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Smart Geotherm

Optimal linearization of complex buildings envelope

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March 2015

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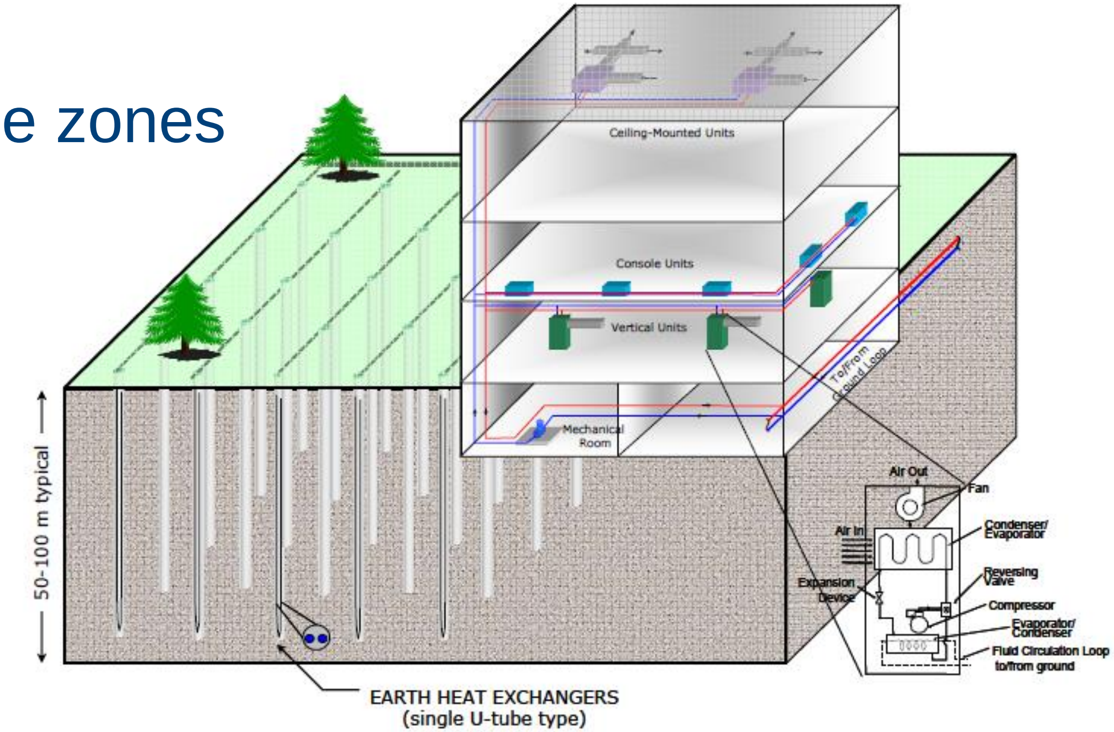
Introduction

Multiple zones

Multiple emission systems

Multiple production systems

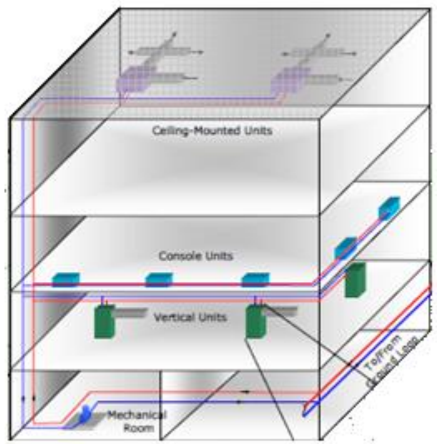


Chiasson (2007) [1]

Minimization of energy cost using (linear) MPC

Introduction

Building Envelope

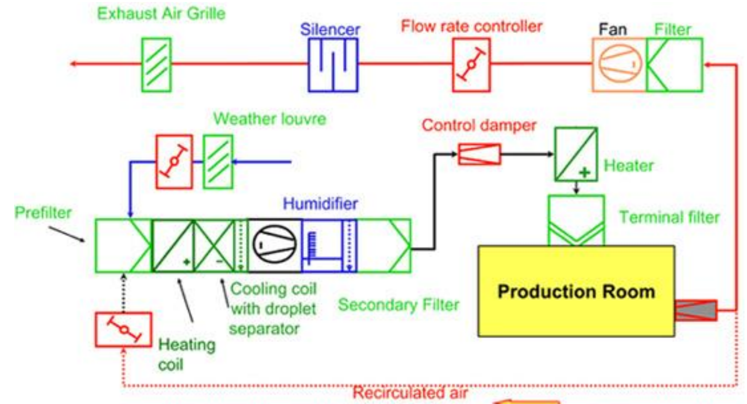


Windows, walls, floors, ...

Slower dynamics (≈ days)
Nearly linear

Dynamic linear controller model

HVAC system



Emission: radiators, ventilation, convectors
Production: heat pump, boiler

Faster dynamics (≈ minutes)
Strongly non-linear

Static correlations in cost function

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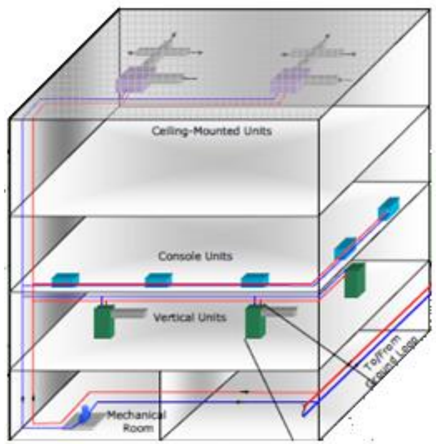
Methods to obtain controller model

System identification:

Excitation



Measurements

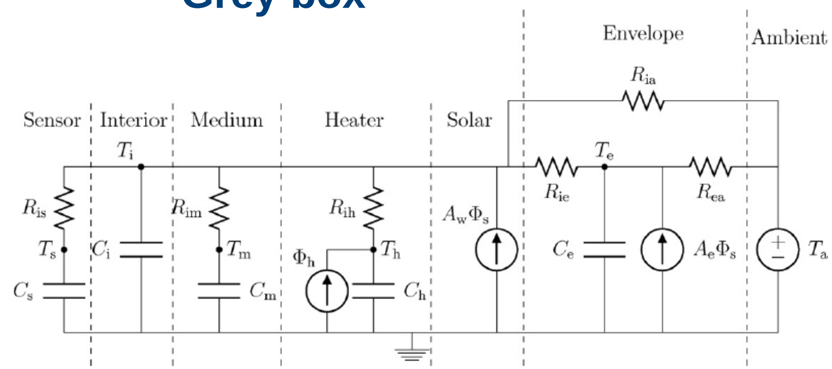
Real building
or
detailed simulation model

- **Black box** (ARMAX, Sub-space, ...)

$$\underbrace{\begin{pmatrix} X(i+1) \\ Y(i|i) \end{pmatrix}}_{\text{known}} = \underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{\text{to be found}} \underbrace{\begin{pmatrix} X(i) \\ U(i|i) \end{pmatrix}}_{\text{known}}$$

Ferkl et Siroky, 2010

- **Grey box**

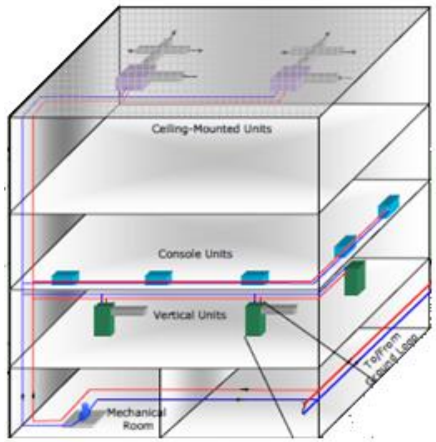


Bacher et Madsen, 2011

Methods to obtain controller model

System identification:

Excitation



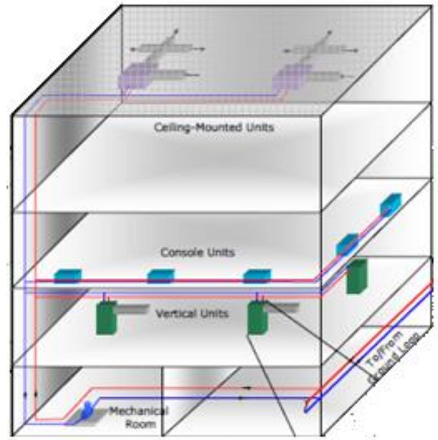
Measurements

Real building
or
detailed simulation model

Difficulties:

1. Data
2. Multi-zones
3. No guarantee of optimal parameter values
4. Case and parameters specific
5. Complexity should be low
6. Optimization problem

Methods to obtain controller model



Linearization

Non-linear (Modelica) system

$$\dot{x} = f(x, u, w)$$
$$y = g(x, u, w)$$

Linearization around (x_e, u_* , w_*)

$$\dot{x} = f(x_e, u_\star, w_\star) + \left.\frac{\partial f}{\partial x}\right|_{(x_e, u_\star, w_\star)} (x - x_e) + \left.\frac{\partial f}{\partial u}\right|_{(x_e, u_\star, w_\star)} (u - u_\star) + \left.\frac{\partial f}{\partial w}\right|_{(x_e, u_\star, w_\star)} (w - w_\star)$$

$$= A\tilde{x} + B_u\tilde{u} + B_w\tilde{w}$$

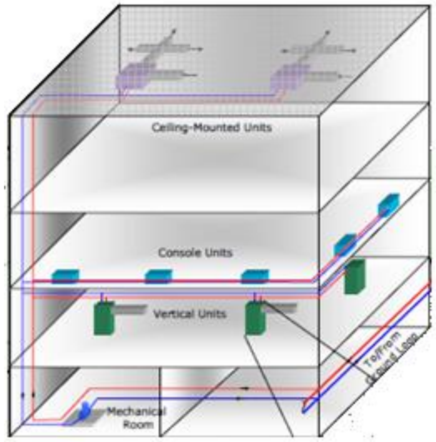
$$y = g(x_e, u_\star, w_\star) + \left.\frac{\partial g}{\partial x}\right|_{(x_e, u_\star, w_\star)} (x - x_e) + \left.\frac{\partial g}{\partial u}\right|_{(x_e, u_\star, w_\star)} (u - u_\star) + \left.\frac{\partial f}{\partial w}\right|_{(x_e, u_\star, w_\star)} (w - w_\star)$$

$$= g(x_e, u_\star, w_\star) + C\tilde{x} + D_u\tilde{u} + D_w\tilde{w}$$

with u = heat inputs (from ventilation, radiators, ...)
w = disturbances (Ambient temperature, solar gains, wind speed, ...)

Methods to obtain controller model

Linearization



Non-linear (Modelica) system

$$\dot{x} = f(x, u, w)$$
$$y = g(x, u, w)$$

Linearization round (x_e, u_* , w_*):

$$\dot{x} = Ax + B_u u + B_w w$$
$$y = Cx + D_u u + D_w w$$

Model order reduction:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \widehat{B_u} u + \widehat{B_w} w$$
$$y = \hat{C}\hat{x} + \widehat{D_u} u + \widehat{D_w} w$$

+ Most accurate linear model **around** (x_e, u_* , w_*)
+ Only most important states
+ Easily automatize in Dymola
+ Dymola function *linearizeModel* returning a state space model

