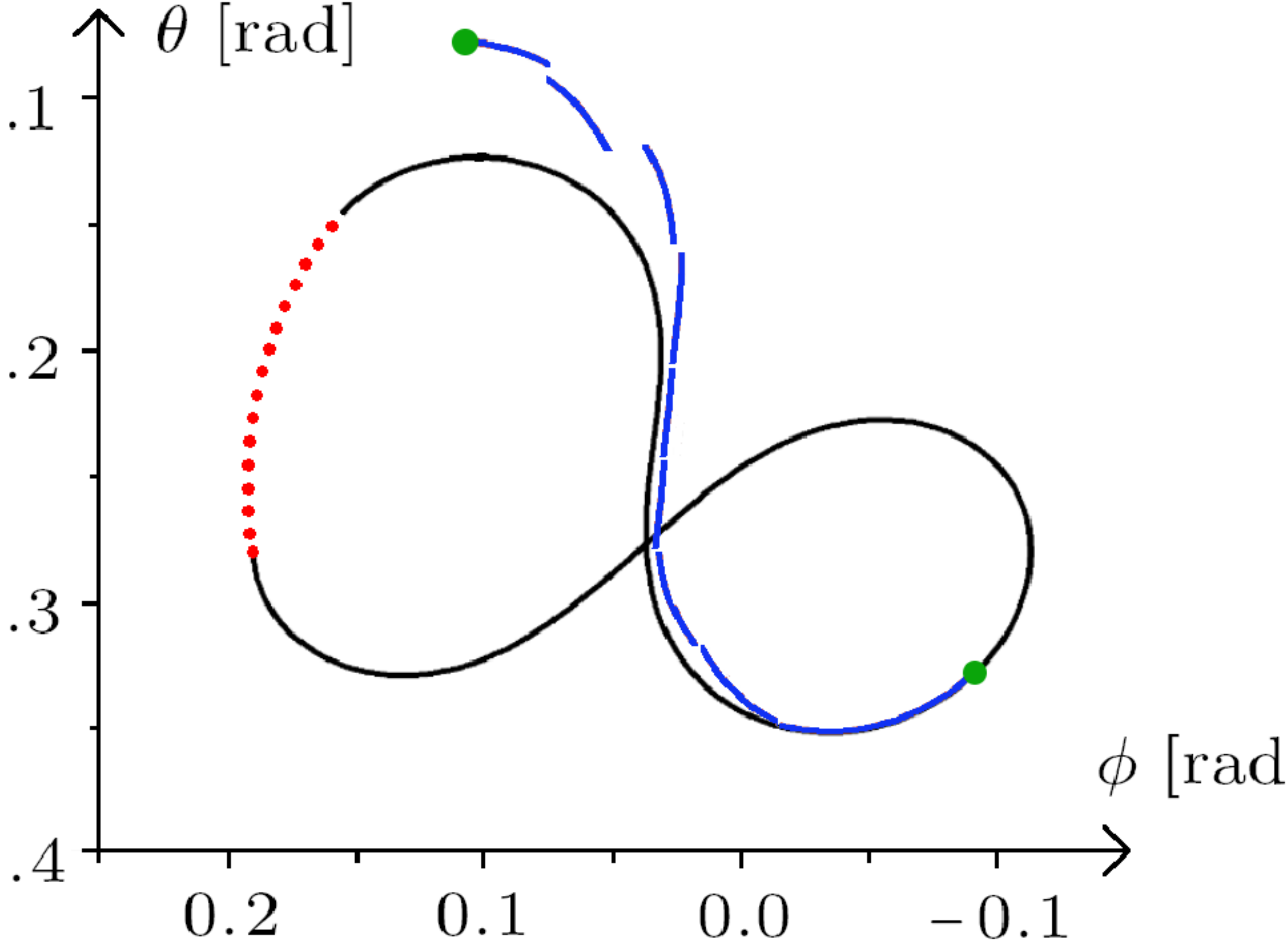
- System and optimizer are coupled: can numerical errors grow and destabilize closed loop?
- Stability analysis combines concepts from both, **NMPC stability theory** and **convergence theory of Newton-type optimization**.
- Stability shown under mild assumptions (short sampling times, stable NMPC scheme) [Diehl, Findeisen, Allgöwer, 2005]
- Losses w.r.t. optimal feedback control are $O(\kappa^2\epsilon^2)$ after ϵ disturbance [Diehl, Bock, Schlöder, 2005]

Kite NMPC Problem solved with ACADO (B. Houska)

- 9 states, 3 controls
- Penalize deviation from "lying eight"
- Predict half period
- zero terminal constraint
- 10 multiple shooting intervals

Solve with **SQP real-time iterations with shift** (implemented in ACADO)

