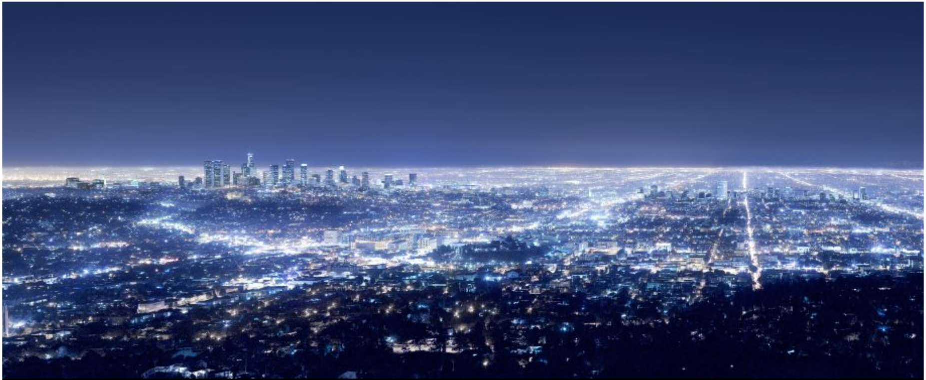

Proper Assessment of QP solvers for Model Predictive Control

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Benchmarking suite

Outline

- Motivation
- Content
  - solvers
  - benchmarks
  - MPC formulation
- Simulations
  - available options
  - preliminary plots
- Future goals

Benchmarking suite

Motivation

Many approaches for solving QP problems that arise in MPC applications

- more than one algorithms for each approach
- performance often illustrated on one or two academic examples

Practitioners often face the challenge to:

- find the best suited algorithm for a specific MPC application
- assess whether there is a single approach that satisfies performance requirements over a given problem class

Idea: Develop a benchmarking suite to conveniently compare the numerical performance of QP algorithms over a large variety of MPC problems

Benchmarking suite

Coupled solvers

Interfaced existing software and prototyped algorithms from literature

- Active set methods
  - qpOASES [1]
  - quadprog
- Interior point methods
  - FORCES [2]
- Gradient Based methods
  - Primal (Fast) Gradient Method [3]
  - Richter's Dual Fast Gradient Method [4]
  - Bemporad's Dual Fast Gradient Method [5]
  - Gisselson's generalized DFGM [6]
- Explicit MPC
  - MPT Toolbox [7]
- Other
  - ADMM [8]
  - Brand's algorithm [9]

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Collection of problems

Benchmark problems originate from:

1. academic examples presented in publications
2. industrial examples and case studies
3. randomly generated examples

Seeking variety with respect to:

- number of inputs, states and outputs
- type of constraints
- horizon length
- open loop stability

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General problem formulation

MPC problem formulation:

$$
\begin{aligned}
\min_{x,\,u}\ & \sum_{k=0}^{N-1} \begin{pmatrix} y_k - y_k^{\mathsf{r}} \\ u_k - u_k^{\mathsf{r}} \end{pmatrix}^T \begin{pmatrix} Q_k & S_k \\ S_k & R_k \end{pmatrix} \begin{pmatrix} y_k - y_k^{\mathsf{r}} \\ u_k - u_k^{\mathsf{r}} \end{pmatrix} + \begin{pmatrix} g_k^y \\ g_k^u \end{pmatrix}^T \begin{pmatrix} y_k - y_k^{\mathsf{r}} \\ u_k - u_k^{\mathsf{r}} \end{pmatrix} \\
& + (x_N - x_N^{\mathsf{r}})^T P \, (x_N - x_N^{\mathsf{r}}) \\
\mathrm{s.\,t.}\ & x_0 \text{ given}, \\
& x_{k+1} = A_k x_k + B_k u_k + f_k \quad \forall\, k \in \{0, \ldots, N-1\}, \\
& y_k = C_k x_k + D_k u_k + e_k \quad \forall\, k \in \{0, \ldots, N-1\}, \\
& y_k^{\mathsf{l}} \le y_k \le y_k^{\mathsf{u}} \quad \forall\, k \in \{0, \ldots, N-1\}, \\
& u_k^{\mathsf{l}} \le u_k \le u_k^{\mathsf{u}} \quad \forall\, k \in \{0, \ldots, N-1\}, \\
& d_k^{\mathsf{l}} \le M_k y_k + N_k u_k \le d_k^{\mathsf{u}} \quad \forall\, k \in \{0, \ldots, N-1\}, \\
& d_N^{\mathsf{l}} \le T x_N \le d_N^{\mathsf{u}} \,.
\end{aligned}
$$

- Kept general to allow easy coupling of new benchmarks
- Data stored in a structure
- Complemented by a control scenario (open or closed loop)
- Moving blocks to allow for different control horizon

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Simulation options

Fair comparison of different solvers is not a trivial procedure

Current simulation options:

1. Solvers can stop with their own termination condition
   - Applying feedback earlier can lead to better closed loop performance
   - Free resources for other processes or for low power consumption
   - Depends on tuning parameters. Difficult to compare solvers
2. Run fixed number of iterations
   - In many applications the amount of time to solve the optimization problem is fixed
   - Checking the termination condition is often more expensive than the iteration itself
   - Maximum iterations of each solver should be weighted based on its complexity
3. Stop using an a priori known optimal solution