- Inaccurate for strongly non-linear systems
- Not applicable for real building

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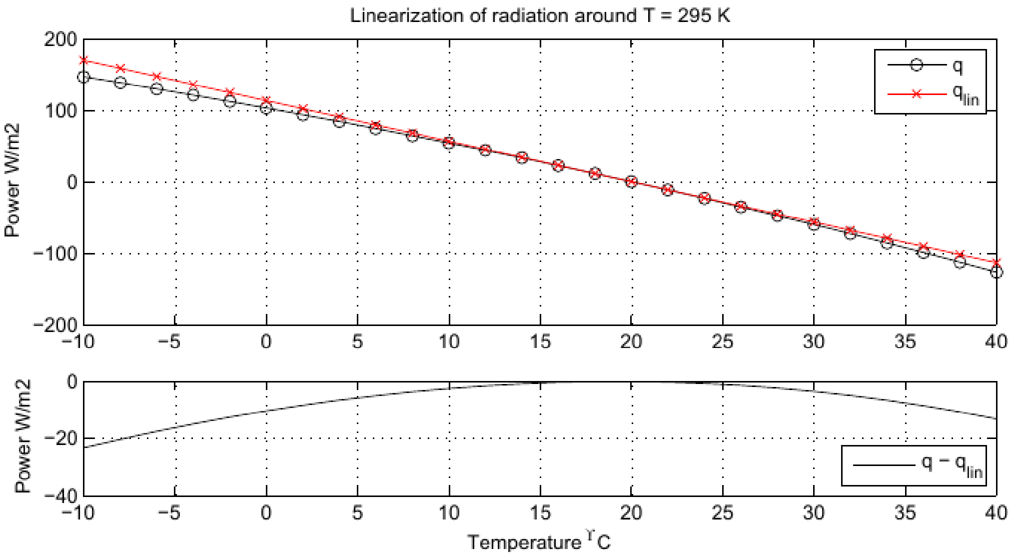
Non-linearities in buildings

Stefan-Boltzmann radiation law:

$$\dot{q}_{rad} = \sigma \epsilon A \left(T_1^4 - T_2^4\right)$$

$$\downarrow$$

$$\tilde{\dot{q}}_{rad} = A \sigma \epsilon 4 \tilde{T}_1^3 \left(T_1 - T_2\right)$$



Exterior heat transfer:

inputs state

Convection:

$$\dot{q}_{cv}^{(k)}(t) = h_{cv}(t)\left(T_{db}(t) - T_s^{(k)}(t)\right)$$

$$h_{cv}(t) = \max\left\{5.01\left(v_{10}(t)\right)^{0.85}, 5.6\right\} \mathrm{W/m^2K}$$

Longwave radiation:

$$\dot{q}_{lw}^{(k)}(t) = \sigma \epsilon_{lw}^{(k)} \left(\left(T_s^{(k)}\right)^4(t) - F_{ce}^{(k)} T_{ce}^4 - (1 - F_{ce}^{(k)}) T_{db}^4\right)$$

$$F_{ce}^{(k)} = \frac{1 + \cos i^{(k)}}{2}$$

$$\downarrow$$

state input

$$\dot{q}_{lw}^{(k)}(t) = 5.67 \epsilon_{lw}^{(k)} \left(T_s^{(k)} - \sqrt[4]{F_{ce}^{(k)} T_{ce}^4 - (1 - F_{ce}^{(k)}) T_{db}^4}\right)$$

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Non linearities in buildings

Interior heat transfer:

Convection:

$$\dot{q}_{cv}^{(k)}(t) = h_{cv}^{(k)}(t)\left(T_{db}(t) - T_s^{(k)}(t)\right)$$

$$h_{cv}^{(k)}(t) = \max\left\{1, n_1^{(k)}\left(D^{(k)}\right)^{n_2^{(k)}}\left|T_{db}(t) - T_s^{(k)}(t)\right|^{n_3^{(k)}}\right\}$$

$$\downarrow$$

$$\dot{q}_{cv}^{(k)}(t) = 3.076\left(T_{db} - T_s^{(k)}\right)$$

Longwave radiation using delta-star transformation with fictive radiant star node:

$$\dot{q}_{lw}^{(k)}(t) = \dot{q}_{lw,g}^{(k)}(t) + \sigma f_{rs}^{(k)}\left(\left(T_s^{(k)}(t)\right)^4 - \left(T_{rs}(t)\right)^4\right)$$

$$\frac{1}{f_{rs}^{(k)}} = \varepsilon_{lw}^{(k)} + \frac{A^{(k)}}{\sum_{j=1}^{n} A^{(j)}} : n \triangleq \left|\mathcal{J}^{(k)}\right|$$

Solar gains:

→ treated as inputs

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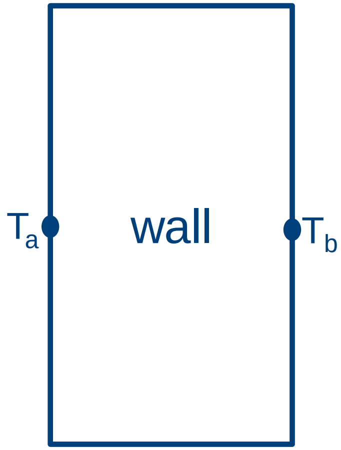
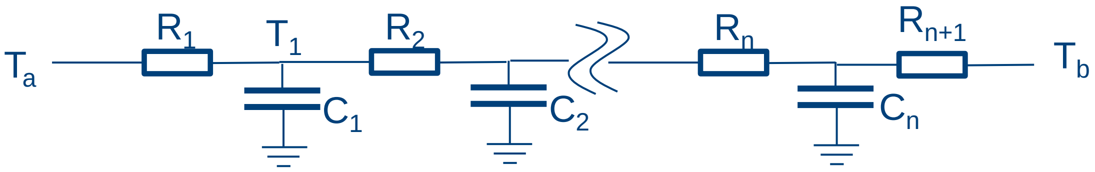
Non linearities in buildings

Heat diffusion: $\left[\frac{1}{\alpha}\frac{\partial}{\partial t} - \nabla^2\right]T = q'''$

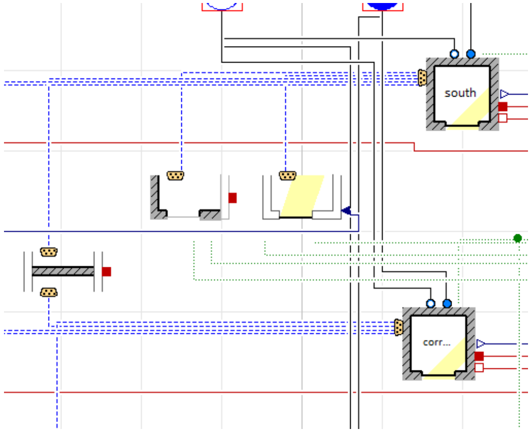




Only the external convection needs to be linearized

Linearization method

1) Automatic linearization of all components of model and gains and disturbances treated as inputs:



2) Apply function *linearizeModel* to obtain state space formulation

3) (Optinal) apply model order reduction

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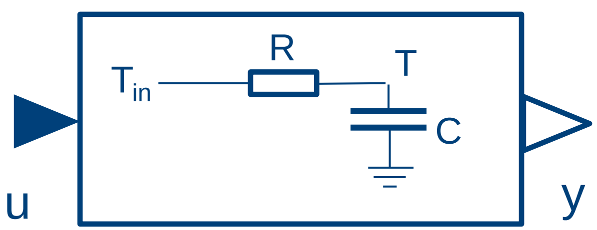
Validation

Simple test

Dymola function: *linearizeModel*

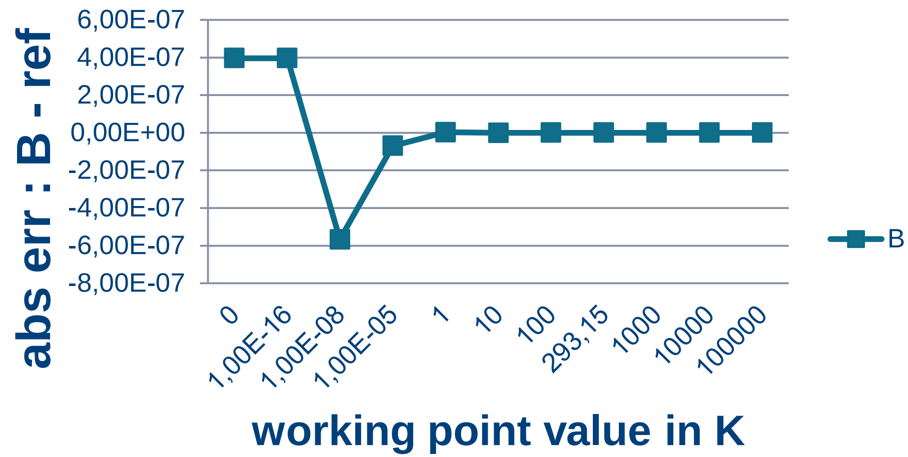


$$C\,\frac{dT}{dt} = \frac{T_{in} - T}{R}$$

$$\dot{x} = \left[-\frac{1}{CR}\right] x + \left[\frac{1}{CR}\right] u$$

T_{in} T

Absolute error on B-matrix



$$B = \left.\frac{\partial F(x,u)}{\partial u}\right|(x_*, u_*)$$

Central difference

$$= \frac{Ax_* + B\,(u_* + \delta) - Ax_* - B\,(u_* - \delta)}{2\,\delta}$$

! Working point (u_*, x_*) for linearization !

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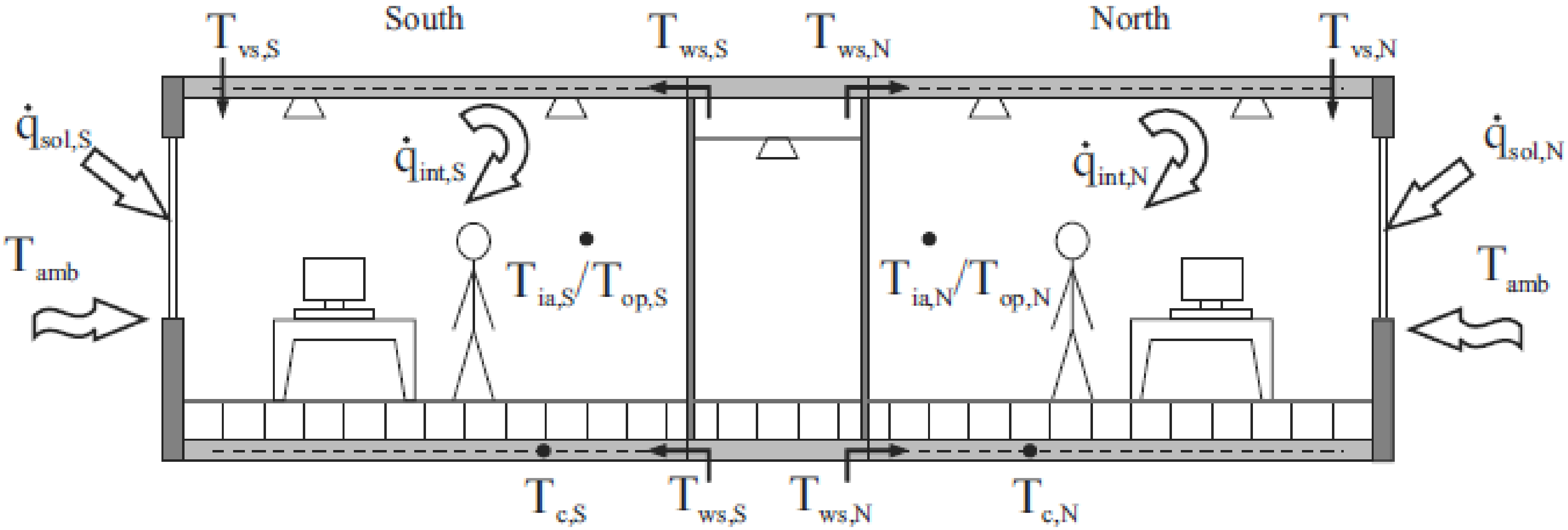
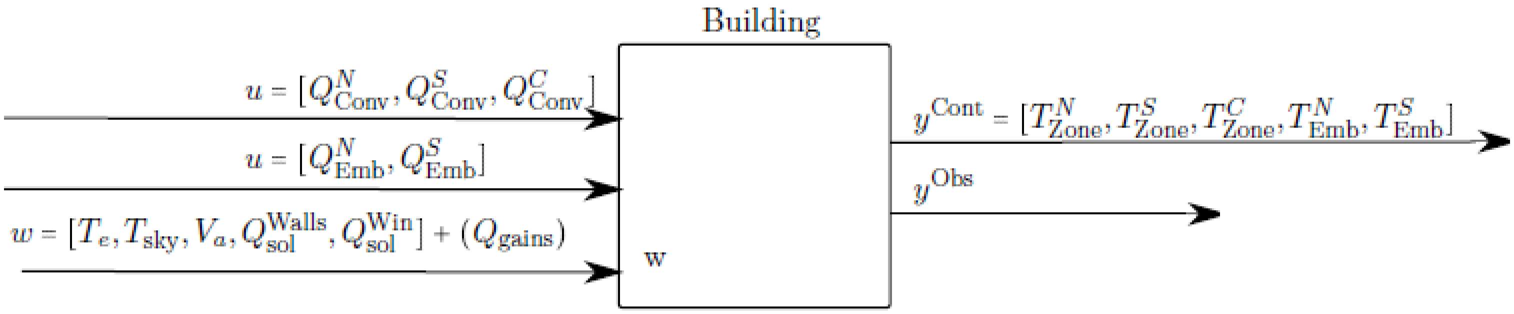
Validation