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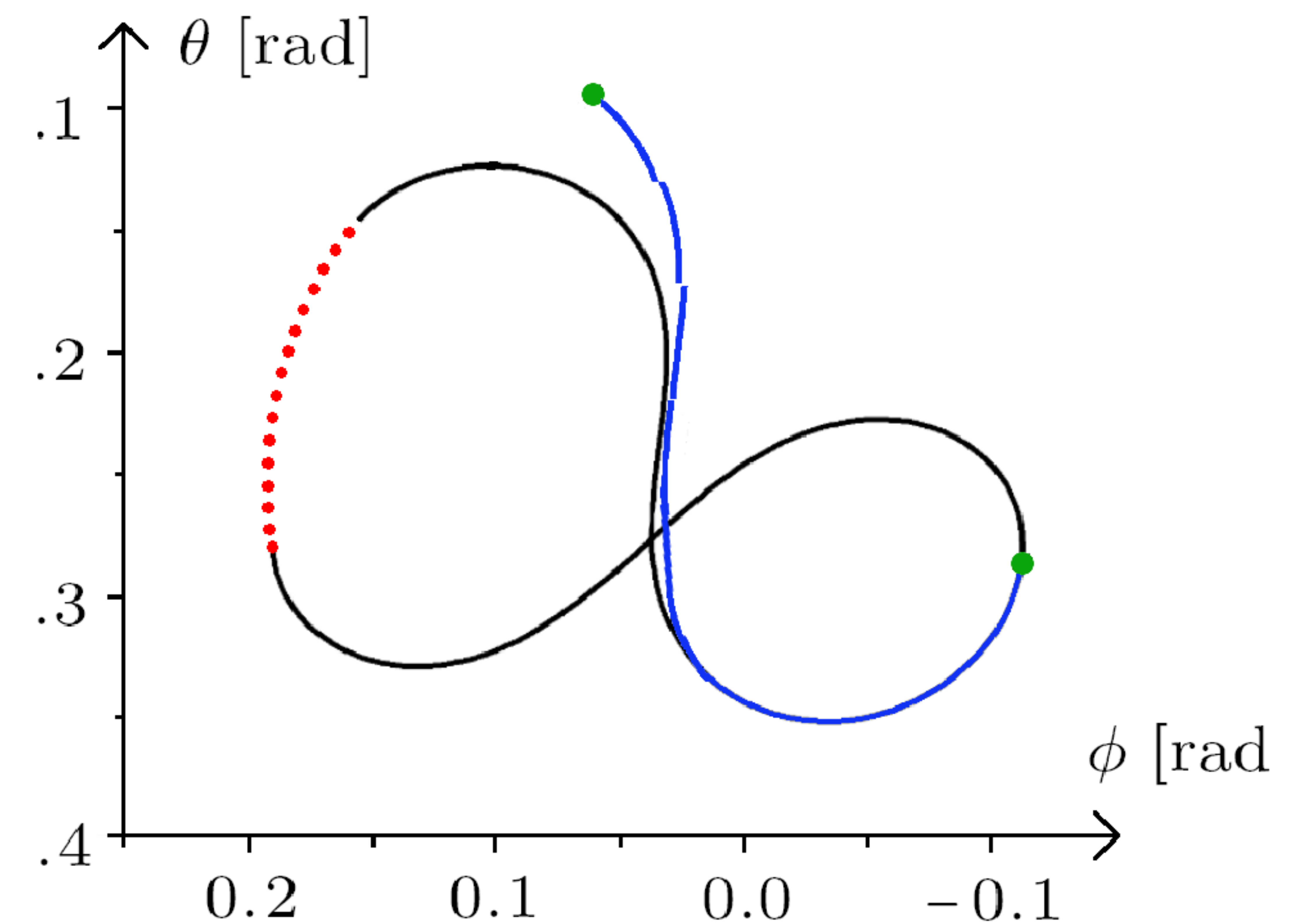

