



# Comparing state estimation techniques for model predictive control

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# Goal

State estimation of states of simple (non-linear) Modelica model

Quantitative and qualitative comparison of different algorithms



# Why

## Model predictive control on real buildings

- Optimizing future control starts from current state
- Current state not fully measured



# How

Compare three state estimation algorithms:

1. Deterministic state estimation
2. Moving horizon estimation
3. Unscented Kalman Filter

in three different cases:

- Ideal (simple model – simple model)
- Non-ideal (simple model – complex model)
- Real (simple model – real building)

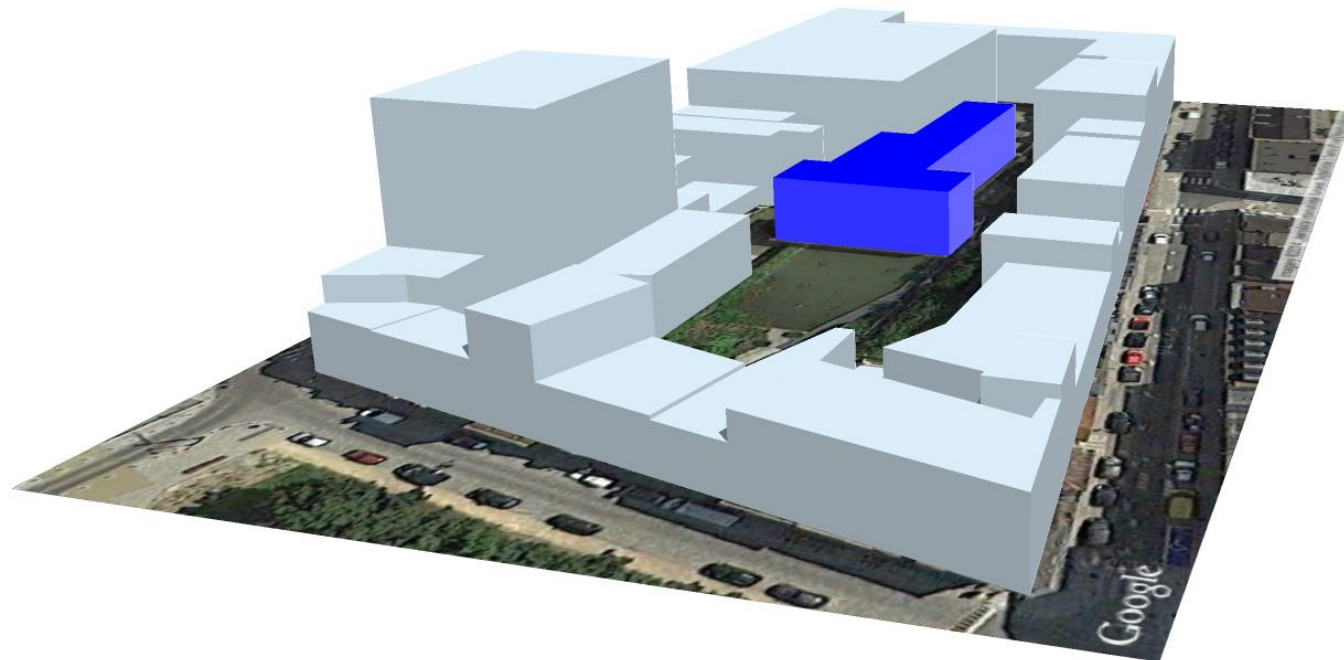


# System

3E headquarters in Brussels

Two floors, 40 – 80 people

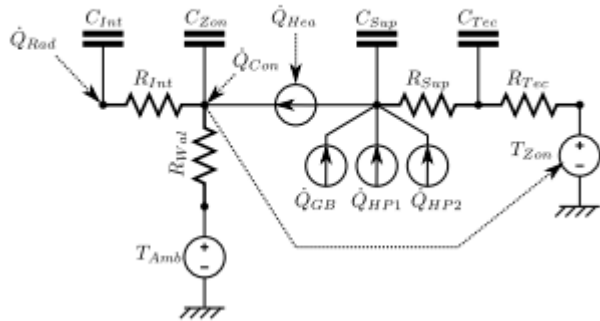