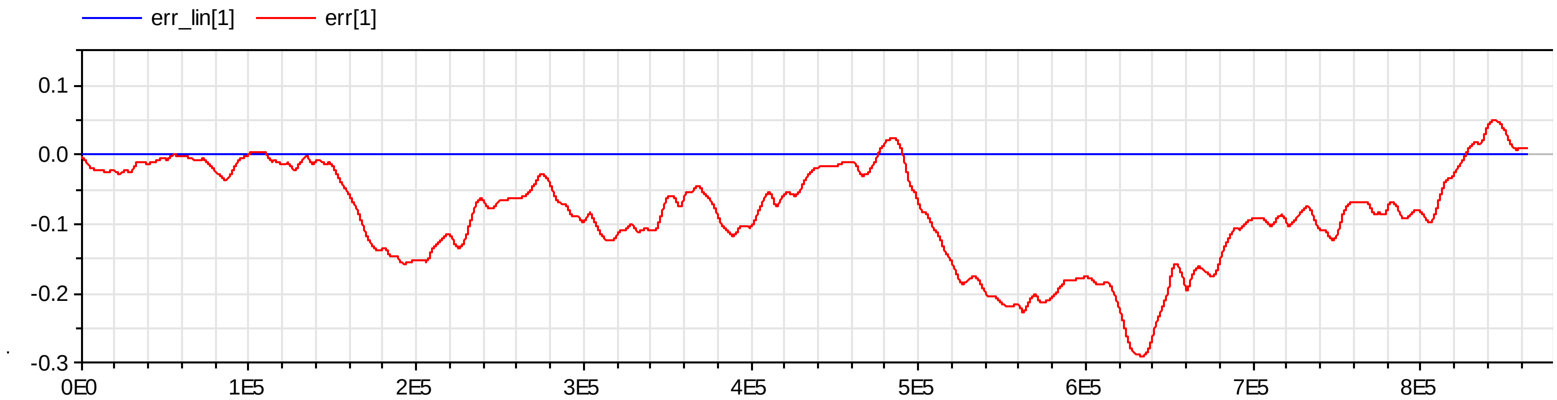
Three zones model

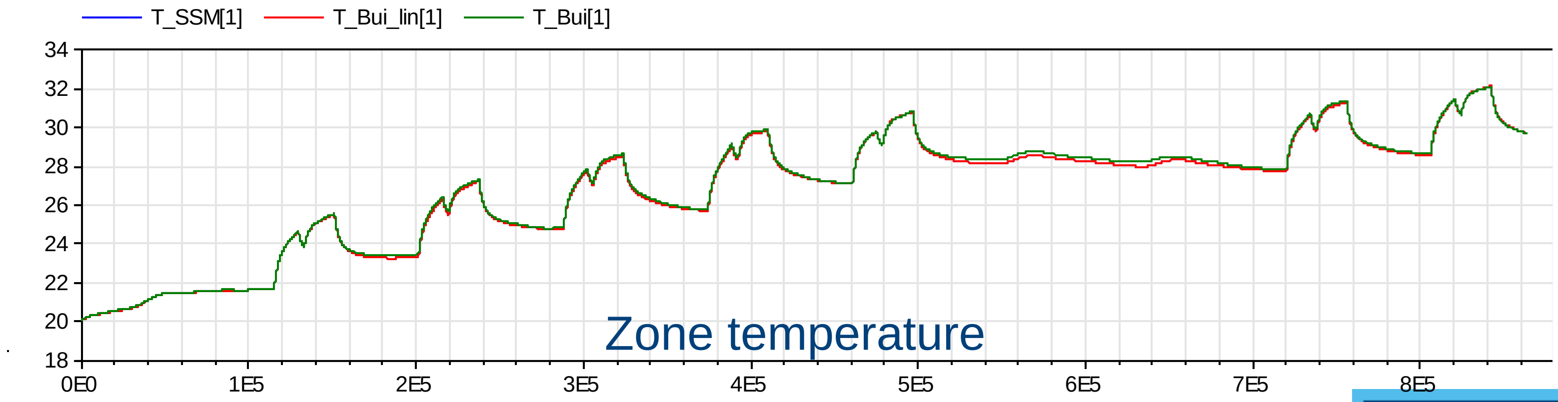




Validation

Simulation performance





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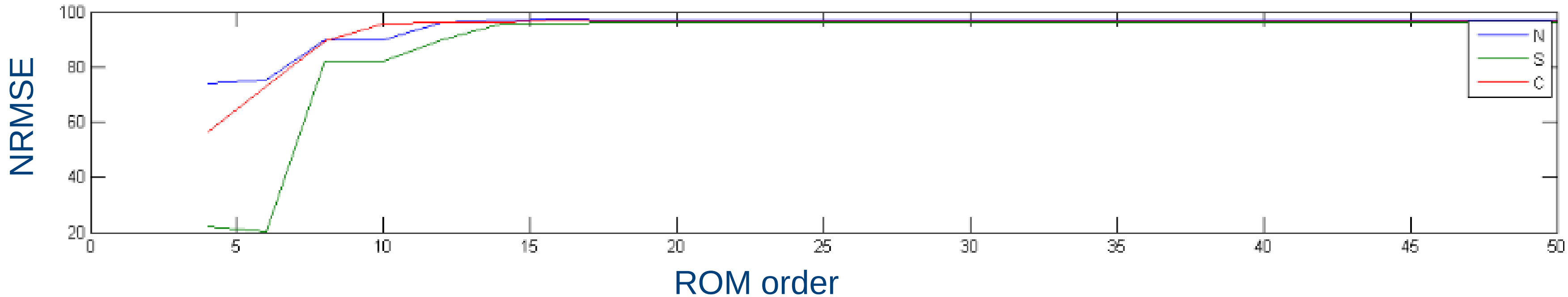
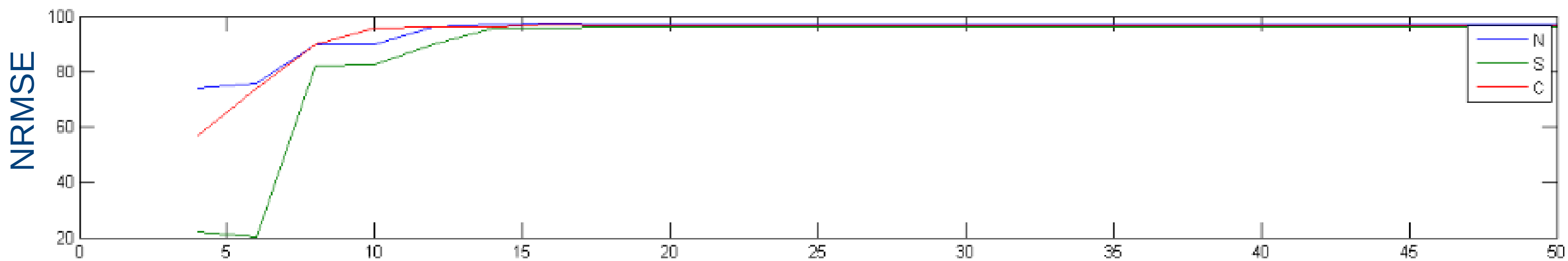
Validation

Prediction performance and 1-day ahead prediction

- Time step of 15’
- Multisine with 30 frequencies + distrubances
- Compute $NRMSE = 100\left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|}\right)$