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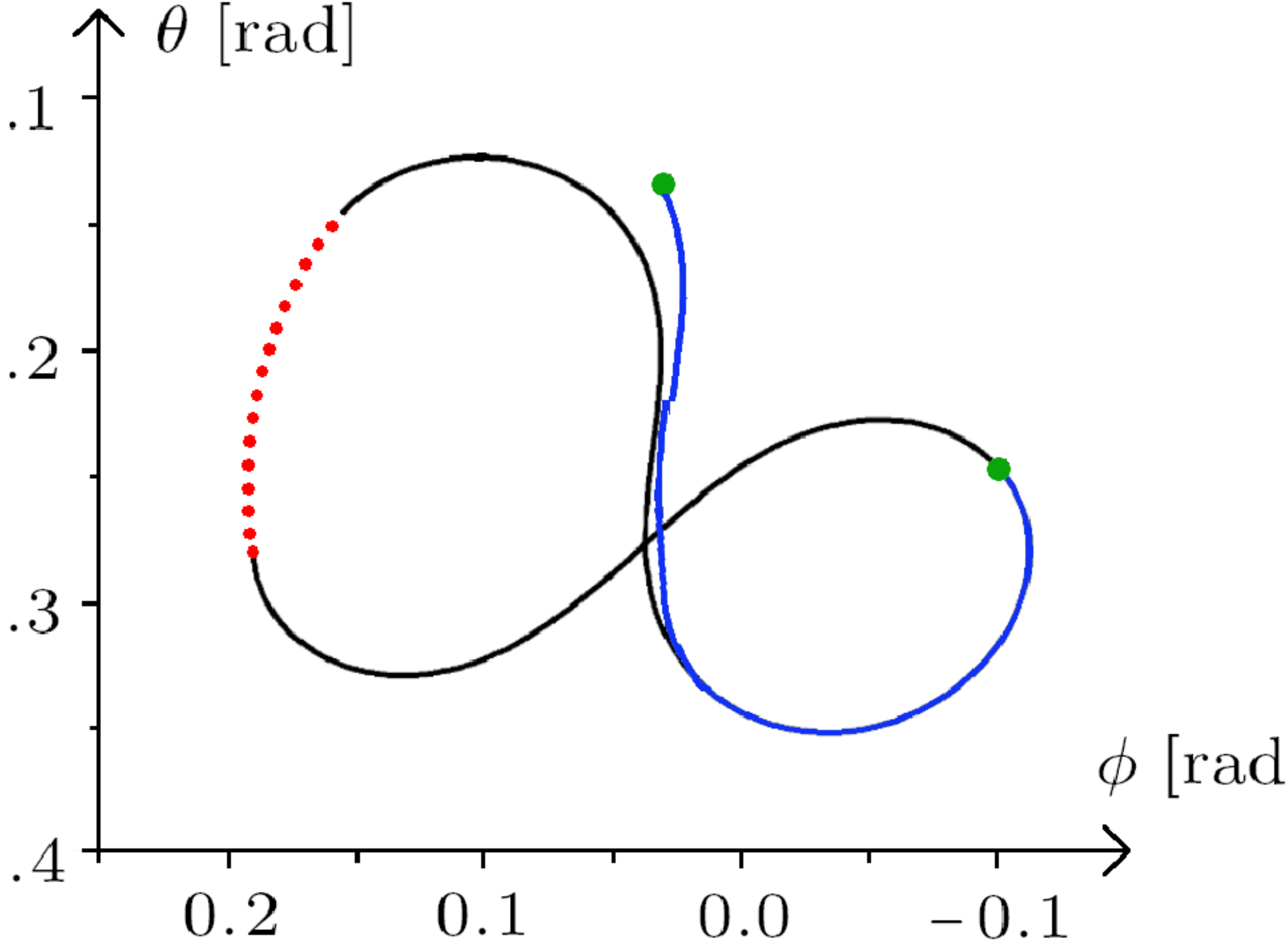