Renewed heating system



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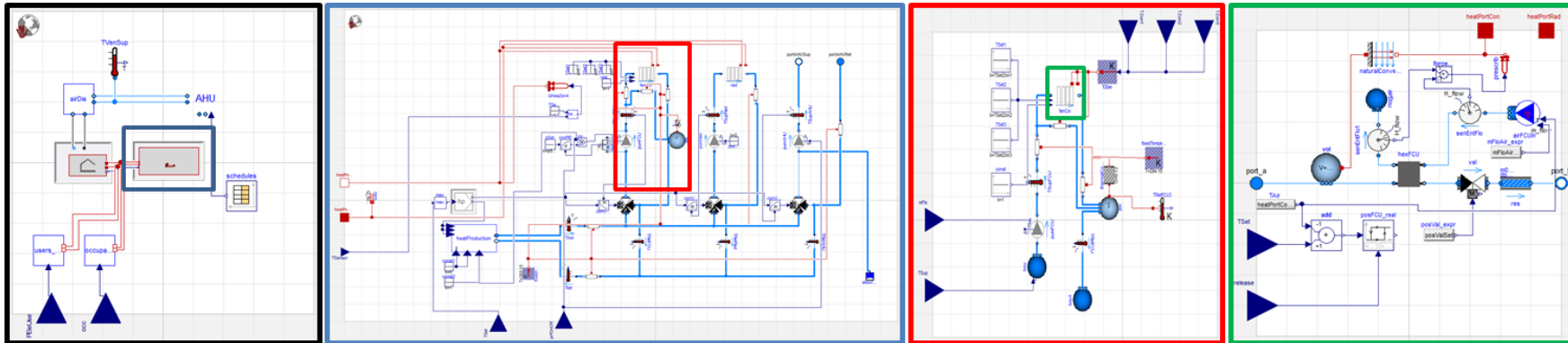
# Models

1



1. Simple building model
2. Complex building model
3. Real building

2



3

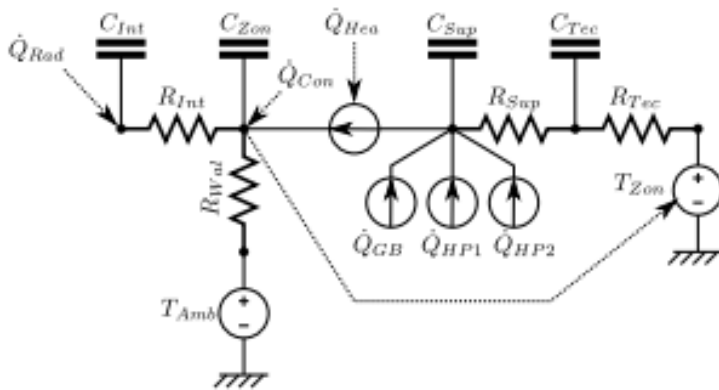


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# 3 different cases, same inputs dataset

Ideal (simple-simple):

- All states known
- Noise properties added/known (Gaussian? Uniform?)
- Use only 'output' states for estimation
- Compare states directly with counterpart (open loop simulation, 1 day?)

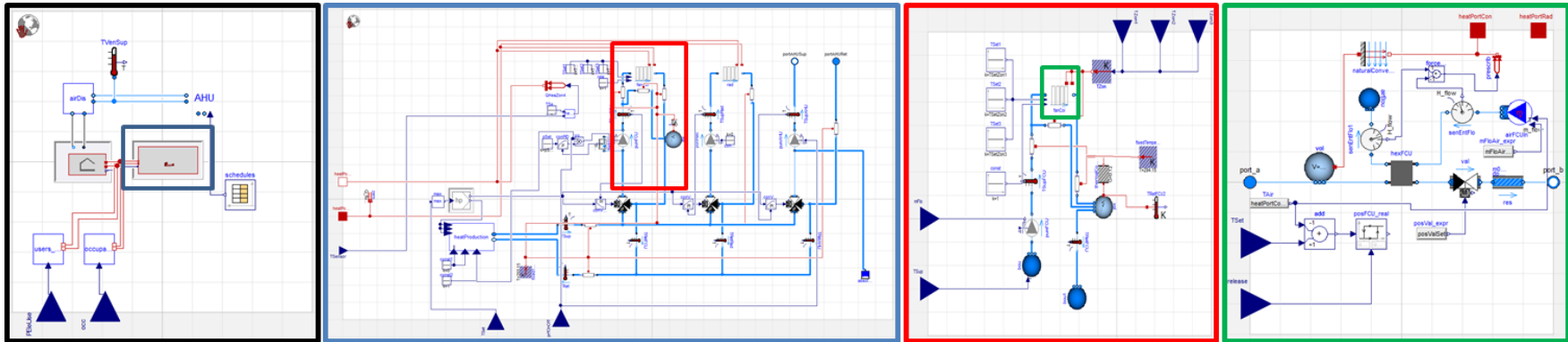


# 3 different cases, same inputs dataset

Ideal (simple-simple)

Non-ideal (simple-complex):

- All variables available
- Noise properties to 'measurements' are known
- Check output state with measured counterpart
- Investigate all states