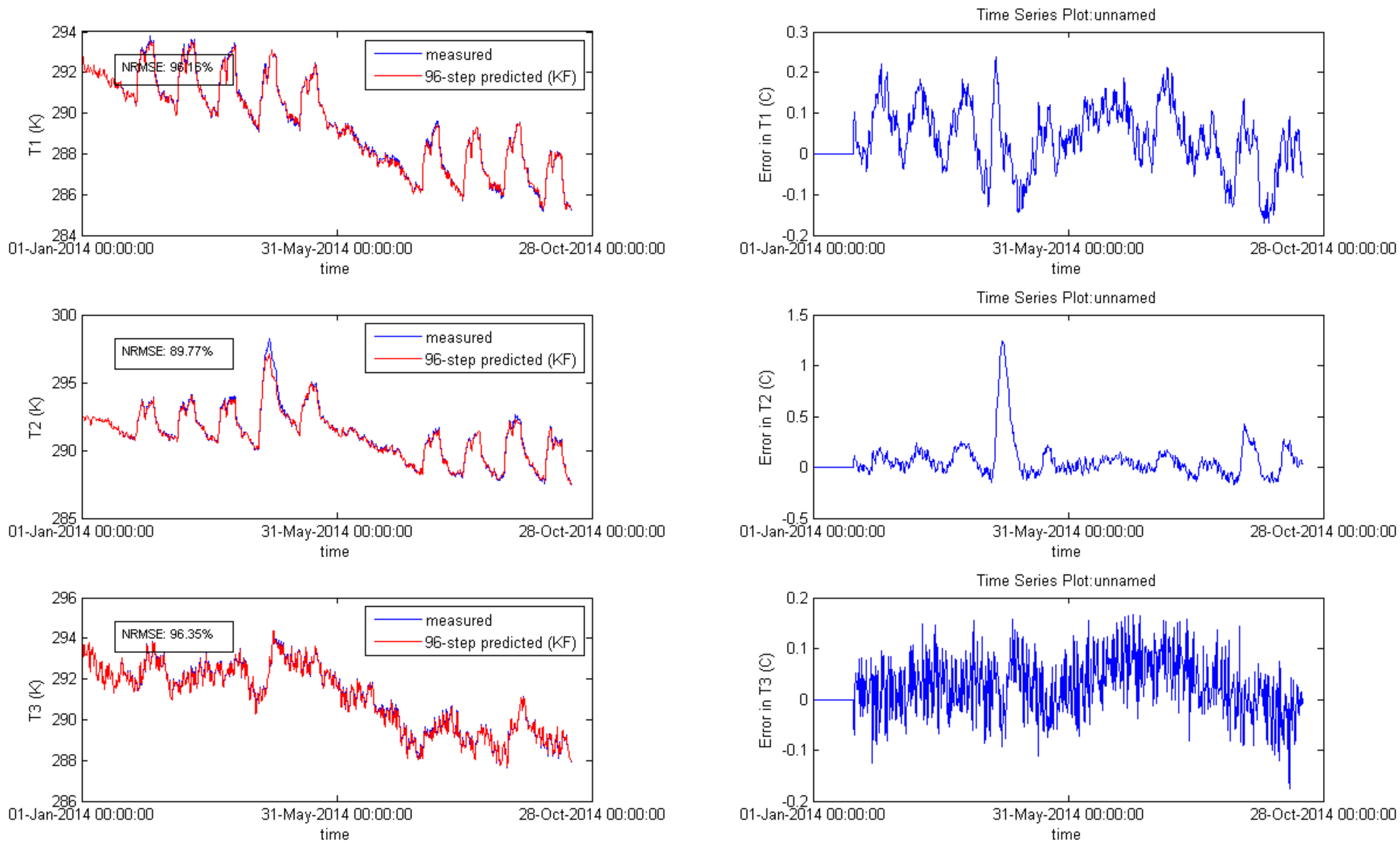




Validation

Prediction performance and 1-day ahead prediction

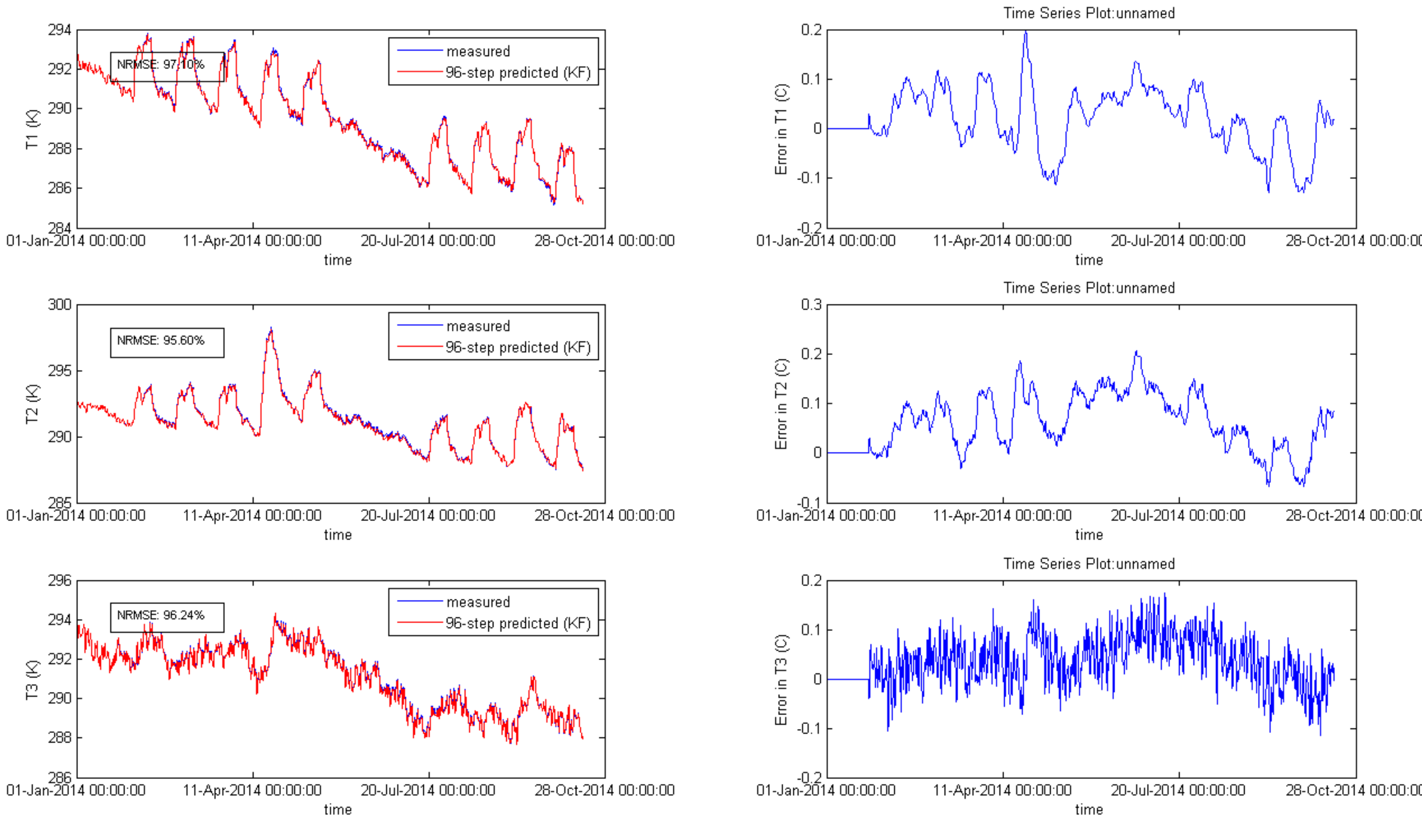
