Numerical Optimal Control, August 2014

Exercise 6: Indirect single-shooting

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Pontryagin's maximum principle

Let us return to the simple OCP from Exercise 5, but with the path constraints on the state removed (these are difficult to handle with indirect methods and as we saw from Exercise 5, they were not active at the solution anyway):

$$
\begin{array}{ll}
\underset{x,u}{\text{minimize}} & \displaystyle\int_0^T x_1(t)^2 + x_2(t)^2 + u(t)^2 \, dt \\
\text{subject to} & \dot{x}_1 = (1 - x_2^2)\, x_1 - x_2 + u, \qquad x_1(0) = 0 \\
& \dot{x}_2 = x_1, \qquad x_2(0) = 1 \\
& -1 \leq u(t) \leq 1,
\end{array}
\tag{1}
$$

where $T = 10$ as before.

Tasks:

6.1 Introduce the *costate* $\lambda(t)$ and write down the Hamiltonian $H(x, \lambda, u)$ of (1):

6.2 Use Pontryagin's maximum principle to derive an expression for the optimal control u^* as a function of x and λ. Note: $u(t)$ may only be a piecewise smooth function. Tip: How does u enter in the Hamiltonian?

6.3 Derive the *costate equations*, i.e. $\dot{\lambda}(t) = \dots$

6.4 Derive the terminal conditions for the costate equations:

6.5 Augment the original equations with the costate equation to form a two-point boundary-value problem (TPBVP) with four differential equations:

6.6 **Extra:** Solve the TPBVP with single-shooting. Use $[0, 0]$ as your initial guess for the adjoint equations. To integrate the system, we can use the CVODES integrator from the SUNDIALS suite. Note that the system is only piecewise smooth, which may cause problems in the integrators. We ignore this fact for simplicity of presentation. When done, compare with the solution script `vdp_indirect_single_shooting.py` available in the CasADi example pack or via the course website.