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# 3 different cases, same inputs dataset

Ideal (simple-simple)

Non-ideal (simple-complex)

Real case (simple-real):

- Measurements have intrinsic error
- Investigate all states and compare to other cases



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# State estimation (example)

Model for state estimation (paper, Rao et al. 2003)

d2c (Ts=1s, matlab)

discrete model → continuous model

$$\dot{x} = \begin{bmatrix} .99 & .2 \\ -.1 & .03 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} x + v_k$$

$$\dot{x} = \begin{bmatrix} .01 & .34 \\ -.17 & -1.16 \end{bmatrix} x + \begin{bmatrix} .2 \\ 1.68 \end{bmatrix} w$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} x + v_k$$

$y =$  output to fit

$w =$  disturbance which is unknown (to the model)

$N(\mu=0, \sigma=1)$ , only positive values



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# No state estimation (no disturbance)



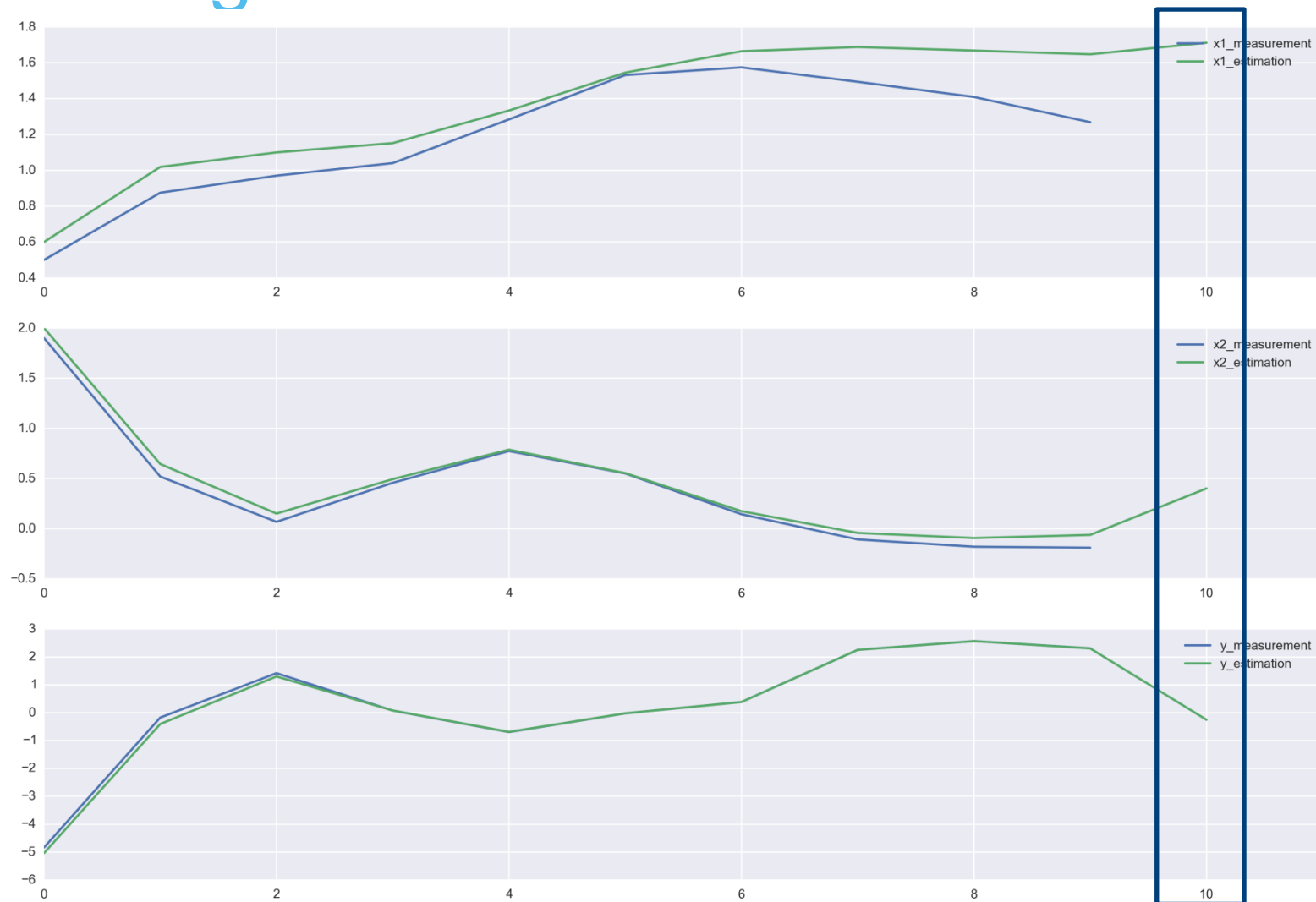
# State estimation

## Mitigate effect of unknown disturbance

- Kalman filter approaches (→3)
  - Prediction (guess)
  - Correction (statistical knowledge of the unknown disturbance)
  - (*Calculation*)
- Moving horizon approach (→1,2)
  - Find optimal values for variables and/or parameters which fit model output to measurements over past horizon
  - Allows constraint formulation
  - (*Optimization*)