12-states



Validation

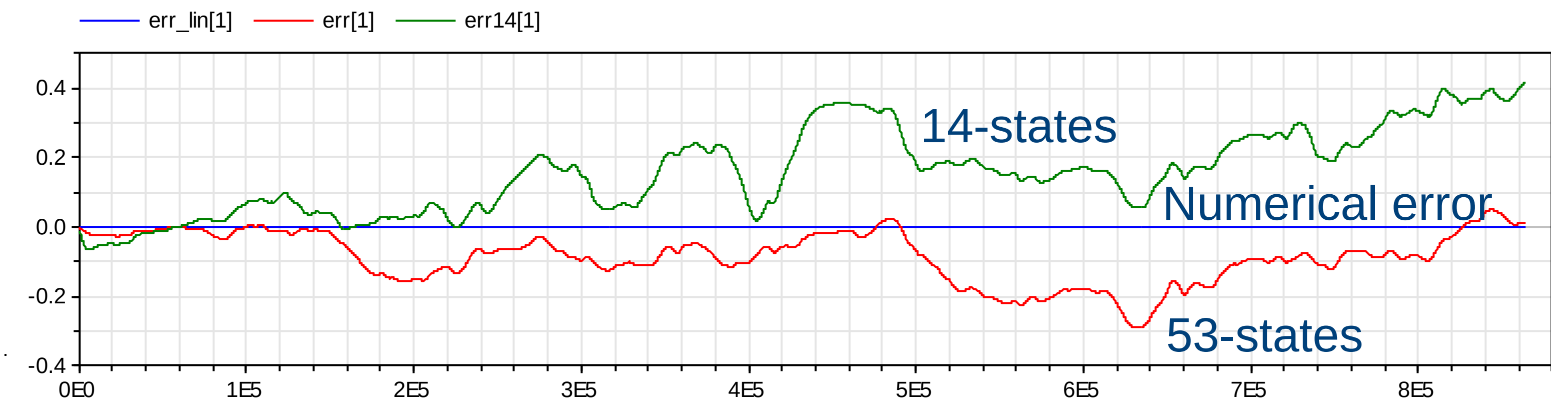
Prediction performance and 1-day ahead prediction