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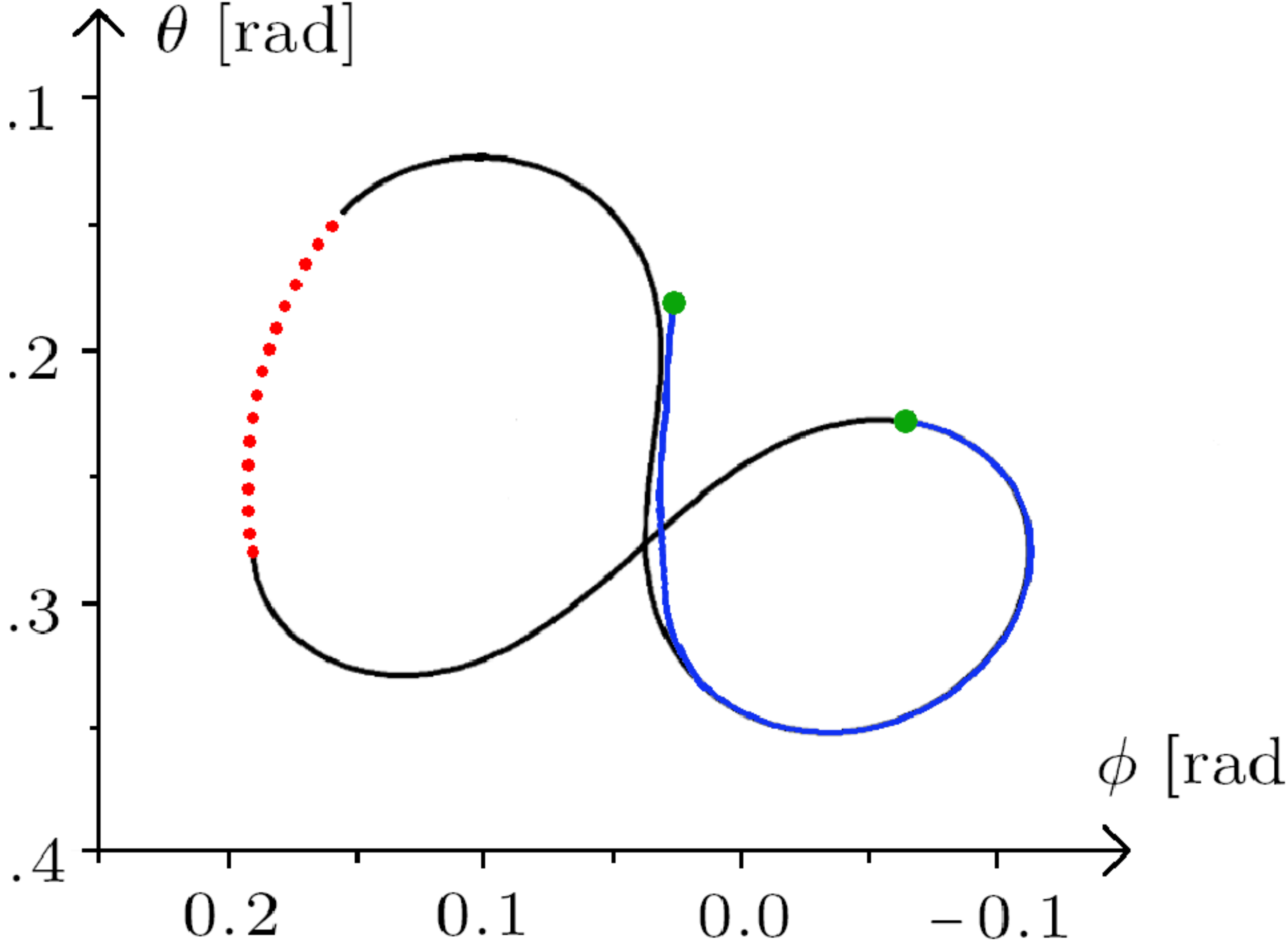

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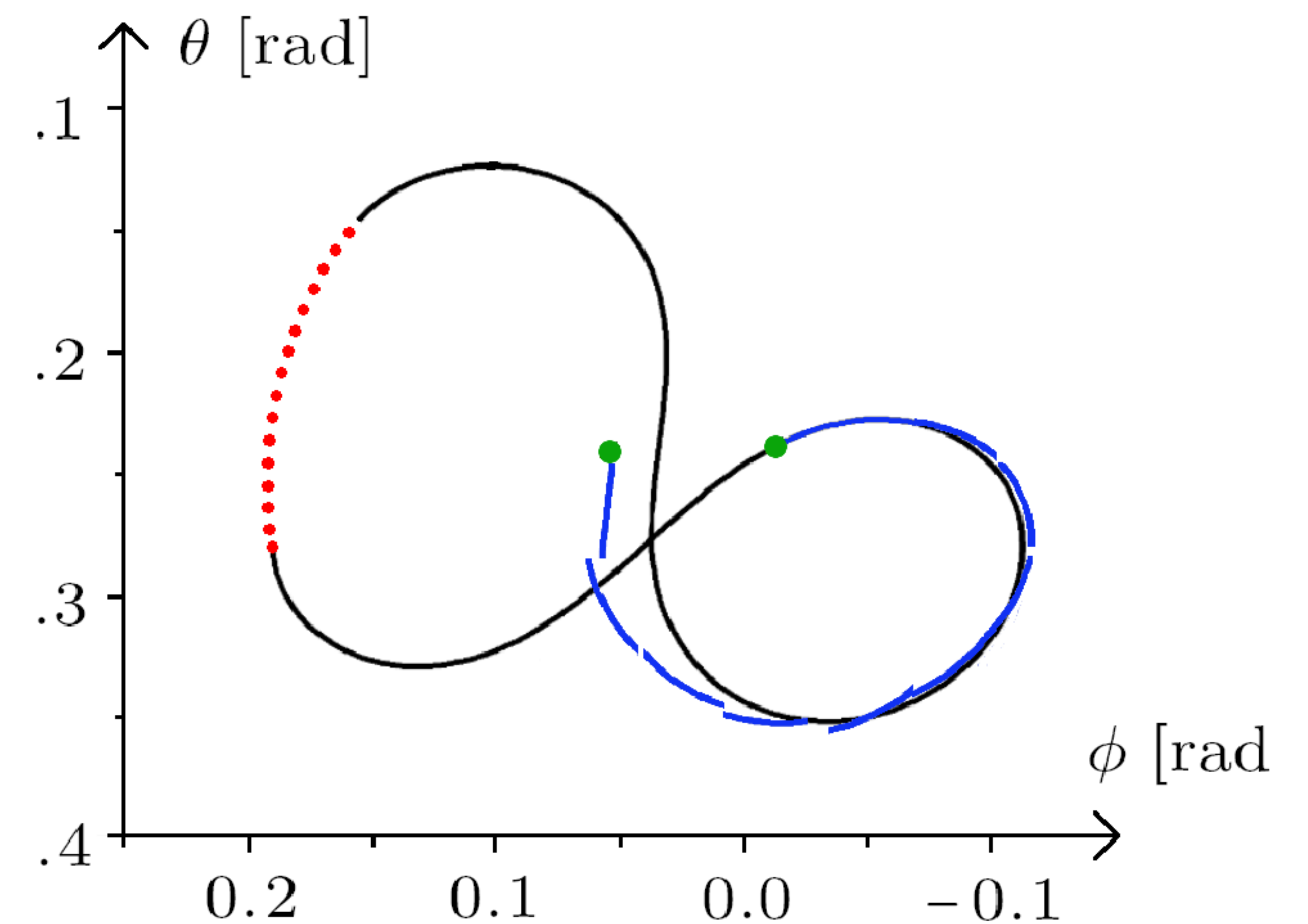