# Moving horizon estimation



# Moving horizon estimation

Deterministic

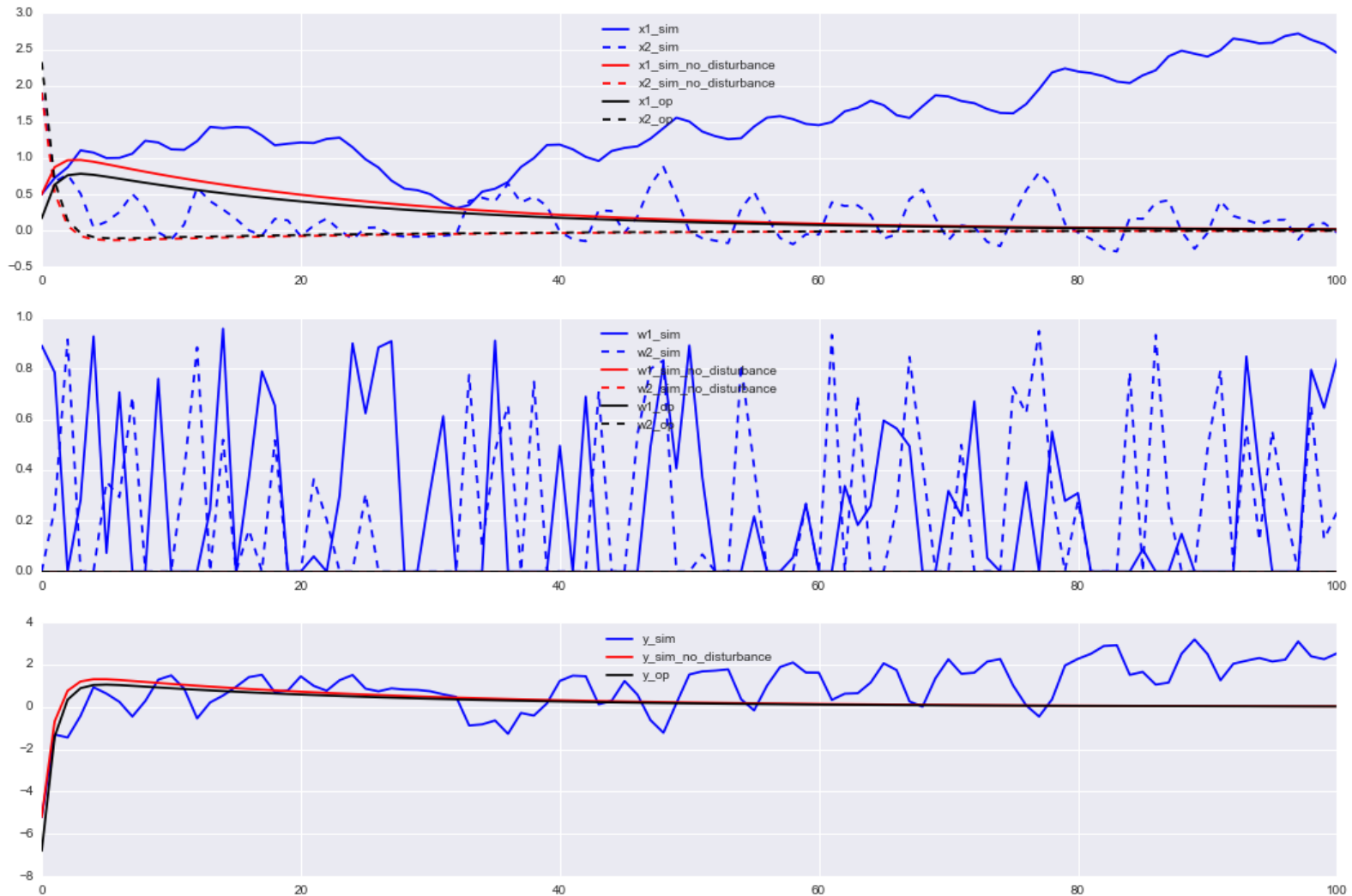
$$\min_{x_0} \sum (y - y_{meas})^T \mathbf{R}^{-1} (y - y_{meas})$$

