Optimal linearization of complex buildings envelope

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Introduction

Multiple zones

Multiple emission systems

Multiple production systems

Minimization of energy cost using (linear) MPC

Chiasson (2007) [1]
Introduction

Building Envelope

Windows, walls, floors, ...

- Slower dynamics (≈ days)
- Nearly linear

Dynamic linear controller model

HVAC system

Emission: radiators, ventilation, convectors

Production: heat pump, boiler

- Faster dynamics (≈ minutes)
- Strongly non-linear

Static correlations in cost function
Methods to obtain controller model

System identification:

- **Black box** (ARMAX, Sub-space, ...)

\[
\begin{pmatrix}
X(i+1) \\
Y(i|i)
\end{pmatrix} = 
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
\begin{pmatrix}
X(i) \\
U(i|i)
\end{pmatrix}
\]

Ferkl et Siroky, 2010

- **Grey box**

Bacher et Madsen, 2011

Excitation

Measurements

Real building or detailed simulation model
Methods to obtain controller model

System identification:

- Excitation
- Measurements
- Real building or detailed simulation model

Difficulties:
1. Data
2. Multi-zones
3. No guarantee of optimal parameter values
4. Case and parameters specific
5. Complexity should be low
6. Optimization problem
Methods to obtain controller model

Linearization

Non-linear (Modelica) system

\[ \dot{x} = f(x, u, w) \]
\[ y = g(x, u, w) \]

Linearization around \((x_e, u^*, w^*)\)

\[
\dot{x} = f(x_e, u^*, w^*) + \frac{\partial f}{\partial x}(x_e, u^*, w^*) (x - x_e) + \frac{\partial f}{\partial u}(x_e, u^*, w^*) (u - u^*) + \frac{\partial f}{\partial w}(x_e, u^*, w^*) (w - w^*) \\
= A\ddot{x} + B_u\ddot{u} + B_w\ddot{w}
\]

\[
y = g(x_e, u^*, w^*) + \frac{\partial g}{\partial x}(x_e, u^*, w^*) (x - x_e) + \frac{\partial g}{\partial u}(x_e, u^*, w^*) (u - u^*) + \frac{\partial f}{\partial w}(x_e, u^*, w^*) (w - w^*) \\
= g(x_e, u^*, w^*) + C\ddot{x} + D_u\ddot{u} + D_w\ddot{w}
\]

with \(u = \) heat inputs (from ventilation, radiators, \(...\))

\(w = \) disturbances (Ambient temperature, solar gains, wind speed, \(...\))
Methods to obtain controller model

**Linearization**

Non-linear (Modelica) system

\[
\begin{align*}
\dot{x} &= f(x, u, w) \\
y &= g(x, u, w)
\end{align*}
\]

Linearization round \((x_e, u_*, w_*)\):

\[
\begin{align*}
\dot{x} &= Ax + B_u u + B_w w \\
y &= Cx + D_u u + D_w w
\end{align*}
\]

Model order reduction:

\[
\begin{align*}
\dot{x} &= \hat{A}\hat{x} + \hat{B}_u u + \hat{B}_w w \\
y &= \hat{C}\hat{x} + \hat{D}_u u + \hat{D}_w w
\end{align*}
\]

+ Most accurate linear model **around** \((x_e, u_*, w_*)\)
+ Only most important states
+ Easily automatize in Dymola
+ Dymola function `linearizeModel` returning a state space model

- Inaccurate for strongly non-linear systems
- Not applicable for real building
Non-linearities in buildings

Stefan-Boltzmann radiation law:
\[
\dot{q}_{rad} = \sigma \epsilon A \left( T_1^4 - T_2^4 \right)
\]
\[
\tilde{q}_{rad} = A \sigma \epsilon 4T_1^3 (T_1 - T_2)
\]

Exterior heat transfer:

Convection:
\[
\dot{q}_{cv}^{(k)}(t) = h_{cv}(t) (T_{db}(t) - T_s^{(k)}(t))
\]
\[
h_{cv}(t) = \max \{ 5.01 (v_{10}(t))^{0.85} , 5.6 \} \text{ W/m}^2\text{K}
\]