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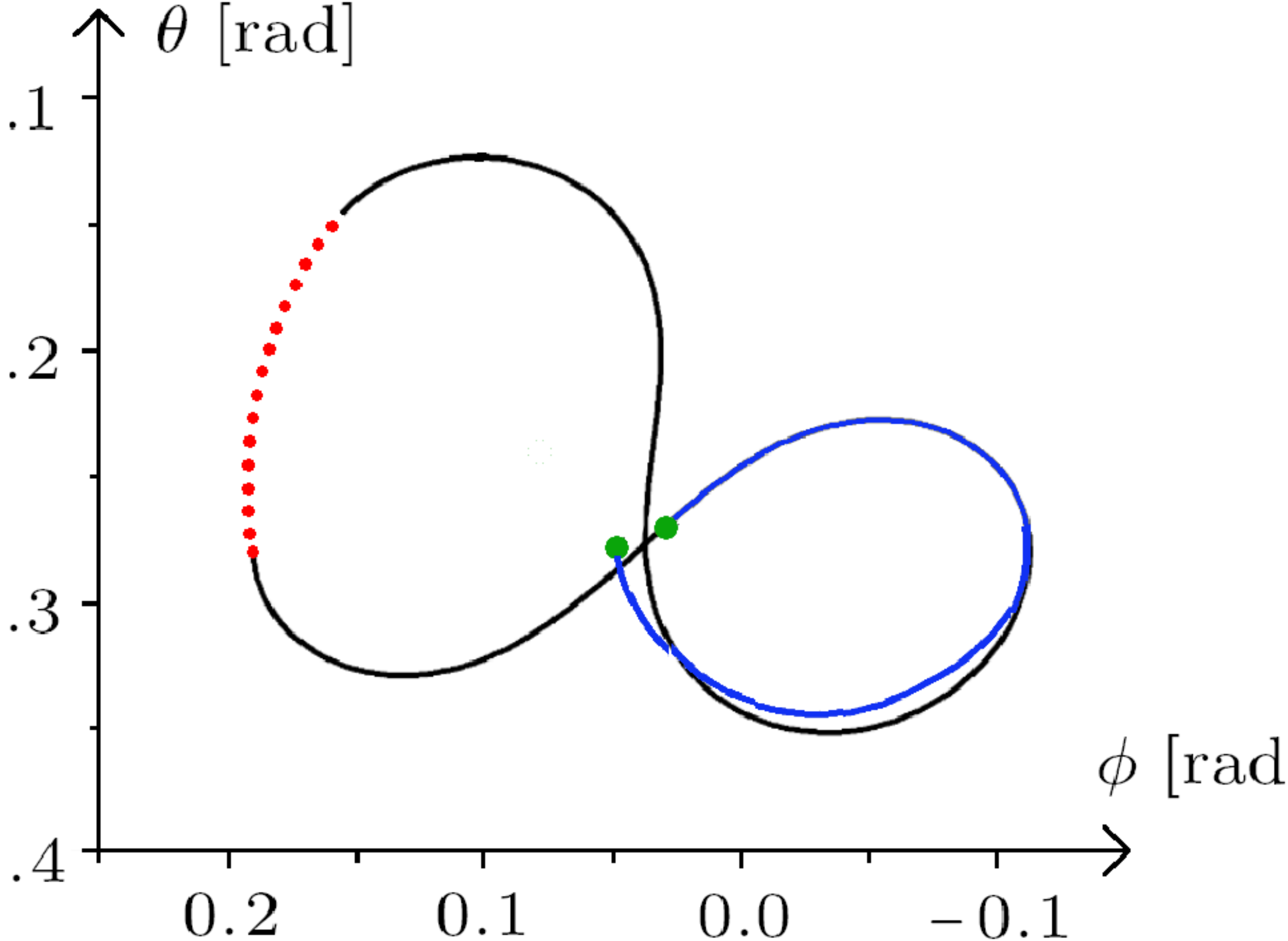
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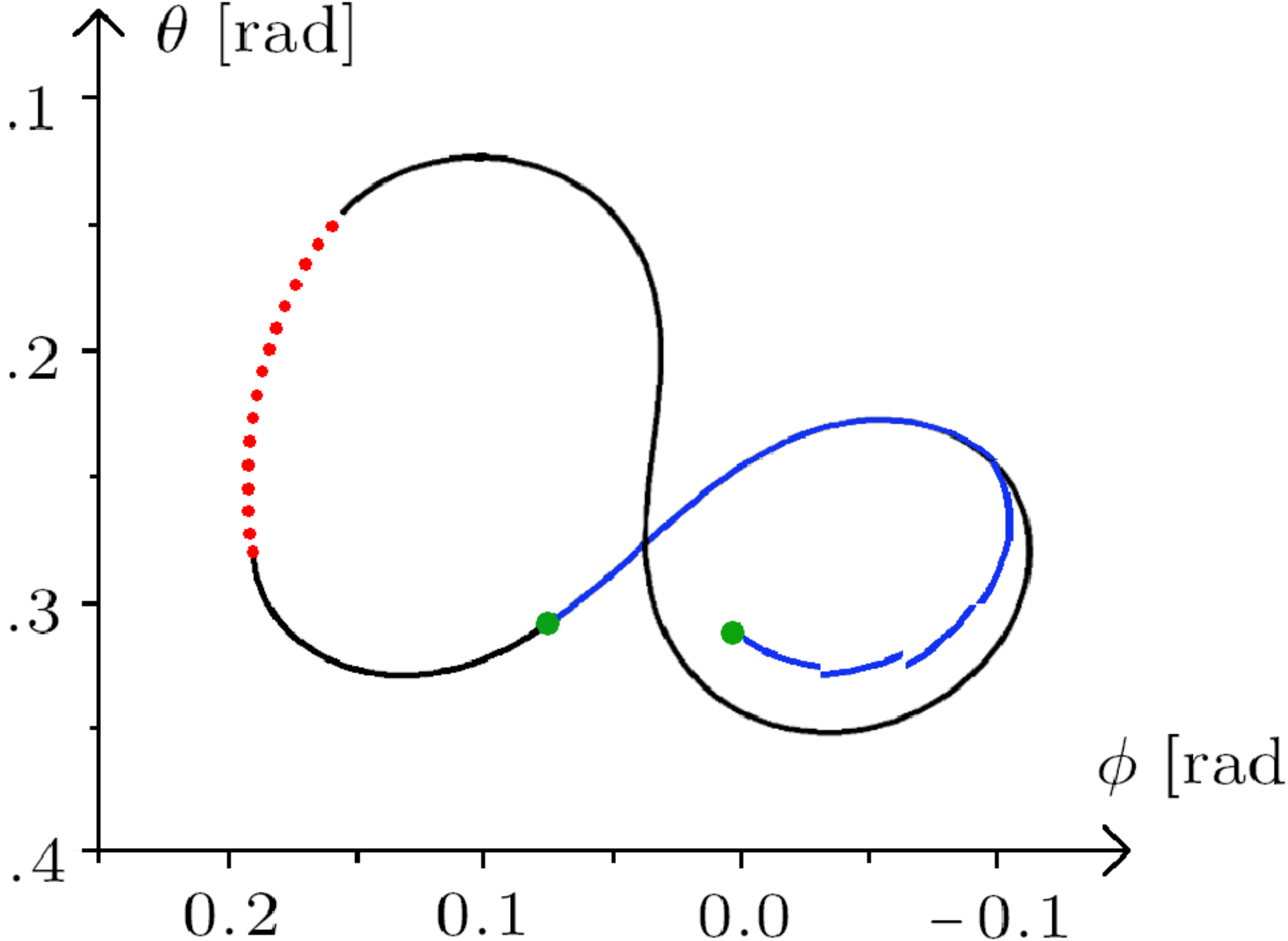