- Initial values of the states to get best output over the past horizon
- Ok if disturbances have low influence
- Easiest implementation



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# Moving horizon estimation (deterministic)



# Moving horizon estimation

Statistical  $\min_{\{x_0\}\{w_k\}} \sum (y - y_{meas})^T \mathbf{R}^{-1} (y - y_{meas}) + (w_k)^T \mathbf{Q}^{-1} (w_k)$

- Fit the output 'y'
- Try to estimate minimal disturbances 'w'  
→ Add unknown state disturbance to the Modelica model

model sim

```
extends partial_sim();  
Modelica.Blocks.Interfaces.RealInput w1;  
Modelica.Blocks.Interfaces.RealInput w2;
```

equation

```
der(x1) = a11*x1 + a12*x2 + b1*w1;  
der(x2) = a21*x1 + a22*x2 + b2*w2;  
end sim;
```

$$\dot{x} = \begin{bmatrix} .01 & .34 \\ -.17 & -1.16 \end{bmatrix} x + \begin{bmatrix} .2 \\ 1.68 \end{bmatrix} w$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} x + v_k$$





# Moving horizon estimation (solutions)

$$\min_{\{x_0\}\{w_k\}} \sum (y - y_{meas})^T \mathbf{R}^{-1} (y - y_{meas}) + (w_k)^T \mathbf{Q}^{-1} (w_k)$$

Use of ExternalData class in JModelica.org

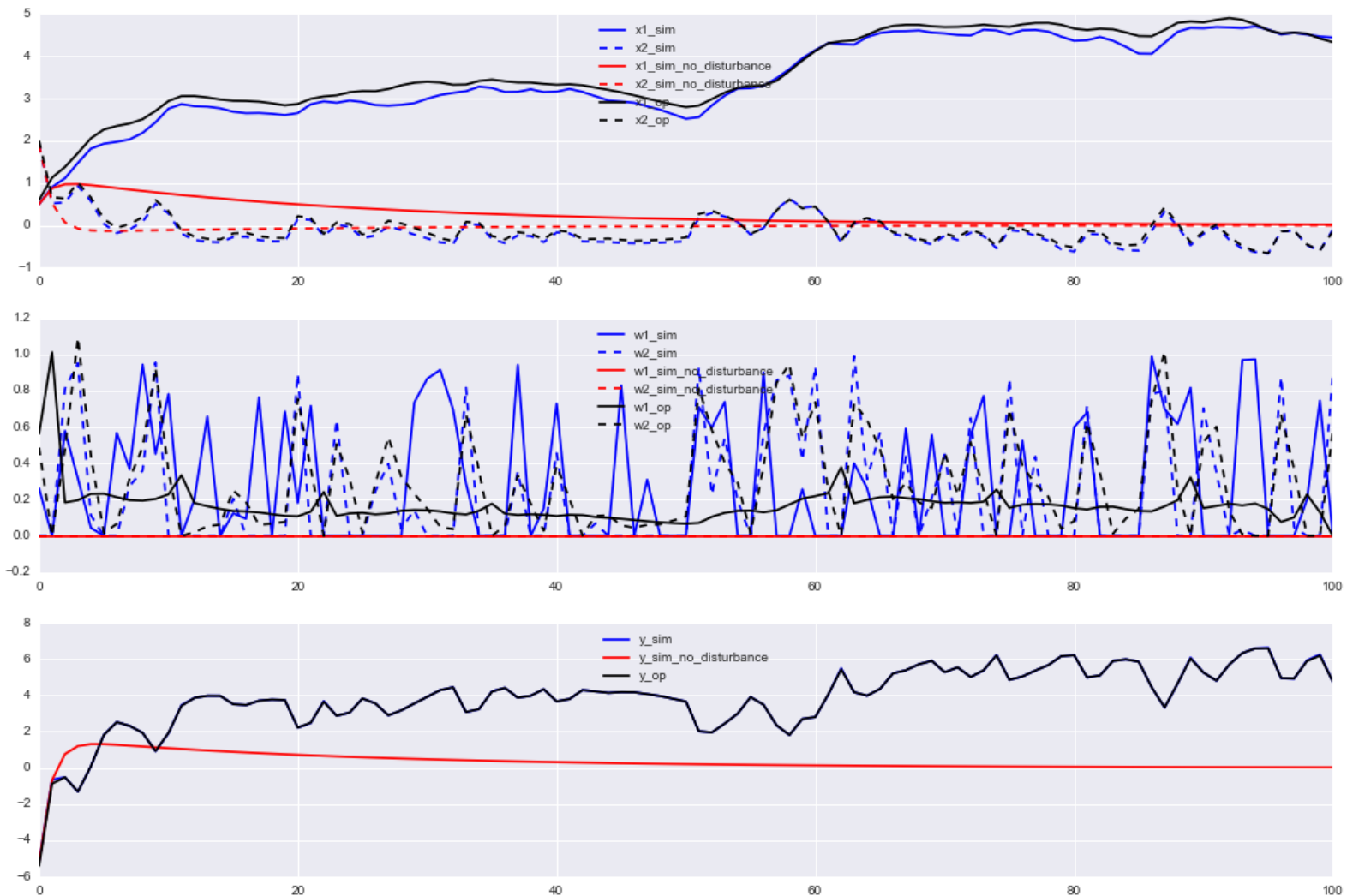
Different solutions for different weights.