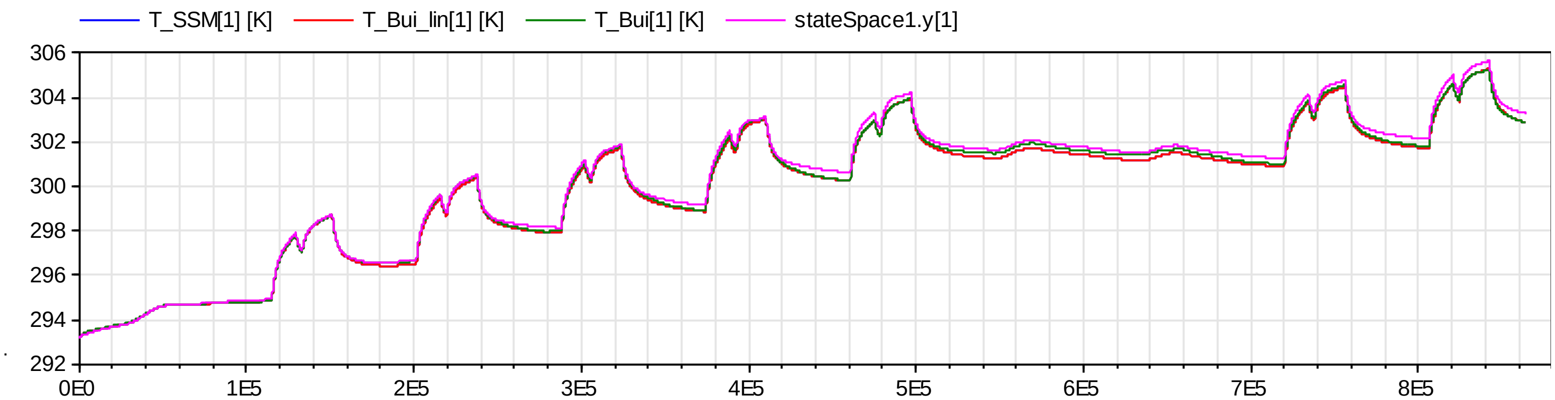
14-states



Validation

Simulation performance





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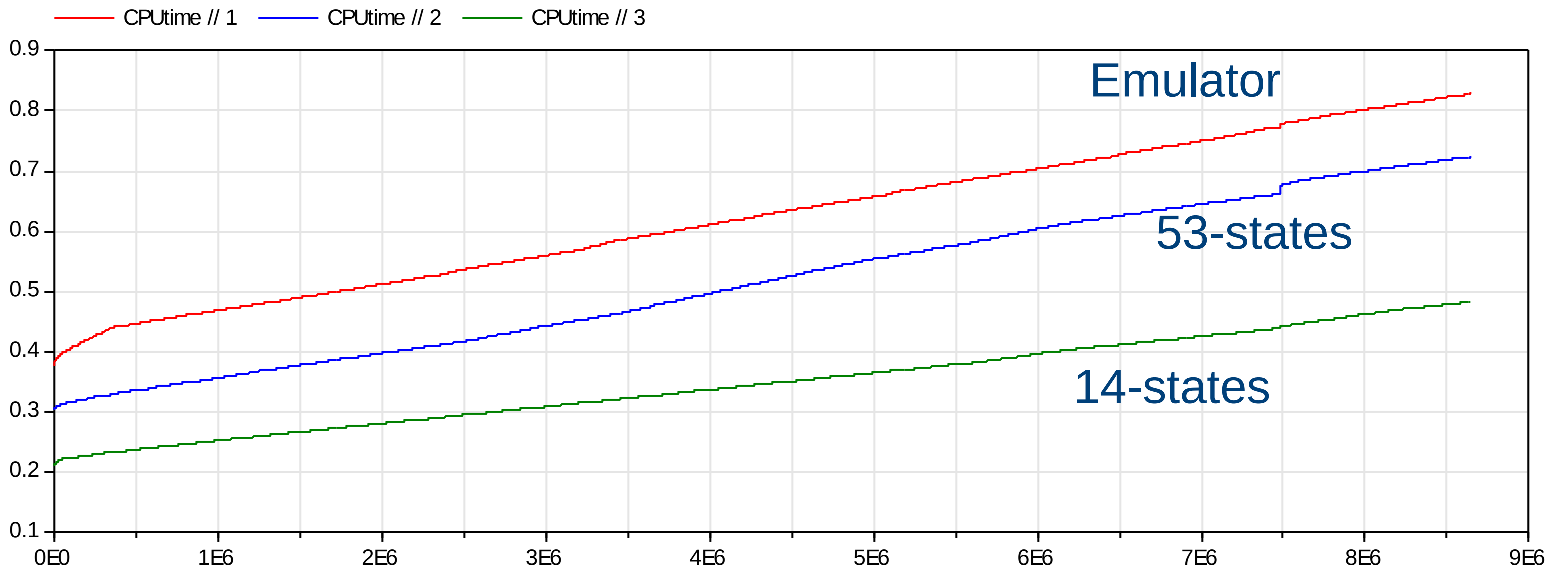
Validation

Simulation performance



- SSM14 1.6 times faster than 53-states
- Faster initialization

Conclusions

- Building are relatively linear
- Automatic linearization is possible
- Loss of accuracy due to convection
- Very good prediction performance with reduced model
- CPU reduction for testing

Further work

- Improve linearization of convection
- Automate model order reduction

Questions?