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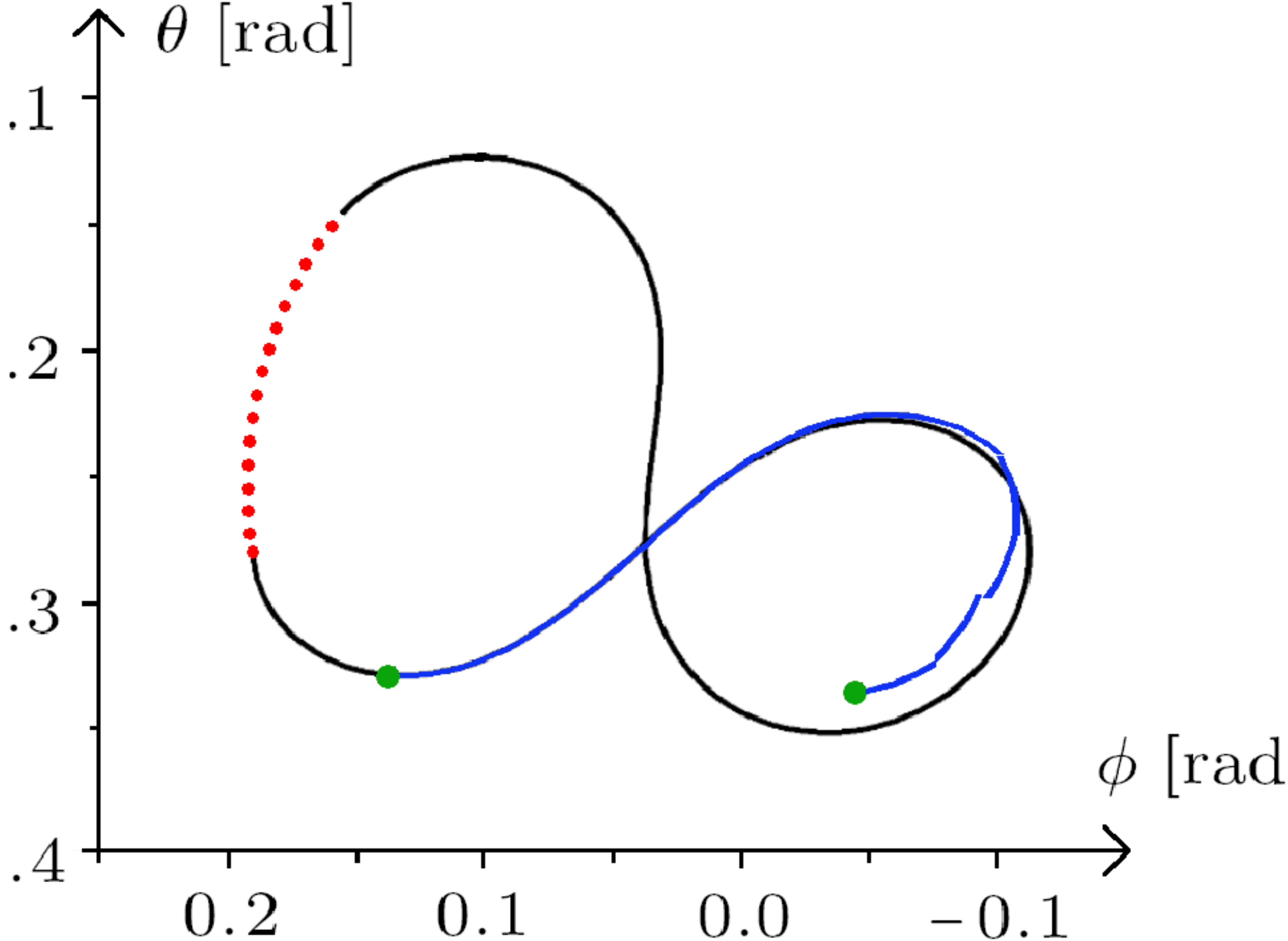