- Large ratio  $R^{-1}$  over  $Q^{-1}$  : disturbances have smaller covariance
- Look at
  - $Q^{-1} = 1, R^{-1} = 1e2$  (best guess, paper)
  - $Q^{-1} = 1, R^{-1} = 1e4$  (high)
  - $Q^{-1} = 1, R^{-1} = 1e-2$  (low)

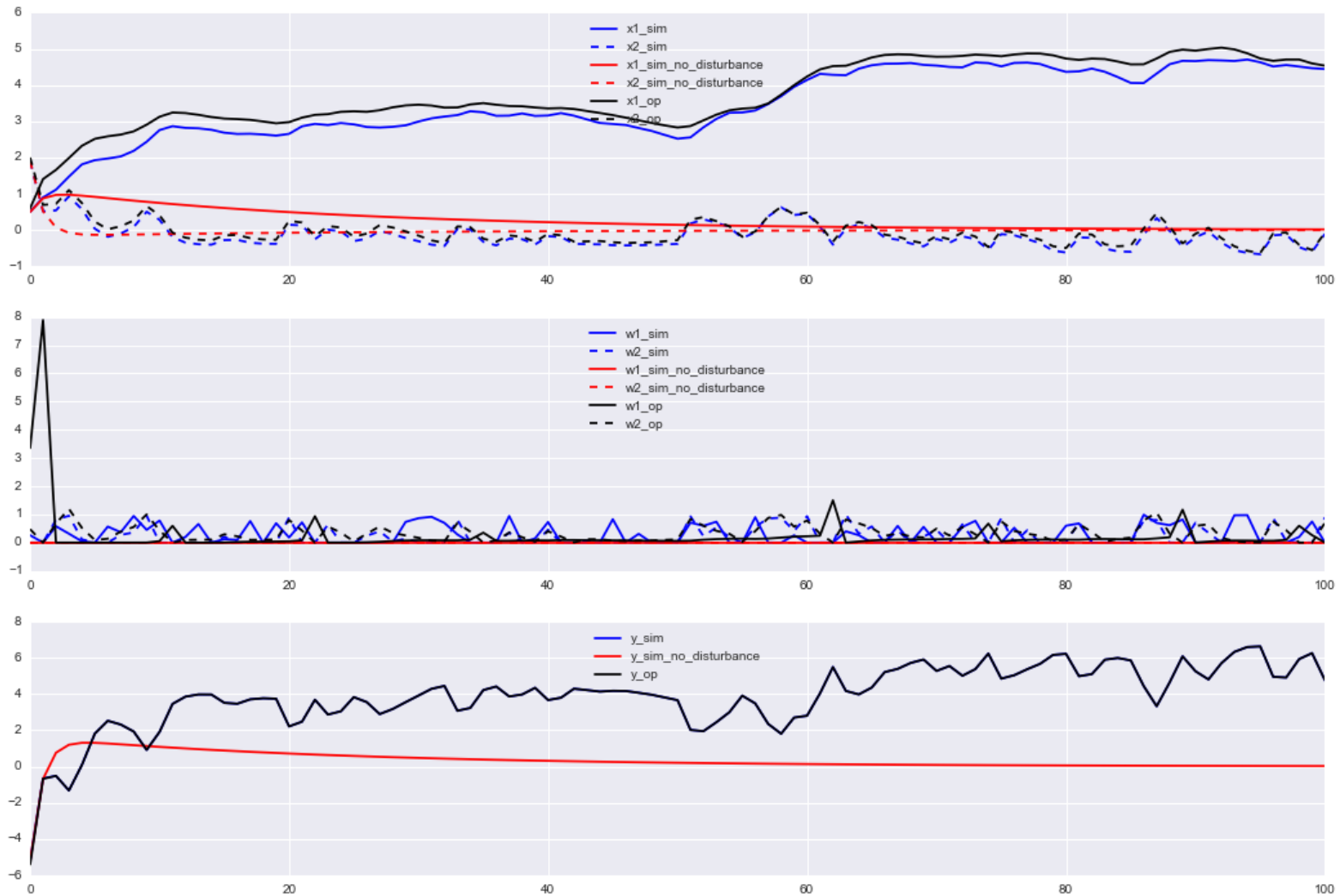


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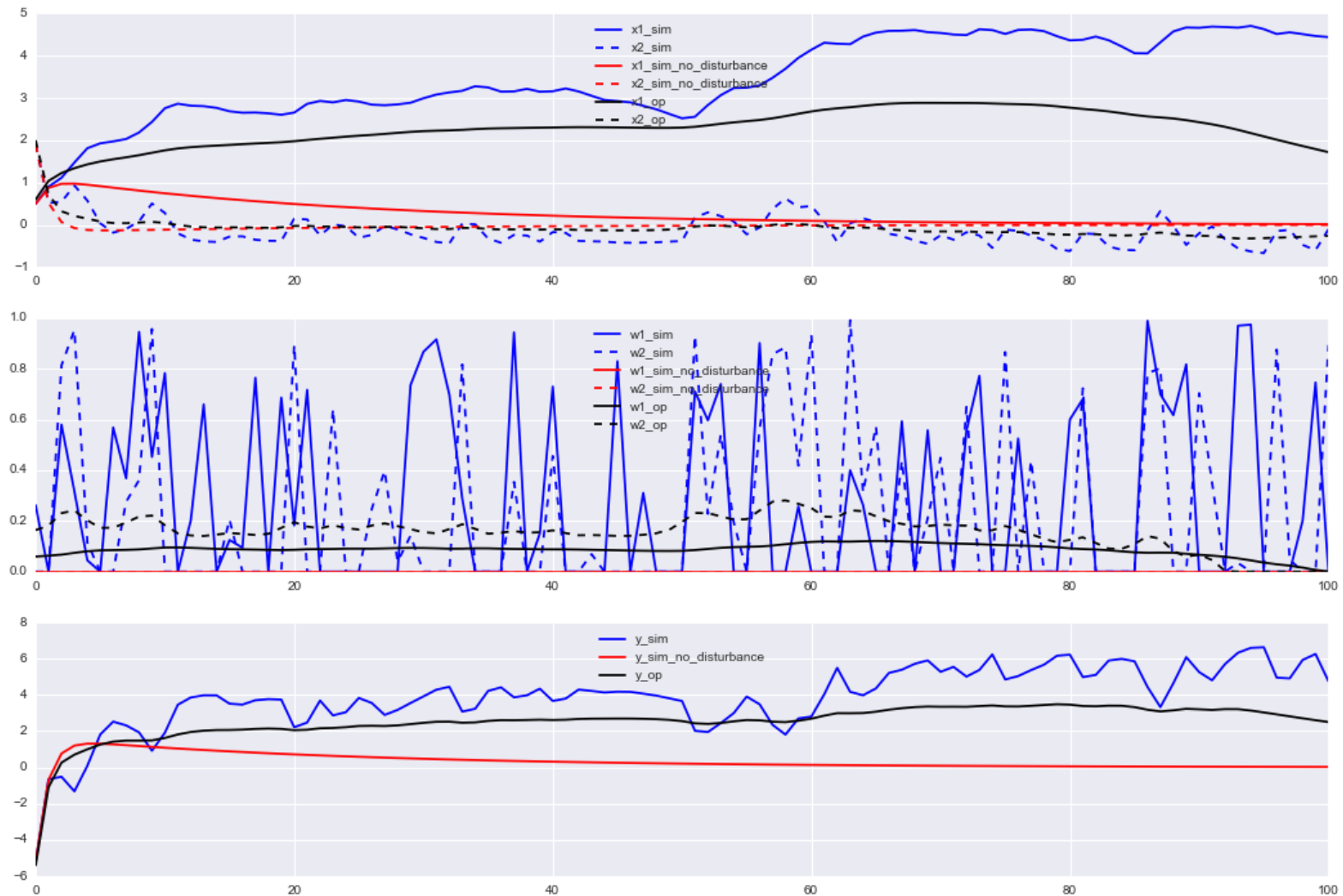
# $Q^{-1} = 1, R^{-1} = 1e2$ (best guess)



# $Q^{-1} = 1, R^{-1} = 1e4$ (high)



$O^{-1} = 1$ .  $R^{-1} = 1e-2$  (low)