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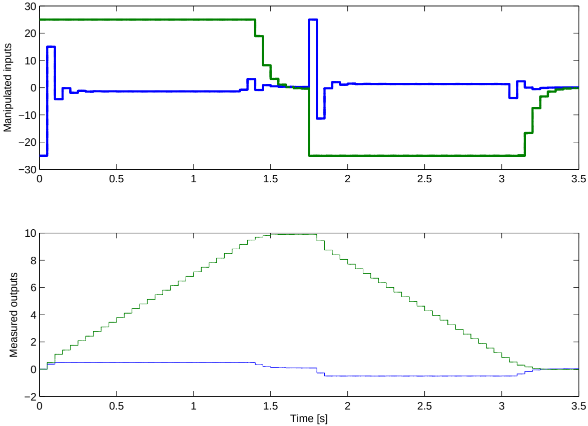
Preliminary results

Simulation of unstable aircraft model [10] in closed loop



Control inputs and measured outputs

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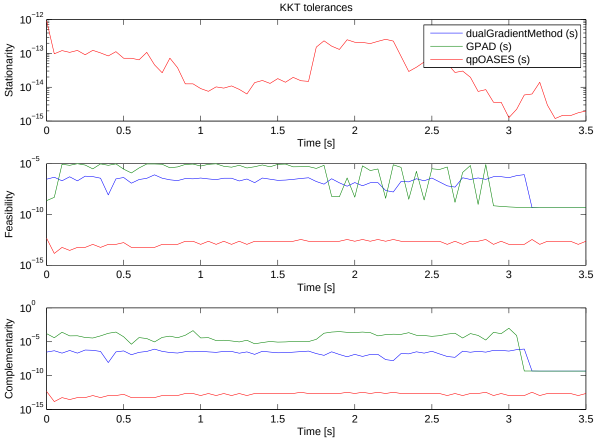
Preliminary results

Simulation of unstable aircraft model in closed loop



Optimality measures based on KKT conditions (log scale)

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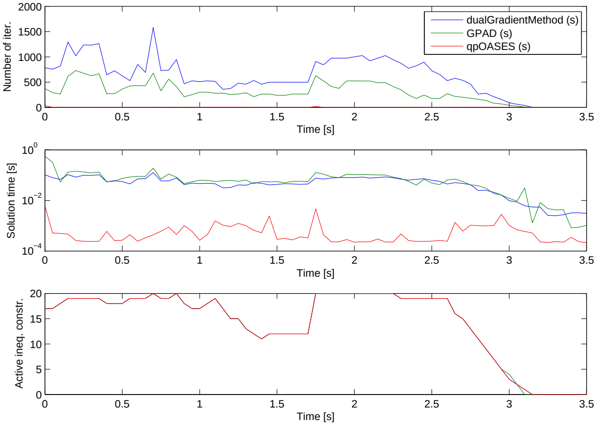
Preliminary results

Simulation of unstable aircraft model in closed loop



Number of iterations, cpu time and number of active constraints

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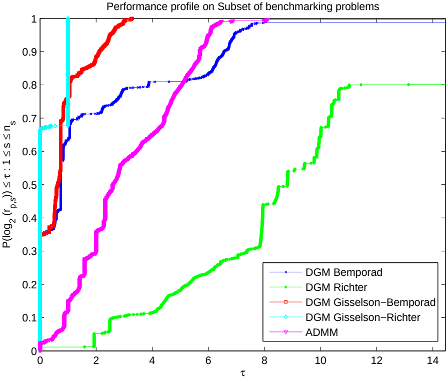
Preliminary results

Performance profile

- $\mathcal{S}$ set of solvers
- $\mathcal{P}$ set of problems
- $t_{p,s}$ time to solve problem p with solver s
- $r_{p,s} = \frac{t_{p,s}}{\min\limits_{\hat{s}\in\mathcal{S}} t_{p,\hat{s}}}$ performance ratio
- $P_s(\tau) = \frac{\text{size}\{p\in\mathcal{P}: r_{p,s}\leq\tau\}}{n_p}$ CDF of performance ratio



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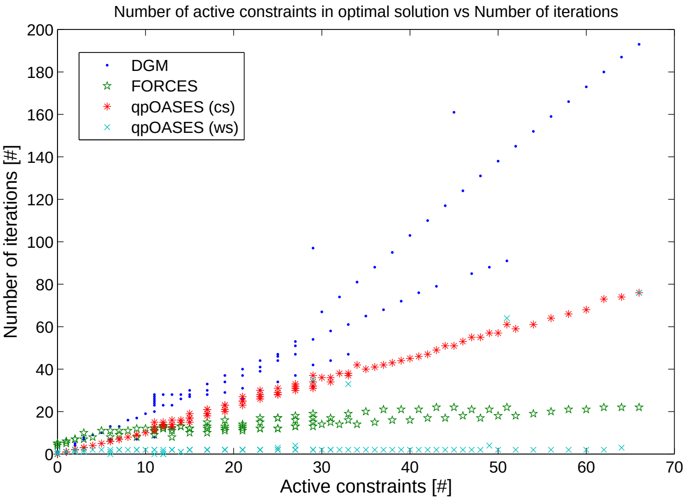
Preliminary results

Number of active constraints versus iterations on one example



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Future work

- Extend benchmanking suite
    - more benchmark problems
    - new QP solvers
- Replace prototype implementations with more efficient ones
- Run simulations on embedded hardware

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