θ [rad]
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Kite NMPC: CPU Time per RTI below 50 ms

- Initial-Value Embedding : 0.03 ms
- QP solution (qpOASES) : 2.23 ms

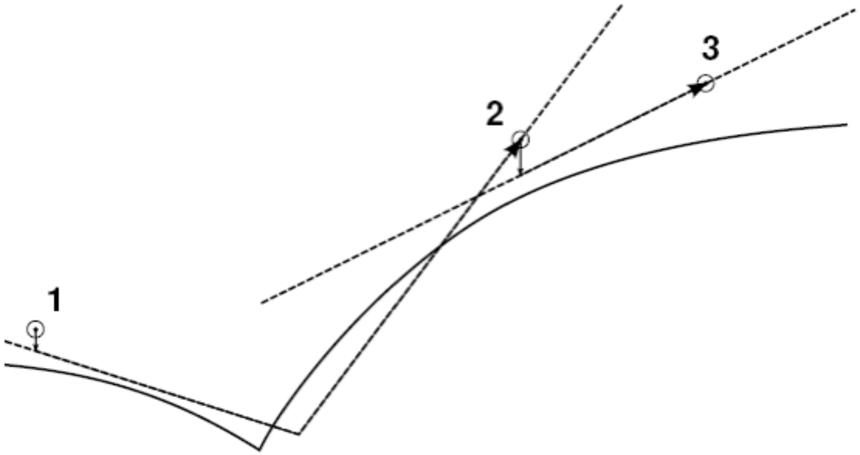
Feedback Phase: 3 ms

(QP after condensing: 30 vars. / 240 constr.)