# Evaluate

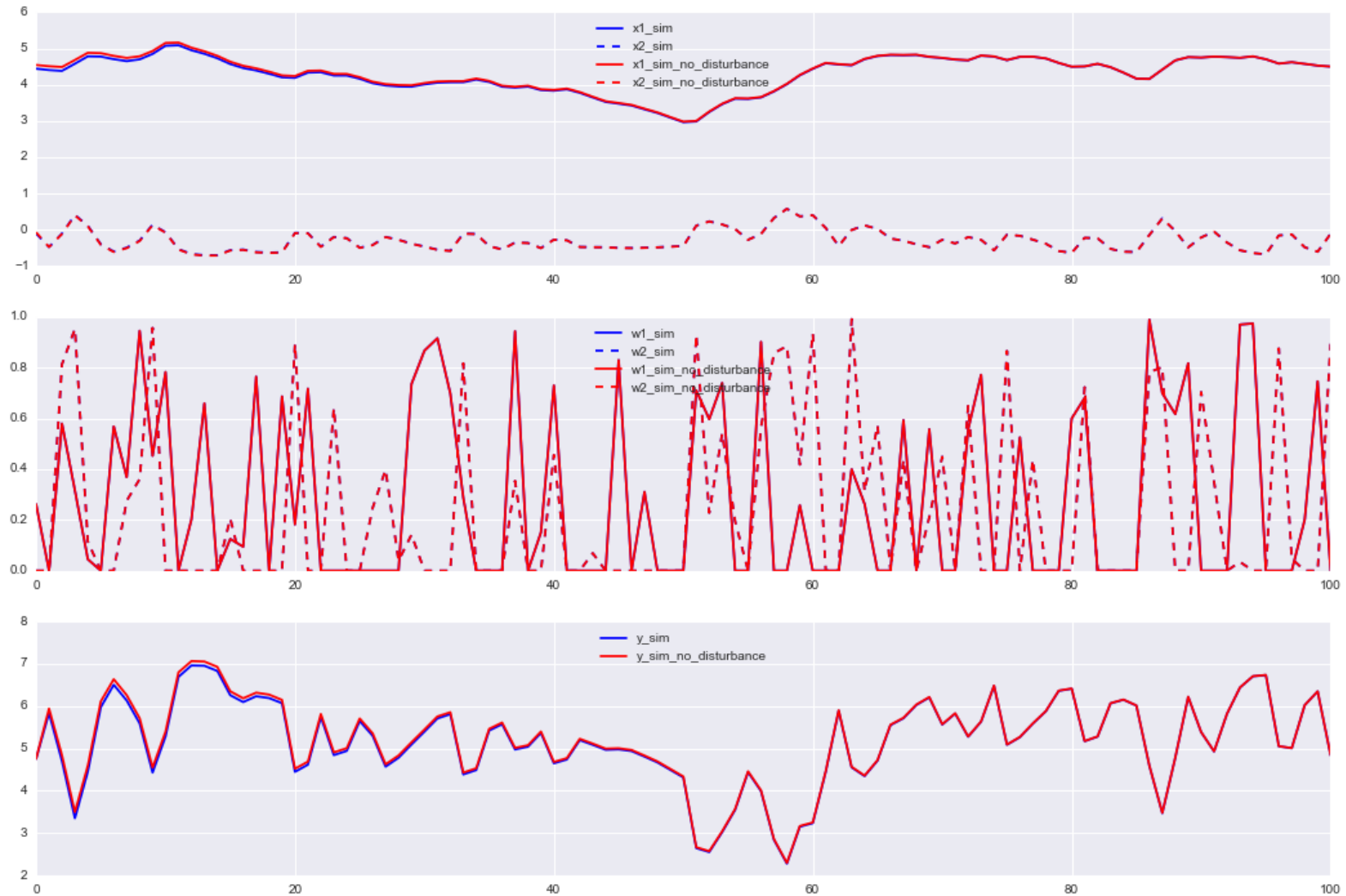
Open loop simulation on next period

Calculate rmse



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# Evaluate



# Evaluate

$Q^{-1} = 1, R^{-1} = 1^e2$  (normal, paper)

- rmse x1 : 0.044
- rmse x2 : 0.007
- rmse y : 0.060

$Q^{-1} = 1, R^{-1} = 1^e4$  (high)

- rmse x1 : 0.040
- rmse x2 : 0.007
- rmse y : 0.056

$Q^{-1} = 1, R^{-1} = 1^e-2$  (low)

- rmse x1 : 1.019
- rmse x2 : 0.143
- rmse y : 1.425



# Challenges

- Best way to implement?
  - Weights?
  - Variable  $w$  discrete or continuous?
- What about end effect?
  - Only output state matters!
  - Similar to mpc
- States in buildings are not equal

$$\dot{x} = \begin{bmatrix} .01 & .34 \\ -.17 & -1.16 \end{bmatrix} x + \begin{bmatrix} .2 \\ 1.68 \end{bmatrix} w$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} x + v_k$$