Longwave radiation:
\[
\dot{q}_{lw}^{(k)}(t) = \sigma \epsilon_{lw}^{(k)} \left( \left( T_s^{(k)}(t) \right)^4 - F_{ce}^{(k)} T_{ce}^4 - (1 - F_{ce}^{(k)}) T_{db}^4 \right)
\]
\[
F_{ce}^{(k)} = \frac{1 + \cos i^{(k)}}{2}
\]
\[
\dot{q}_{lw}^{(k)}(t) = 5.67 \epsilon_{lw}^{(k)} \left( T_s^{(k)}(t) - \sqrt[4]{F_{ce}^{(k)} T_{ce}^4 - (1 - F_{ce}^{(k)}) T_{db}^4} \right)
\]
Non linearities in buildings

Interior heat transfer:

Convection:

\[ \dot{q}_{cv}^{(k)}(t) = h_{cv}^{(k)}(t) \left( T_{db}(t) - T_s^{(k)}(t) \right) \]

\[ h_{cv}^{(k)}(t) = \max \left\{ 1, n_1^{(k)} \left( D^{(k)} \right)^{n_2^{(k)}} \left| T_{db}(t) - T_s^{(k)}(t) \right|^{n_3^{(k)}} \right\} \]

\[ \dot{q}_{cv}^{(k)}(t) = 3.076 \left( T_{db} - T_s^{(k)} \right) \]

Longwave radiation using delta-star transformation with fictive radiant star node:

\[ \dot{q}_{lw}^{(k)}(t) = q_{lw,g}^{(k)}(t) + \sigma f_{rs}^{(k)} \left( \left( T_s^{(k)}(t) \right)^4 - (T_{rs}(t))^4 \right) \]

\[ \frac{1}{f_{rs}^{(k)}} = \varepsilon_{lw}^{(k)} + \frac{A^{(k)}}{\sum_{j=1}^{n} A(j)} : n \triangleq |\mathcal{J}^{(k)}| \]

Solar gains:

\[ \rightarrow \text{treated as inputs} \]
Non linearities in buildings

Heat diffusion:
\[
\left[ \frac{1}{\alpha} \frac{\partial}{\partial t} - \nabla^2 \right] T = q'''
\]

Only the external convection needs to be linearized
Linearization method

1) Automatic linearization of all components of model and gains and disturbances treated as inputs:

2) Apply function $\text{linearizeModel}$ to obtain state space formulation

3) (Optional) apply model order reduction
Validation

Simple test

Dymola function: *linearizeModel*

\[
\begin{align*}
C \frac{dT}{dt} &= \frac{T_{in} - T}{R} \\
\dot{x} &= \left[-\frac{1}{CR}\right]x + \left[\frac{1}{CR}\right]u
\end{align*}
\]

! Working point \((u_*, x_*)\) for linearization!

Absolute error on B-matrix

\[
B = \frac{\partial F(x, u)}{\partial u} \bigg|_{(x_*, u_*)}
\]

Central difference

\[
= Ax_* + B (u_* + \delta) - Ax_* - B (u_* - \delta)
\]

\[
= \frac{Ax_* + B (u_* + \delta) - Ax_* - B (u_* - \delta)}{2 \delta}
\]
Validation

Three zones model
Validation

Simulation performance

![Graph showing validation results](image-url)
Validation

Prediction performance and 1-day ahead prediction

- Time step of 15’
- Multisine with 30 frequencies + disturbances
- Compute $NRMSE = 100 \left( 1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|} \right)$
Validation

Prediction performance and 1-day ahead prediction

12-states
Validation

Prediction performance and 1-day ahead prediction

14-states
Validation

Simulation performance

- Numerical error

14-states
53-states
Validation

Simulation performance

- SSM14 1.6 times faster than 53-states
- Faster initialization
Conclusions

• Building are relatively linear
• Automatic linearization is possible
• Loss of accuracy due to convection
• Very good prediction performance with reduced model
• CPU reduction for testing

Further work

• Improve linearization of convection
• Automate model order reduction
Questions?