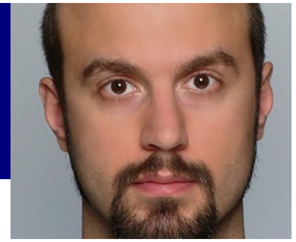
- Expansion of the QP : 0.10 ms
- Simulation and Sensitivities : 44.17 ms
- Condensing (Phase I) : 2.83 ms

Preparation Phase: 47 ms

(on Intel Core 2 Duo CPU T7250, 2 GHz…
without code generation yet)



Nonlinear MPC and MHE on Flight Carousel



Milan Vukov

(sampling time 50 Hz, using ACADO Code Generation)

Closed loop experiments with NMPC & NMHE

KU LEUVEN

UNI FREIBURG

ERC HIGHWIND

Further algorithmic developments in opposite directions

Multi-Level Real-Time Iterations

[Bock, D. et al. NMPC 05, Wirsching 2007]

Make real-time iterations cheaper.

Four Levels:

A) mp-QP at innermost level

B) Feasibility improvement

C) Optimality Improvement

D) Full re-linearization, only rarely in outer loop

- Allows extremely fast sampling rates at innermost level A (feedback phase).
- Level C allows to converge to NLP solution WITHOUT NEW JACOBIAN EVALUATIONS.

Advanced Step NMPC

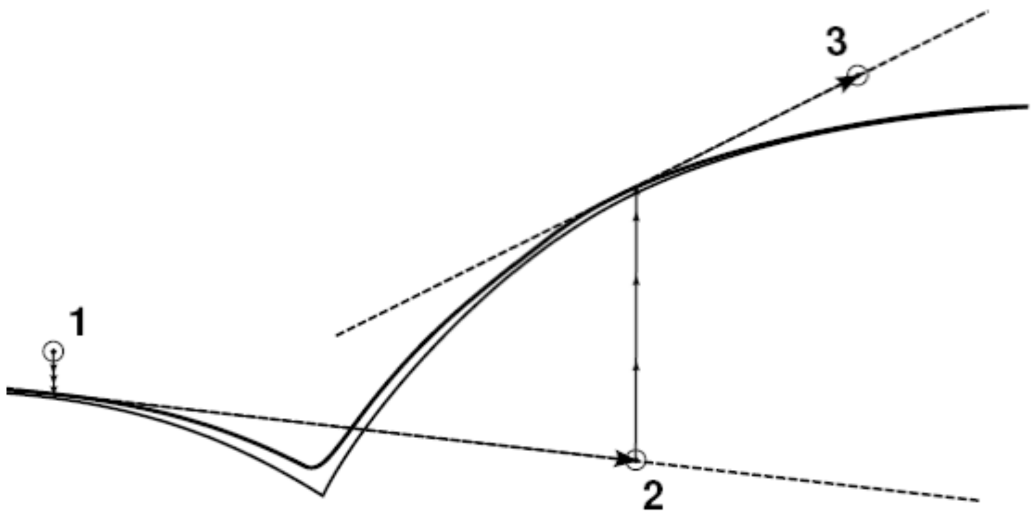
[Zavala and Biegler 2007]

Combine two well-tested ideas [D. 2001]

- Preparation vs. Feedback Phase
- Tangential Predictor in Feedback

with two new building blocks

- For preparation, iterate next problem to convergence via IP method
- use IP predictor in feedback phase



Summary: six ideas for fast nonlinear MPC

- **simultaneous** optimisation: keep states in problem
- **real-time iteration**: use linearisation in non-converged points
- **fast feedback phase** to avoid delays, and longer preparation phase
- **tangential predictor** by initial value embedding
- **solve full QP** to make predictions across active set changes
- **code generation** to minimise overhead (cf afternoon talk R. Quirynen)

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Overview Paper

Efficient Numerical Methods for Nonlinear MPC and Moving Horizon Estimation

Moritz Diehl, Hans Joachim Ferreau, and Niels Haverbeke

L. Magni et al. (Eds.): Nonlinear Model Predictive Control, LNCIS 384, pp. 391–417.
springerlink.com